2/1/07

Physics of Compact Objects (con't)

Pressure inside stars:

Know: n, T, ρ, etc. $\Rightarrow$ Calculate $P = P(n, T, \ldots)$

Equation of state

Possible combinations of conditions of gas:

classical ("non-degenerate")	quantum ("degenerate")
non-relativistic (NR)	ultra-relativistic (UR)
$\epsilon_{kin} \ll m_0 c^2$	$\epsilon_{kin} \gg m_0 c^2$

Case 1: classical + NR ("normal" → atmosphere of Earth)

- gas is NR $\Rightarrow P = \frac{2}{3} U_{kin}$

$$U_{kin} = \frac{N}{V} \langle \epsilon_{kin} \rangle = n \langle \epsilon_{kin} \rangle$$

$$= \frac{3}{2} n k_B T$$

Boltzmann constant

($\frac{1}{2} kT$ per degree of freedom per particle)

$$\Rightarrow P = \frac{2}{3} U_{kin} = n k_B T$$ ("ideal gas law")

Classical / Quantum Boundary

Q: When does quantum mechanics (QM) become important?

Define the average distance between particles:

$$n = \frac{N}{V} \simeq \frac{1}{\ell^3} \Rightarrow \ell \simeq n^{-1/3},$$

where ℓ is the average density between particles

Consider the QM wave associated with a particle:

$$\lambda_{deB} = \frac{h}{p},$$ where h is Planck's constant.

QM important when $\ell \lesssim \lambda_{deB}$.

$$\Rightarrow n^{-1/3} \lesssim \frac{h}{p} \Rightarrow p \lesssim h n^{1/3}$$

$\Rightarrow$ Slow-moving particles, which are dense, can become quantum gases.

Now, slow-moving ⇒ cold, as

$$\frac{3}{2} k_B T = \frac{p^2}{2m_0}$$

$$\Longrightarrow k_B T \approx \frac{p^2}{m_0} \lesssim \frac{h^2 n^{2/3}}{m_0}$$

So, the classical / quantum boundary exists along

$$T \propto n^{2/3} \text{, or}$$

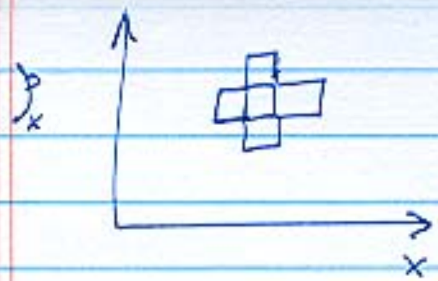
$$T = \text{constant} \times n^{2/3}.$$

<u>Case 2: Quantum gas</u>

Consider phase-space (6-dimensional)

(x, y, z, p_x, p_y, p_z)

Look at a slice of phase space:



quantum cells

$$\underbrace{dx\,dy\,dz}_{dV}\;dp_x\,dp_y\,dp_z = h^3$$

From Heisenberg's
Uncertainty Principle: $dx\,dp_x \gtrsim h$