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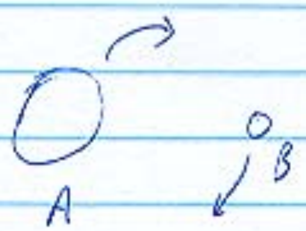
White Dwarfs

Discovery → from observation (in 19th century)

- Sirius - a double star system



- Sirius B : strange properties

$$\left.\begin{matrix} M \sim 1\, M_\odot \\ R \sim \frac{1}{100} R_\odot \end{matrix}\right\} \Rightarrow \langle \rho \rangle \simeq 10^9\ {}^{kg}/_{m^3}$$

$$\left(\text{Sun has } \langle \rho \rangle \simeq 10^3 \frac{kg}{m^3}\right)$$

Typical parameters for White dwarfs
→ small (dwarf), hot (white)

Estimate strength of GR effects:

- important concept in General Relativity (GR)

"Schwarzschild Radius"

$$R_s = \frac{2GM}{c^2} \qquad \left(R_{s,\odot} \simeq 3\text{ km} \right)$$

Consider the escape velocity from the surface of a star:

$$E = \frac{1}{2}mv^2 - \frac{GMm}{R}$$

$$E \to 0, \quad r \to \infty$$

$$\Rightarrow \quad v_{esc}^2 = \frac{2GM}{R}$$

For $v_{esc} = c$,

$$R_s = \frac{2GM}{c^2}.$$

Real meaning: circumference of BH

$$C_{BH} = 2\pi R_s$$

Strength of GR effects:

(i) Sun $\quad \frac{R_{s,\odot}}{R_\odot} \sim 10^{-6}$

White Dwarf: $\frac{R_s}{R} \sim 10^{-4}$, so GR is not so important here

Neutron star: $\frac{R_s}{R} \sim 0.5$, so GR will be important

<u>Basic Properties of White Dwarfs:</u>

For hydrostatic equilibrium,

$$\langle P \rangle = -\frac{1}{3} \frac{E_{pot}}{V} \approx \frac{-1}{3} \frac{GM^2}{R^4}$$

For WD that are not too massive, pressure is mainly due to

NR <u>electron</u> degeneracy pressure

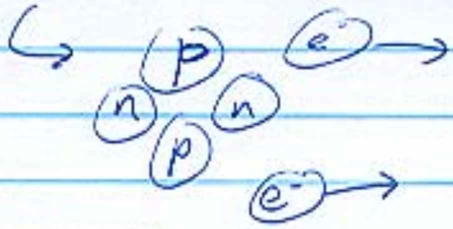
$$\langle P \rangle = K_{NR} \langle n_e \rangle^{5/3}$$

Electrons provide the degeneracy pressure, due to their large de Broglie wavelength, $\lambda = \frac{h}{p}$.

Relate n_e to mass density:

Mass of a white dwarf mainly provided by $^{4}_{2}He$, $^{12}_{6}C$, $^{16}_{8}O$

e.g. He has 2 nucleons per 1 free electron



$$\boxed{\rho = 2 m_H n_e}$$

Thus, $$\frac{K_{NR} \langle \rho \rangle^{5/3}}{(2 m_H)^{5/3}} = \frac{1}{3} \frac{G M^2}{R^4}$$

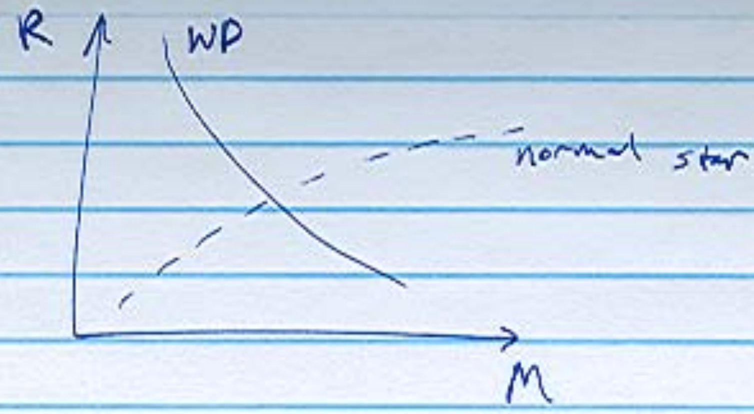
Now, this shows that $\langle \rho \rangle^{5/3} \propto \frac{M^2}{R^4}$.

Using also $\langle \rho \rangle \sim \frac{M}{R^3}$, this gives

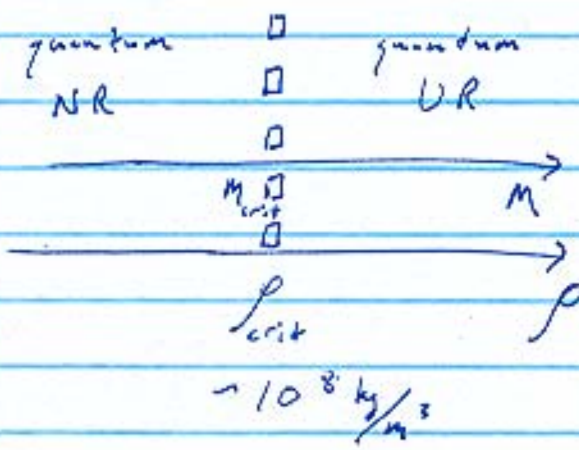
$\boxed{\langle \rho \rangle \propto M^2}$ or $\boxed{R \propto M^{-1/3}}$.

→ The "Mass radius relation for NR WD"

$$R = 0.01\, R_\odot \left(\frac{M}{M_\odot} \right)^{-1/3}$$



NR - UR boundary:



As $M \to M_{crit}$, $\langle P \rangle \to \langle P_{UR} \rangle$:

$$\langle P_{UR} \rangle = \frac{K_{UR}}{(2m_H)^{4/3}} \langle \rho \rangle^{4/3}, \quad K_{UR} = \frac{hc}{4}\left(\frac{3}{8\pi}\right)^{1/3}$$

$$\simeq \frac{hc}{8}$$

$\langle P_{UR} \rangle > \frac{1}{3} \frac{GM^2}{R^4}$, to counterbalance gravity.

- Use $\langle \rho \rangle = \frac{M}{\frac{4\pi}{3} R^3} \sim \frac{M}{R^3}$

$$\Longrightarrow \frac{K_{UR}}{(2m_H)^{4/3}} \frac{M^{4/3}}{\cancel{R^4}} \gtrsim \frac{1}{3} \frac{GM^2}{\cancel{R^4}} \quad \left(\text{radius cancels!}\right)$$

$$\Longrightarrow \underbrace{M \lesssim \left(\frac{3}{8}\right)^{3/2} \left(\frac{hc}{G}\right)^{3/2} \frac{1}{(2m_H)^2}}_{M_{CH}}$$

"The Chandrasekhar limit"

$$M_{ch} = \frac{1}{4}\left(\frac{3}{8}\right)^{3/2}\left(\frac{hc}{G}\right)^{3/2}\frac{1}{m_H^2} \simeq 1.7\, M_\odot$$

→ More precisely, $M_{ch} \sim 1.4\, M_\odot$

No WD can exist with a mass $M > M_{ch}$.