for light emitted from $r = R_s$:

$\nu_\infty = 0$, whatever the emitted frequency ν_{em}!

But: photon with zero frequency (or infinite $\lambda = \frac{c}{\nu}$, or $E = h\nu = 0$)

ceases to exist (becomes invisible)

$\Rightarrow$ Schwarzschild BH $\quad R \le R_s$

$2\pi R_s$ ~ "critical circumference"

- Also: clocks at $r = R_s$, as seen from far-away observer (A)

  completely freeze ("infinite time dilation")

  $\rightarrow d\tau = 0$

Structure of Critical Surface (or Circumference) at $r = R_s$

- Notice: there are 2 pathological regions ("singularities") in

  Schwarzschild metric

(i) $r = 0$ $\rightarrow$ physical singularity ("real")

$\rightarrow$ where GR breaks down $\rightarrow$ need quantum gravity

(ii) $r = R_s$ $\rightarrow$ coordinate singularity (not real)

$\rightarrow$ can be transformed away

by choosing suitable coordinates

- To study geometry near $r = R_s$, introduce new time coordinate

$$\boxed{\bar{t} = t + \frac{R_s}{c} \ln\left|\frac{r}{R_s} - 1\right|}$$

$$d\bar{t} = \frac{\partial \bar{t}}{\partial t} dt + \frac{\partial \bar{t}}{\partial r} dr$$

$$\frac{\partial \bar{t}}{\partial t} = 1$$

$$\frac{\partial \bar{t}}{\partial r} = \frac{R_s}{c} \frac{1/R_s}{\frac{r}{R_s} - 1} = \frac{1}{c} \frac{R_s}{r} \frac{1}{1 - \frac{R_s}{r}}$$

- eliminate old time coordinate: $\quad dt = d\bar{t} - \frac{\partial \bar{t}}{\partial r} dr$

$$\Rightarrow \quad \boxed{ds^2 = -c^2\left(1 - \frac{R_s}{r}\right) d\bar{t}^2 + 2c \frac{R_s}{r} d\bar{t}\, dr + \left(1 + \frac{R_s}{r}\right) dr^2 + r^2 d\Omega^2}$$

- Note: non-singular at $r = R_s$!

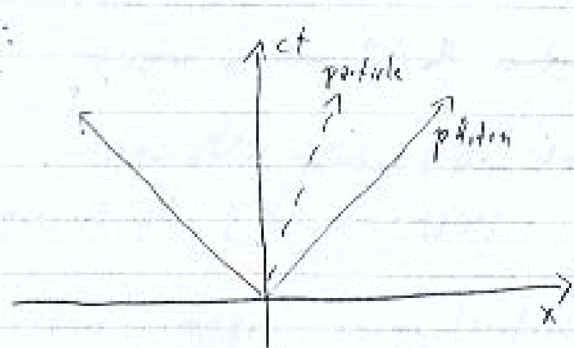
- Now consider radial light-rays:

  - radial $\Rightarrow$ $d\theta = d\varphi = 0$ $\quad (d\Omega^2 = 0)$

  - light-rays: travel on null geodesics $\quad ds^2 = 0$

- <u>light-cones:</u>

e.g. in SR



$$ds^2 = -c^2 dt^2 + dx^2 = 0$$

$$\Rightarrow \frac{dx}{dt} = \pm c$$

- Note: particles with non-zero rest mass must travel inside of lightcone $\quad \frac{dx}{dt} = v < c$

- light-cones in Schwarzschild geometry:

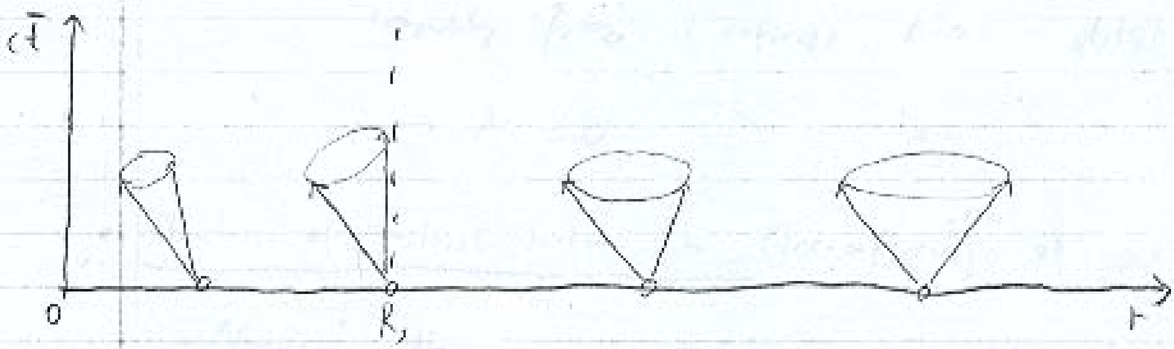
$$\left(1+\frac{R_s}{r}\right)\left(\frac{dr}{d\bar{t}}\right)^2 + 2c\,\frac{R_s}{r}\left(\frac{dr}{d\bar{t}}\right) - c^2\left(1-\frac{R_s}{r}\right) = 0$$

→ quadratic equation for $\frac{dr}{d\bar{t}}$

→ 2 solutions:

$$\left(\frac{dr}{d\bar{t}}\right)_1 = -c$$

$$\left(\frac{dr}{d\bar{t}}\right)_2 = c\,\frac{1-\frac{R_s}{r}}{1+\frac{R_s}{r}}$$



- Note: at $r = R_s$ → no photon can escape to outside ("event horizon")

for $r < R_s$: all radiation is drawn to $r=0$ singularity!

→ same is true for all material particles!

→ no static solution ($dr = d\varphi = d\theta = 0$) possible

- Conjecture of Cosmic Censorship (Penrose 1969)

"Thou shalt not have naked singularities!"

→ every (real) singularity is surrounded by an event horizon