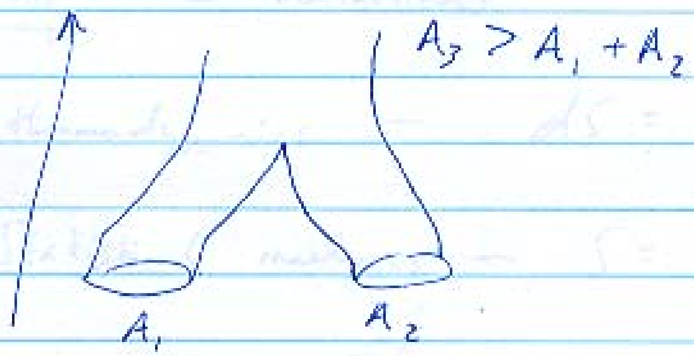
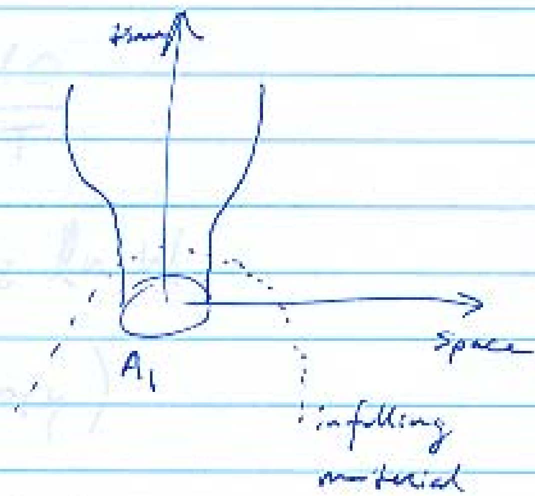
4/26/07

Hawking Radiation

Q: How do BH's evolve?

- Hawking's big idea:



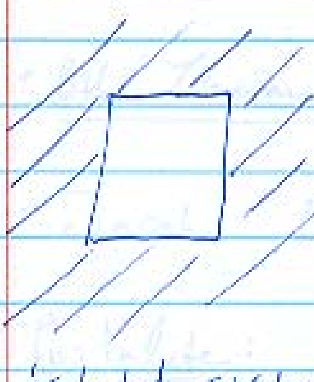


• **Area-increase theorem**

- Event horizon always increases (or stays constant)

$$\frac{dA_{BH}}{dt} \geq 0$$

• **Brief review of entropy**



- 2nd law of thermodynamics

$$\boxed{\frac{dS}{dt} \geq 0}$$

- **Notice**: Curious similarity between entropy and BH horizon area!

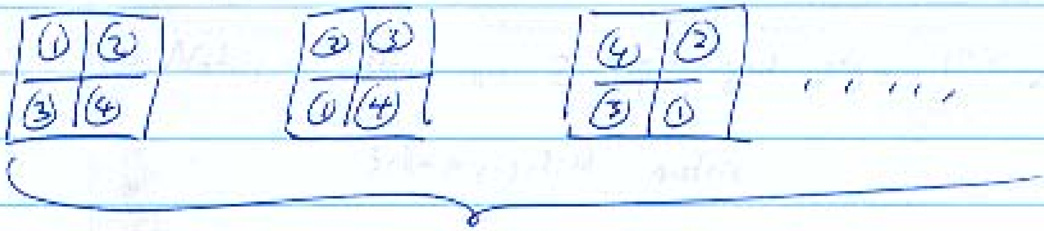
Entropy: 2 definitions

(1) thermodynamics — $dS = \frac{dQ}{T}$

(2) Statistical mechanics — $S = k_B \ln W$

(Boltzmann's entropy)

$W \equiv$ number of microstates that give rise to same macrostate



N microstates

• **BH Thermodynamics**

- Jacob Bekenstein (1972)

Postulate: $S_{BH} = k_B \frac{1}{4} \frac{A_{BH}}{A_{Pl}}$

$\Rightarrow$ "Horizon area is entropy"

$$A_{BH} = 4\pi R_s^2$$

- "Planck area" — $A_{Pl} \simeq l_{Pl}^2 = \frac{G\hbar}{c^3} = 10^{-70}\,m^2$

$$l_{Pl} \simeq 10^{-35}\,m$$

$$\Rightarrow \quad \boxed{S_{BH} = k_B \frac{4\pi G}{\hbar c} M^2}$$

<u>Problem</u>: If BH's have entropy, they should also have non-zero temperature!

<u>Use</u>: $T_{BH}\, dS_{BH} = dQ_{BH} = d(Mc^2)$

$$= c^2\, dM$$

and

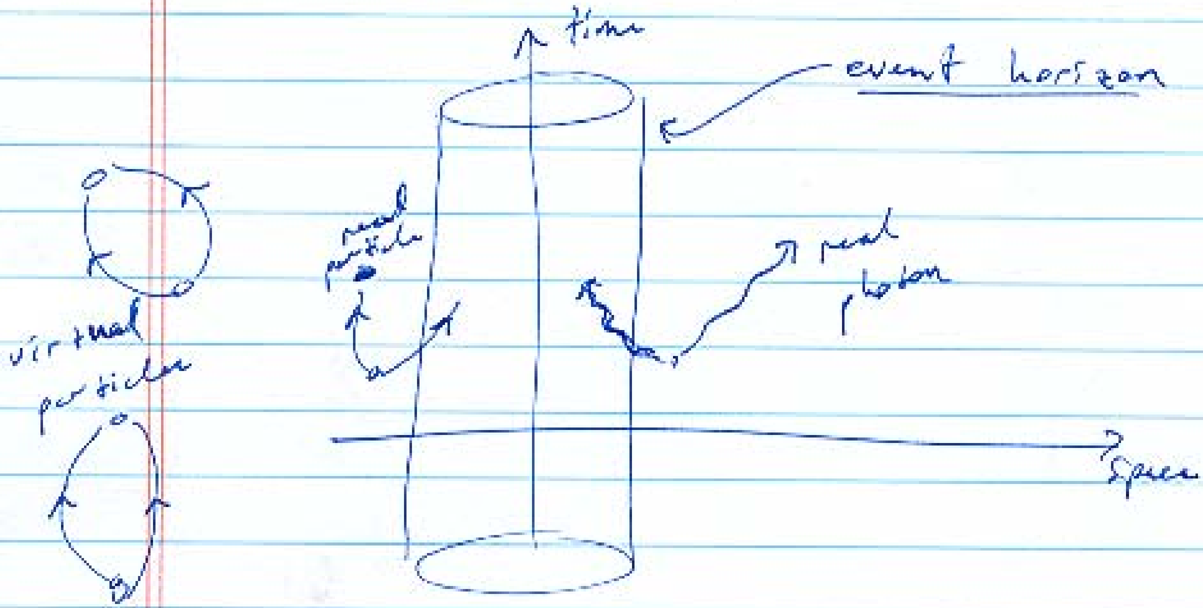
$$dS_{BH} = k_B \frac{8\pi G}{\hbar c} M\, dM$$

$$\Rightarrow \quad T_{BH} \overset{?}{=} \frac{\hbar c^2}{8\pi G k_B} M^{-1} \simeq 10^{-7}\,K \left(\frac{M}{M_\odot}\right)^{-1}$$

Big problem: Any object with $T \neq 0$ emits radiation, but (classical) BH's were supposed to not emit anything!

• Hawking Effect (QM + GR)

→ Considering pair creation near horizon:



- Energy conservation - real particle (outside horizon) $\epsilon_1 = \epsilon > 0$
  ⇒ Particle inside horizon must have negative energy $\epsilon_2 = -\epsilon < 0$

- Create photons and non-zero rest mass particles

- For photons: black-body radiation (thermal equilibrium)

→ Photon density $n_\gamma = \underbrace{\frac{\pi^2}{15} \frac{k_B^3}{\hbar^3 c^3}}_{\text{constant}} T^3$

- estimate mean distance between photons:

$$l_{\gamma\gamma} = n_{\gamma\gamma}^{-1/3} = \frac{\hbar c}{k_B T}$$

Equate to size of BH:

$$\lambda = R_s = \frac{GM}{c^2} \Rightarrow T = \frac{\hbar c^3}{G k_B} M^{-1} = T_{BH}$$

- Estimate "BH luminosity"

$$L_{BH} = 4\pi R_s^2 \sigma_{SB} T_{BH}^4 \quad \text{(black-body law)}$$

$$L_{BH} = 10^{-27}\, W \left(\frac{M}{M_\odot}\right)^{-2}$$

- BH evaporation / explosion

→ Estimate how long it takes for BH to evaporate:

→ 'Hawking time'

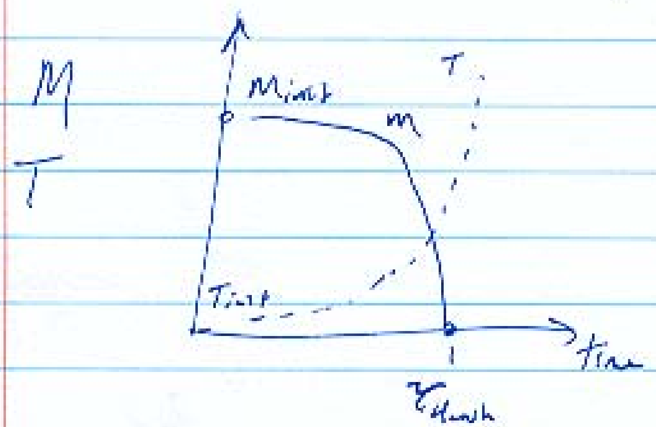
$$\frac{d(Mc^2)}{dt} = -L_{BH} = -10^{-27}\, W \left(\frac{M}{M_\odot}\right)^{-2}$$

→ Solve by separation of variables:

$$\tau_{Hawk} \simeq 10^{66}\ yr. \left(\frac{M(t=0)}{M_\odot}\right)^3$$

<u>Notice</u>: 'Normal (solar mass) BH would take forever to evaporate!

- But eventually this <u>will</u> happen



- Ask: What BH mass needed for evaporation to occur within age of Universe (Hubble time) ≃ 13.7 Gy

→ Find $M \sim 10^{11}$ kg

→ 'Mini-BHs' → maybe created in early universe