4/24/07

First Stars: Fundamental Properties

- **Basic Parameters:**

  - Mass (predicted by theory), $M \sim 100\, M_\odot$

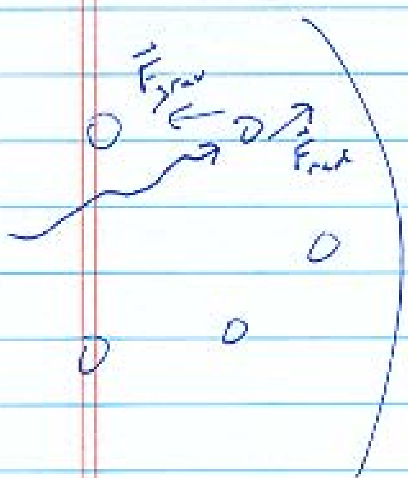
  - Mass - radius relation, $R \simeq 5 R_\odot \left(\frac{M}{100 M_\odot}\right)^{1/2}$

$$\rightarrow \langle \rho \rangle = \frac{M}{\frac{4\pi}{3} R^3} \sim 1{,}000\ \mathrm{kg\ m^{-3}} \sim \langle \rho \rangle_\odot$$

  - To estimate T, need to know something of the pressure

- **Radiation Pressure**

  - In massive stars ($\gtrsim 50\, M_\odot$), radiation pressure (P_{rad}) is important



  - photons (γ) : UR particles $\Rightarrow P_{rad} = \frac{1}{3} U_{rad}$

  - need U_{rad} (radiation energy density)

$$\rightarrow U_{rad} = \langle n_\gamma \epsilon_\gamma \rangle$$

Assume: radiation is in thermal equilibrium

→ Roughly, $\epsilon_\gamma \simeq k_B T = \frac{hc}{\lambda}$

$$n_\gamma \simeq \frac{1}{\lambda^3}$$

$$\Rightarrow n_\gamma \simeq \frac{1}{\lambda^3} \simeq \left(\frac{k_B}{hc}\right)^3 T^3$$

$$U_{rad} \simeq \frac{k_B^4}{h^3 c^3} T^4 = \frac{8\pi^5}{15} \frac{k_B^4}{(hc)^3} T^4 \equiv a_{rad} T^4$$

↑ precise calculation

where $a_{rad} = 7.57 \times 10^{-16}\ \mathrm{J\,m^{-3}\,K^{-4}}$ (radiation constant)

$$\Rightarrow P_{rad} = \frac{1}{3} a_{rad} T^4$$

- Hydrostatic Equilibrium for Massive Stars

$$\frac{dP_{rad}}{dr} = -\rho \frac{Gm}{r^2}$$

Using $\frac{dP_{rad}}{dr} \simeq \frac{P_{rad}}{R}$, roughly,

$$\frac{a_{rad} T_I^4}{R} \simeq \langle \rho \rangle \frac{GM}{R^2}$$

$$\Rightarrow T_I \simeq \left(\frac{\langle \rho \rangle}{a_{rad}} \frac{GM}{R} \right)^{1/4} \sim 10^8 K$$

↑ 'internal temperature', on average

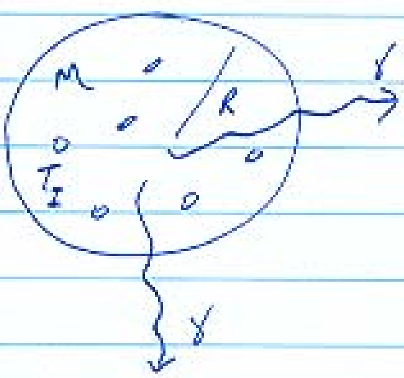
<u>Note</u>: $T_I \propto \frac{M^{1/2}}{R}$

Q: How is this related to temperature at surface? (or 'effective' temperature)

A: Need to consider <u>transport of photons</u>!

• <u>Radiative Diffusion</u>

- (incorrectly) assume that photons were able to freely escape from the stellar interior



$$\Rightarrow \tau_{direct} \simeq \frac{R}{c}$$

<u>Energy output</u>

$$L = \frac{\Delta E}{\Delta t} \simeq \frac{U_{rad} R^3}{\tau_{direct}}$$

$$\Rightarrow L \simeq \frac{c\, a_{rad}\, T_I^4 R^3}{R}$$

Notice : $a_{rad} = \frac{4\sigma_{SB}}{c}$ $\left(\sigma_{SB} = 5.67 \times 10^{-8}\ W m^{-2} K^{-4}\right)$

Q : Why is this incorrect ?

A : Because photons are 'trapped' inside star!

M


$\tau_{diff} \equiv$ "diffusion time"

$\leftarrow$ Photons escape by slow diffusion

$$L = \frac{\Delta E}{\Delta t} = \frac{\Delta E}{\tau_{diff}} = 4\pi R^2 \sigma_{SB} T_{eff}^4$$

$$\Rightarrow T_{eff} \simeq \left(\frac{\tau_{direct}}{\tau_{diff}}\right)^{1/4} T_I$$

Random walk: mean-square displacement

$$D^2 = \langle(\vec{l}_1 + \vec{l}_2 + \dots + \vec{l}_N)^2\rangle$$

$$= \langle l_1^2\rangle + \langle l_2^2\rangle + \dots + \langle l_N^2\rangle + 2\left(\langle \vec{l}_1\cdot\vec{l}_2\rangle^{0} + \dots + \langle \vec{l}_N\cdot\vec{l}_N\rangle^{0}\right)$$

$\Rightarrow D^2 = N l^2$, $l \equiv$ mean free path

$\Rightarrow \boxed{D \simeq \sqrt{N}\, l}$

- In stellar interiors, $l_\gamma = 1$ cm (Stars are very opaque!)

$$\Rightarrow t_{diff} \simeq \frac{N l}{c} = \frac{R^2}{l c}$$

$$\Rightarrow T_{eff} = \left(\frac{l}{R}\right)^{1/4} T_I \simeq 10^{-3}\, T_I \simeq 10^5\, K$$

↑ use $R \simeq \sqrt{N}\, l$

- Luminosity

$$L = 4\pi R^2 \sigma_{SB} T^4$$

- Notice: $L \propto R^2 \frac{M^2}{R^4} \propto \frac{M^2}{R^2} \propto M$

(use $R \propto M^{1/2}$)

$$\Rightarrow L = 10^{33}\, W \left(\frac{M}{100 M_\odot}\right) \simeq 10^6 L_\odot \left(\frac{M}{M_\odot}\right)$$

→ Close to theoretical upper limit

radiation pressure $\leq$ gravity

→ Eddington luminosity

$$L_{Edd} = 1.3 \times 10^{31}\ \mathrm{W} \left(\frac{M}{M_\odot}\right)$$

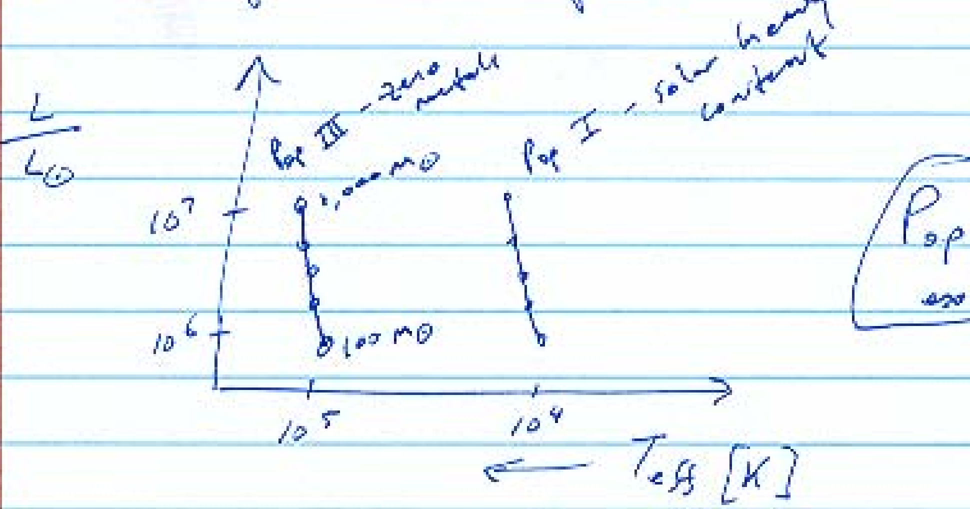
For Pop III stars, $L_{Pop\,III} \sim L_{Edd}$.

- **Stellar Lifetime**

$$t_* = \frac{0.007\ Mc^2}{L_{Edd}} \simeq 3\ \mathrm{Myr.} \quad (3 \times 10^6\ \mathrm{yr.})$$

For the sun, $t_{*,\odot} = 10^{10}$ yr.

- **Hertzsprung-Russel diagram**



Pop III stars are extremely blue