Black Holes in Theory

Into the Abyss

1. Why Black Holes?

Black holes have become a cultural icon. Although few people understand the physical and mathematical innards of the black holes that Einstein's equations reveal, nearly everyone understands the symbolism of black holes as yawning maws that swallow everything and let nothing out. Can there be any compelling reason to understand more deeply a trivialized cultural metaphor? The answer, for anyone interested in the nature of the world around us, is an emphatic yes! Black holes represent far more than a simple metaphor for loss and despair. Although black holes may form from stars, they are not stars. They are objects of pure space and time that have transcended their stellar birthright. The first glimmers of the possibility of black holes arose in the eighteenth century. Two hundred years later, they are still on the forefront of science. In the domain of astronomy, there is virtual certainty that astronomers have detected black holes, that they are a reality in our Universe. In the domain of physics, black holes are on the vanguard of intellectual thought. They play a unique and central role in the quest to develop a "theory of everything," a deeper comprehension of the essence of space and time, an understanding of the origin and fate of our Universe.

There is a certain inevitability to black holes in a gravitating Universe. Einstein's theory says that for sufficiently compressed matter, gravity will overwhelm all other forces. The reason lies in the fundamental equation, $E= mc^2$. Because mass and energy are interchangeable, one of the implications of this equation is that energy has weight. The very energy that is expended to provide the pressure to support a star against gravity increases the pull of the gravitational field. The more you resist gravity, the more you add to its strength. The result is that if an object is compressed enough, gravity becomes overwhelming. Any force that tries to resist just makes the pull all the greater. When gravity exceeds all other forces, the object will collapse to form a black hole.

The first people to contemplate the notion that gravity could become an overwhelming influence were John Mitchell, a British physicist, and the Marquis de LaPlace, a French mathematician. Mitchell in 1783 and LaPlace in 1796 based their arguments on Newton's theory of gravity. They used the concept of an *escape velocity*. The notion is that to escape from the surface of a gravitating object, a sufficiently large velocity must be imposed to overcome the pull of gravity and "escape" into space. If the velocity is too small, the launch will fail. If it is just right, a launched vehicle will just coast to a halt as it gets far away from the gravitating object. With more velocity, a launched vehicle will still have a head of steam as it breaks free of gravity and it will continue to speed away. That is the whole idea behind tying two big, solid-fueled boosters and an external liquid fuel tank to the space shuttle when it goes up from Cape Canaveral. The shuttle must achieve escape velocity, or near it, to get into orbit, and that means lifting off the launch pad really fast!

Mitchell and LaPlace used this idea of an escape velocity to argue that an object could be so compact that the escape velocity from the surface would exceed the speed of light. By some coincidence, an algebraic formulation of this escape velocity condition in the context of Newton's theory of gravity gives the correct result for the "size" of a black hole using the correct theory of gravity, general relativity. Mitchell did not, apparently, coin a zippy shorthand name for his intellectual creation. LaPlace called his hypothetical compressed entities *corps obscurs*, or hidden bodies. (The modern French equivalent is *astres occlus*, or closed stars. The literal translation, *trous noirs*, has also gained acceptance after some initial resistance because of its suggestion of double entendre.)

With some hindsight, we can see that Newton's theory of gravity was flawed. This theory predicted that, if two masses got infinitesimally close together, the force would go to infinity. A general lesson of physics is that, when infinities arise, there is a problem with the mathematical formulation that reflects some omission in the physics. Another problem with Newton's law of gravity is that, although it prescribed how the strength of gravity scaled with the mass of a gravitating object (to the first power) and the distance between objects (inversely as the square of the distance), it did not say how gravity varied in time. Consider two orbiting stars. A literal use of Newton's law of gravity says that, as one star moves, the other instantaneously responds to the fact that the motion has occurred. Thus according to Newton's law of gravity, the effect of gravity propagates infinitely fast. This second troublesome infinity violates the idea that nothing can move faster than the speed of light. Finally, and perhaps most compelling from a strictly practical point of view, Newton's gravity did not work.

Newton's law of gravity is spectacularly successful in most normal circumstances, when distances are large and speeds are slow. Astronomers still use it to great effect to predict the orbits of most stars. Rocket scientists use it to plot the paths of spacecraft even as they do complex orbits that carry them around planets, getting a boost from the interaction. The *Galileo* spacecraft went through a remarkable series of bank shots around the inner planets, picking up speed in the various encounters with Venus and Earth, before being flung to Jupiter. The recently launched *Cassini* spacecraft completed the first stage of its voyage to Saturn by first looping inward to circle Venus. *Cassini* received a kick from the orbital motion of Venus that gave it the momentum to sail out to Saturn. The success of gravitational multiple-bank shots shows that Newton's gravity works very well in this regime.

For very fine measurements, however, Newton gives the wrong answer! The predictions of Newton do not agree with observation, with the way nature works. Classic examples are the rate of rotation of the perihelion of Mercury and the deflection of light by the Sun. In contrast, Einstein's theory of gravity has passed every test of observation. A modern example is the use of global positioning systems (GPS) in boating, camping, and driving as well as military and industrial uses. This system works by timing the signals from an array of orbiting satellites. It is based on the mathematics of the curved space and warped time of Einstein, not the simple law of gravitation of Newton. If the silicon chips in the GPS detectors knew only about Newton, boaters in the fog and soldiers in the field would get lost!

As we shall see, giving up Newton for Einstein does not represent merely swapping one set of mathematics for another. Rather, Einstein brought with him a revolution in the fundamental concept underlying gravity. Newton crafted his mathematics in the language of a force of gravity as the underlying concept. Physicists and astronomers still use the notion of a gravitational force in casual terms, even though it has become outmoded in a fundamental way. Einstein's view was radically different. For Einstein, there is no force of gravity. Instead, Einstein's theory represents gravity as a manifestation of curved space. A gravitating object curves the space around it. A second object then responds by moving as straight as it can in that curved space. The curved space results in deflections of motion that are manifested as gravity even though the object is in free fall, sensing no force whatsoever. Much of this chapter will be devoted to exploring this conception of gravity.

The progress of our understanding of gravity is not over, however. We have come to understand that, even though it has passed every experimental test, Einstein's theory has flaws. It has its own nasty infinities that represent some omission in the physics. Ironically the hints of a new, better theory are again cast in the language of force, but not the force of Newton. In notions being developed today, the force is quantum in nature and may play on a field of ten or eleven dimensions, not the three of space and one of time that sufficed for both Newton and Einstein. We will begin with an exploration of black holes as portrayed in Einstein's theory and see how deeper issues arise. Some of those issues will be explored in Chapter 12.

2. The Event Horizon

As described by general relativity, a black hole is a region of space-time bordered by a one-way membrane called an *event horizon*, as shown schematically in Figure 9.1. Matter or light can pass inward through the event horizon, but nothing that travels at or less than the speed of light, even light itself, can get back out. The term "event horizon" comes from the notion that if an "event," like a firecracker exploding, occurs just outside the event horizon, the light can reach an observer, and the fact that the event occurred can be registered. If the firecracker goes off just inside the event horizon, however, no information that the event occurred can reach the observer. The event takes place beyond a horizon so that it cannot be seen. Once inside the event horizon, escape is impossible without traveling faster than the velocity of light. The location of the event horizon is thus intimately related to the fact that the speed of light is a speed limit for all normal stuff. The simple argument of Mitchell and LaPlace concerning the formation of a *corps obscurs* relates to the size of the event horizon. The size of the event horizon scales with the mass of the black hole. For a black hole with ten times the mass of the Sun, it would have a radius of 30 kilometers, about 20 miles in radius. The nature of the event horizon in the context of curved space and time will be explored in more depth in Section 5.

3. Singularity

When Newton was pondering the means by which apples bonked him on the head and, more particularly, how the Earth kept the Moon trapped in orbit, he intuited an important aspect of gravity. He realized that the gravity of the Earth must act from the center of the Earth, not, for instance, from its surface. This was not a trivial conclusion, and he needed to prove that it was true. Newton knocked off his gravity studies for a while and invented the mathematics of calculus in order to prove his conjecture. With his new mathematical tools, Newton was able to prove that, although the mass of the Earth is distributed throughout its volume, each little piece of the Earth acts in concert as if it were in the center. The result is that for any object beyond the Earth's surface, the gravitational attraction of the Earth will act as if all the mass of the Earth were concentrated at a point in the center. This is true for any spherical gravitating body. The gravitational attraction depends only on the distance from the center of the body, not on the radius or volume of that body. Armed with this mathematically proven conclusion, Newton went on to formulate his theory of gravity with a mathematical expression that said that the force of gravity between two spherical objects depended only on the masses of the two objects and on the inverse square of the distance between their centers.

As an example to make this property concrete, imagine that the Sun were suddenly compacted to become a neutron star of the same mass. It would get cold and dark on the Earth, but the Earth would continue in exactly the same orbit because the gravitational pull it feels from the Sun depends only on the mass of the Sun, not on how big it is. Another implication is that we are in no danger of falling into a black hole. All the black holes we know or suspect are far away. The gravity would be frightful if we were to get near their centers, but at a large distance from their centers, the gravity gets weak as it does at a large distance from any object and vanishingly small if the distance is very large. In this context, there is one interesting difference between normal stars of any kind – suns, white dwarfs, or neutron stars – and black holes. The former act as if all their mass were concentrated at a point in the center. For black holes, this is literally true.

Inside the event horizon, all mass that falls into a black hole is trapped. Even though there is no material surface at the event horizon, the matter within the black hole still signifies its presence by exerting a gravitational pull. The gravitational acceleration exists outside the event horizon and causes the formation of the event horizon. Although the black hole still exerts a gravitational pull, the matter itself is crushed out of all recognizable existence. General relativity predicts that the matter compacts into a region of zero volume and infinite density at the center of the black hole. Even more profound, space and time cease to exist at this point. Such a region is called a *singularity* and is illustrated schematically as a point in Figure 9.1. For a black hole, all the mass that creates the gravity is literally at this point in the center, at the singularity.

The infinities associated with the singularity are clues that Einstein's theory is not a complete theory of gravity, despite its great success. We know in principle what is lacking. Einstein's theory does not contain any aspects of the quantum theory. The uncertainty principle of the quantum theory tells us that it is not possible to specify the position of anything exactly, including the position of an infinitely small singularity. The notion of a singularity as it arises in Einstein's theory is thus an intrinsic violation of the quantum theory. With a theory of gravity that properly incorporated quantum effects, which general relativity does not, the singularity would probably be altered to be a region of exceedingly small volume and immense, but not infinite, density. It is the nature of that exceedingly small volume, the singularity that forms inside a black hole, the singularity from which our Universe was born, that is the heart of the quest for a new, deeper understanding of physics.

4. Being a Treatise on the General Nature of Death Within a Black Hole

The manner in which a black hole crushes matter out of existence, save for its gravitational field, is rather graphic. Consider something falling into a black hole, say a human body – feet first. In this case, at every instant the feet are going to be closer to the center of the black hole than is the head. Gravity is thus going to be stronger at the feet and will pull the feet away from the head. The natural forces on an extended body tend to stretch it along the direction toward the center of the gravitation. At the same time, all parts of the body are trying to fall toward the center. The left shoulder is trying to fall toward the center. The right shoulder is trying to fall toward the center. As the body gets closer to the center, the distance between separate paths directed at the center gets ever smaller. The shoulders get shoved together, and whatever is in between must suffer the consequences. A body falling into a black hole will be stretched feet from head and crushed side to side. This is known jocularly as the "noodle effect." Anything falling into a black hole will be noodleized, as shown in Figure 9.2.

The technical name for this simultaneous radial stretching and lateral crushing is the *tidal force*. It is precisely the same effect as causes the tides on the Earth. Here the Moon pulls on the Earth and its oceans, pulling them toward the Moon and pushing them in sideways to form the tidal bulges in the oceans, the faintest form of noodle. As a body falls into a black hole, the tidal forces increase drastically. First the body stretches into a noodle and breaks apart. Then the individual cells stretch into noodles and are destroyed. Next gravity overcomes the electrical forces that bind matter into molecules and atoms. Atoms will be wrenched out of molecules and electrons pulled from atoms. As infall proceeds, the rising tidal forces will overcome the nuclear force, stretching out the atomic nuclei and breaking them apart into individual protons and neutrons. In their turn, the protons and neutrons will break up into quarks, and the quarks into whatever comprises them. These building blocks will in turn be subject to supernoodlization until the singularity is reached and matter as we know it ceases to exist. Another way of characterizing the singularity in Einstein's theory is that the tidal forces become infinite. Physicists are gaining the first hints of what conditions may be in the singularity that will prevent that infinity. A discussion of this topic is postponed to Chapter 12.

5. Black Holes in Space and Time

5.1. Curved Space and Black Holes

Black holes are in the most fundamental way a beast of curved space. Visualizing this curvature that occupies all of three dimensions is very difficult for creatures such as us who are limited to a three-dimensional perspective. Even the experts have difficulty picturing the immense complexity of curved space. They have invented tricks to help with the perception. We will describe these tricks because they help, but even they represent only a shadow, and a fairly complicated one, of the truth.

The notion of curved space raises a general question. How do we characterize it? A line inscribed in a wavy two-dimensional space may be straight in some sense from our three-dimensional perspective, but not truly straight at all. Likewise, a properly "straight" line in a curved two-dimensional space may look strangely curved from another perspective. The ability to define and construct straight lines in curved space is fundamental to understanding how curved space works.

What do we mean by a straight line in curved space? There is a rigorous way to decide which lines are straight in a given space, a way that is intuitively reasonable as well. To obtain a straight line in a curved space, start with a small portion of the space where it is, for all practical purposes, flat. Think of any measurement you would normally make on the surface of the Earth, ignoring the fact that the Earth is really a closed spherical surface. In this small, nearly flat portion, use two short straight sticks. Lie one stick down. Now extend the second stick so that it partially overlaps the first, so that you know it is pointed in the same direction as the first, but so that it also extends out a way. Now hold down the second stick and slide the first along, keeping it parallel to the second stick until it extends out a way. Continue in this manner, extending each stick in turn a little way in such a manner that you are always assured that each extension goes in precisely the same direction as the last. As you proceed, draw a line using each stick in turn as a straight edge. Never look off at a distance to orient yourself. This technique depends on the fact that you are looking only at the local little patch of very nearly flat space in which you find yourself at any given instant. This method of drawing a straight line is called *parallel propagation* because each step consists of extending one of the sticks parallel to the other. One can prove mathematically that the line you draw as a result of this tedious operation is the shortest distance between any two points along it. What more could you want from a truly straight line? The operation of parallel propagation is what you approximate every time you sketch a freehand straight line. You do not make two marks on a paper and then try to make the distance between them as short as possible. Rather you start your pencil off in some direction and then, trying to keep your hand steady, continue the line parallel to itself. That is what makes parallel propagation so intuitive. It is what you really do to sketch a straight line.

In a flat space, parallel propagation will give the ordinary straight lines known and loved by tenth-grade geometry teachers. Parallel lines constructed in this fashion will never cross. Triangles made of three such lines will have 180 degrees as the sum of their interior angles. This is the geometry of Euclid, the geometry of flat space. In an arbitrarily curved space, watch out! Viewed from above, lines drawn as straight as possible by the method of parallel propagation will appear wackily curved if the surface is curved; but parallel propagated lines are as straight as possible and will be the shortest distance between two points even if the space is curved.

A particular trick the mathematicians have developed for picturing curved space is to project a three-dimensional curved space onto two dimensions in a special way, like casting a shadow. One dimension is suppressed, and the resulting two-dimensional figure is displayed as a two-dimensional surface in three-dimensional space. It becomes something we can look over, around, and under from our three-dimensional perspective and get a feel for the real thing. The technical name for the image that results from projecting the two-dimensional representation into ordinary flat, three-dimensional space is called an *embedding diagram*, because the two-dimensional "shadow" is embedded in the three-dimensional space.

To perform this trick for a black hole, one of the dimensions of rotation is suppressed. The resulting figure looks like a cone, or as if you were to poke your finger into a rubber sheet, as shown in Figure 9.3. The distant, still flat, parts of the sheet are the simple two-dimensional projection of flat, uncurved, three-dimensional space. The cone made with your finger is a technically proper representation of the curved space around a black hole (at least in qualitative shape, the mathematics of Einstein's theory tells the precise shape of the cone).

Full appreciation of the manner in which this cone represents the curved space of a black hole takes some time and quiet contemplation. One feature of the cone is immediately apparent and quite important. Consider the construction of a circle on the surface around the cone. This operation must be done in the confines of the two-dimensional surface. To go off this surface into three dimensions is cheating because that would be like going from the real three dimensions of a black hole into an unphysical honest-to-gosh fourth spatial dimension. To draw a circle, start at the center of the "black hole," at the bottom of the depression of the cone. Draw a line out along the curved surface directly away from the center. This line is a radius line, despite the fact that, from our three-dimensional view of the operation, it follows the funny curved surface of the cone. Now stop at some point along the surface of the cone and draw a circle, a line connecting all those points that are equally distant from the center.

Now imagine that you measure the length of the radius line and the circumference of the corresponding circle. Do you see that the radius in this curved surface must always be longer than normal? The ratio of the circumference to 2&pgr; times the radius is always less than one. The process of constructing the cone preserves this aspect of the original curved space, and the resulting embedding diagram lets it be seen graphically. In this curved space, the distance inward as represented by the radius is somehow stretched and lengthened. If you were to go off to a flat portion of the rubber sheet and do the same operation, start at a point, go out a certain distance along a radius, make a circle, you would get the standard result – the circumference is 2&pgr; times the radius. That is the test for flat space.

Let us apply the technique of parallel propagation to the curved space around a black hole as portrayed by the projected two-dimensional cone, as illustrated in Figure 9.4. Figure 9.4 shows two scientists drawing lines by parallel propagation in the two-dimensional space they occupy. Both start at some distance out in the “flat” portion. One draws a parallel-propagated line that passes far from the black hole. This line looks straight to an imaginary three-dimensional hyperspace observer, the perspective we take whenever we look down from our three-dimensional hyperspace onto a two-dimensional embedding diagram. The other scientist draws a parallel-propagated straight line that skirts the deepest portion of the cone (we do not want anyone crushed by the infinite tidal forces!). As this line nears the lowest portion of the cone, think what happens. A small portion of the space surrounding this point is oriented differently than a small portion of the space out in the flat, away from the cone. The line drawn in this location is going around the axis of the cone, responding to the “aroundness” of the surface, despite the fact that it is going as straight as it can in the curved space of the cone. From this part of space, the line must head off in a direction different from the direction along which it originally aimed in flat space. As this line continues, it will eventually emerge into flat space once more, but in a different direction from the original line segment that started in flat space. This line is also a straight line in the two-dimensional curved space. From the superior three-dimensional position of the hyperspace observer the line looks curved. It is bent toward the center of the cone where the curvature is severe.

Looking from the point of view of the hyperspace observer is useful for perspective, but we must bear in mind that our reality is closer to that of the two-dimensional scientists. We must draw lines, do geometry, and figure out the curvature of space around gravitating objects as three-dimensional people in a three-dimensional space. We do not have the luxury of stepping out into some four-dimensional hyperspace and looking back to see how our space curves. We can determine that two initially parallel light rays passing by a star will diverge, just as the two scientists drawing the parallel-propagated lines in Figure 9.4 will determine a real divergence of initially parallel lines. The two-dimensional scientists cannot see the conical space around the gravitating object, as it is revealed to the hyperspace observer, but they can deduce its nature by doing careful geometry. They can, for instance, deduce that the radius of a circle in that part of space is long compared to its circumference.

We can explore the nature of space around a gravitating object a bit more. Think of an equilateral triangle composed of three straight lines surrounding the deepest point of the cone in Figure 9.4. Each line will look like an arc bowed outwards to a three-dimensional hyperspace observer. All observers will agree that the lines will not meet at 60-degree angles, and the sum of the interior angles will be greater than 180 degrees. How about parallel lines? Two lines drawn parallel initially will curve differently as they pass near the cone, and the one closer to the center will be bent more severely. The lines will not be parallel in the flat space to which they emerge. Lines drawn by parallel propagation will be the shortest distance between two points. A line that does not dip down in the cone must travel farther to reach a given point on the far side. Likewise, a line that goes too deeply within the cone will have wasted some motion and will have farther to climb out. There is a shortest distance between any two points, and the line that is shortest is straight, but there may be more than one straight line between two given points. Think of a line that misses the bottom of the cone narrowly to the left. It will be bent to the right. A line that misses the bottom to the right will be bent to the left. These two lines will cross. From the point of beginning to the point of intersection, there will be two straight lines.

All this is rather abstract, but it applies to Einstein's theory of gravity in general, not just in the vicinity of black holes. Think of the straight line that just encircles the neck of the cone and closes on itself, as shown in Figure 9.5. A straight line cannot do that in flat space, but the cone shows that it is not just possible but demanded of certain straight lines in the curved space. That closed curved straight line in curved space is an orbit! In Einstein's theory, orbits are not caused by the action of a gravitational force as they are in Newton's theory. For Einstein, the gravitating body causes a curvature in space – of which our cone is a representation – and orbiting bodies are moving with no force as straight as they can in that curved space. The Moon is moving as straight as it can in the curved space around the Earth, and the Earth is moving as straight as it can in the curved space around the Sun. For such problems as planetary orbits, both Newton's theory and Einstein's give virtually the same numerical results, despite the vastly different concepts on which they are based. That Einstein's theory explains everything that Newton did in the regime of weak gravity is one of the powers of the theory. In addition, Einstein's theory predicts the nature of black holes that Newton's is powerless to describe.

Now, perhaps, you are prepared for the mind-bending exercise of attempting to picture the nature of curved space in its three-dimensional glory, with our toy two-dimensional cone as a guide. Figure 9.6 is an attempt to help do that. Draw a radial line out along the cone in the two-dimensional representation. At intervals, draw circles of constant radius, each with its own stretched-out radius. That will characterize the two-dimensional conelike surface as perceived by the three-dimensional hyperspace observer. What sort of three-dimensional curved space does the three-dimensional observer see in his own space? That's us! Imagine, if you can, rotating each of those circles in the two-dimensional space so that the swept-out locus of the rim of the circle is a two-dimensional sphere encompassing a three-dimensional volume. Now you have a set of nested spheres, but the distance from the center to the periphery of each sphere is "stretched out." The distance to the center of each sphere in the empty space around a gravitating object is larger than it would have been in flat space.

This exercise is an attempt to represent the curvature of the three-dimensional gravitating space. Neither the three-dimensional observer in Figure 9.6 nor we can directly perceive this curvature as a cone or anything else. For that, we would have to be a denizen of some fourth-dimensional hyperspace to look down on our three-dimensional space. We simply cannot do that. We can do careful three-dimensional geometry in the confines of our own three-dimensional space and work out the nature of the curvature of our space without ever being outside of it. If you were to measure the circumference of a given sphere around a gravitating object and then measure the distance to the center, you would find that the circumference was in every case less than 2&pgr; times the radius and that the smaller the sphere, the larger would be the discrepancy, just the property preserved in two dimensions and manifested in our cone representation. A three-dimensional scientist cannot, however, perceive where the extra length of the radius goes. All the scientist can or needs to know is that the radius is long compared to the circumference.

The important thing on which to concentrate is that such curvature exists in the space around the Earth, not just near a black hole. If you could draw a huge circle in the space around the Earth and then measure the radius of the circle, you would find that the radius was longer than you would expect if the space were flat. If you were to construct a triangle in the space around Earth consisting of three segments that are the shortest distances between the vertices, you would find that the angles added up to more than 180 degrees. All gravitating bodies curve the space around them! A black hole is only the most extreme example.

With this newfound perspective, let us return to the nature of black holes. Picture again a flat flexible sheet as a two-dimensional representation of flat, empty three-dimensional space. A star would cause a depression in the sheet. The star would be reduced to a two-dimensional spot of finite area (representing volume in the full three dimensions; check the Earth and Moon in Figure 9.5), and the depression representing curved space would extend beyond the star into the surrounding empty space. At no point within the star or beyond its surface is the curvature especially severe.

Suppose that the star were compacted to become a neutron star. This would be represented by making the spot smaller and the depression in the sheet much deeper. At rather large distances from the neutron star, the curvature of the sheet would be about the same. Near the neutron star, the walls of the depression will be nearly vertical (how one needs that three-dimensional, higher perspective to describe the goings-on!). As in the gravity of Newton, the strength of gravity depends on the distance to the center of the object. At the same relatively large distance, the gravity is the same. A neutron star has greater gravity than a normal star, not in the sense that it reaches out farther but in the sense that, because it is smaller in radius, one can approach much closer to the center of the gravitating star. A measure of the stronger gravity of the neutron star is the severity of the curvature of the flexible sheet at the bottom of the deep depression. The sheet would change directions rapidly at the bottom, a measure of the large curvature.

When a black hole forms, all the matter is crushed into the singularity. The mass of the star is no longer represented by an area but by a point. The flexible sheet is stretched to extremes. The curvature undergoes a discontinuity at the bottom of the cone. The sheet changes directions by 180 degrees in an infinitesimal length. One can go around the neck of the cone in an infinitesimal distance (see Figure 9.3). This is a representation of the infinite tidal forces that accompany a real singularity. Somewhere down inside the depression of the cone, a circle represents the location of the event horizon. To get the full effect, you should picture the space as an escalator moving rapidly inward, flowing down toward the singularity. To move outward, you have to run up the down escalator. At the event horizon, the escalator moves inward at the speed of light. Because you cannot run faster than the speed of light in the piece of space you occupy, you are dragged down to the singularity once you cross within the event horizon.

The singularity is a region of mystery, where our present laws of physics break down.

That does not mean black holes cannot exist. Einstein's theory is still quite valid at the event horizon, which is the only part of a black hole anyone will ever observe and live to tell about. The British mathematician Roger Penrose has proved what is called the *singularity theorem*. This theorem says that once an event horizon forms by any means, some singularity must form. The theorem does not prove that *all* matter must fall into the singularity once a black hole forms, but that conclusion seems somehow inevitable.

5.2. Black Holes and the Nature of Time

Black holes cannot really be understood without a discussion of the nature of time in their vicinity. Like curved space, the flow of time is warped near and within a black hole. This makes temporal events difficult to picture in ordinary terms. One of the fundamental problems with a discussion of time in curved space is that everything depends on whose time you are discussing.

When two things are moving apart at a large relative velocity, the great Doppler shift means that all frequencies are observed to be lower. These frequencies include not only the frequency of light but also the tick of a clock, even the biological clock. Two people rocketing away from each other at great speeds will each see the other aging more slowly than they themselves are. In the case of large gravity, there is a related effect. To an observer who is not in a large gravitational field, a clock that sits deep within the gravitational pull of some compact star will be seen to run more slowly. A person orbiting around the compact star will be seen to age more slowly. The photons that climb out of the region of highly curved space and strong gravity require some time, so that the rate of arrival of the photons at a distant observer is slow. There is a long gap between the arrival of one photon and the next. Each photon carries information concerning the “age” of the object that emitted it. Because the photons take longer to get out, they arrive when the outside observer has aged considerably. The outside observer detects the photons and sees the object in the gravitational field as younger.

Consider two investigators. One volunteers to fall down a black hole, giving her life for science. The other, the project scientist, volunteers to remain at a safe distance and monitor the proceedings. The first volunteer falls straight down into the black hole and by her own watch and biological clock passes through the event horizon, is noodleized, and dies in a few seconds. The project scientist, watching through his telescope, sees the watch of the falling volunteer running ever more slowly, and the volunteer herself aging more slowly. As the falling volunteer approaches the event horizon, time stops flowing from the vantage point of the distant observer, and he never sees the falling volunteer cross the event horizon. The reason is that the last photon emitted by the volunteer before crossing the event horizon takes a very long time to reach the distant observer. The distant observer can, in principle, always see some photons from the falling person, no matter how long he waits. When those laggard photons finally arrive, the distant observer sees the falling volunteer before she crossed the event horizon.

In practice, the photons that arrive at distant times in the future are highly red-shifted and difficult to detect. In addition, the time between their individual arrivals is very long. Most of the time the distant observer sees absolutely nothing. Because of the large red shift and the delay between arrival of photons, the actual perception is that anything falling into the black hole turns black very rapidly.

The term “frozen star” was invented to describe the mathematical solution of Einstein's theory that corresponded to the result of the absolute collapse of a star. This term focused on the fact that a distant observer can never see the surface of the star fall through the event horizon. There is thus a suggestion that the surface of the star somehow lingers at the event horizon to be touched, and probed and explored. The term “black hole” was coined by John A. Wheeler in 1968 at a meeting in New York City on pulsars. Wheeler tried to come up with a graphic term to encourage his colleagues to contemplate even more extreme states of gravitational compaction than white dwarfs and neutron stars. The name “black hole” concentrates on the collapse and the fact that the star rapidly turns completely black and on the fact that, after collapse ensues, no part of the star can ever be recovered. If you tried to fly down and grab some of this frozen star, you would find that the surface receded from your grasp as your time became its time and you could see it fall once more.

The term “black hole” is much more pertinent to the real situation because it directs attention to the actual collapse and to the interior of the black hole. The case is difficult to prove, but there is a sense that the term “black hole” itself spurred some of the marvelous work that followed. With this new term and new mode of thinking came complete mathematical solutions of the interior of black holes, where people's minds can reach even if their bodies cannot.

6. Black Hole Evaporation: Hawking Radiation

As remarked earlier, Einstein's theory, for all its magnificence and success, is not complete. This theory is a so-called classical theory in that it incorporates none of the principles of the quantum theory. In Einstein's theory, as in Newton's, all motion and changes are smooth, and all positions can, in principle, be specified exactly. Einstein's theory is not compatible with our understanding of microscopic physics as described accurately by the quantum theory.

6.1. Quantum Event Horizons

The first successful attempt to include some of the principles of the quantum theory was done by the brilliant theoretical physicist from the University of Cambridge, Stephen Hawking. The process by which energy is converted into equal parts matter and antimatter is intrinsically a quantum mechanical process. Hawking's genius was to see how to add a little of the quantum process into the otherwise classical realm of Einstein's theory. He showed that the gravitational energy associated with the curved space in the vicinity of an event horizon will create particles and antiparticles. In principle, electrons and positrons, or even protons and antiprotons, could be generated. The easiest particle to make, however, is the photon because it has no mass (technically speaking, a photon and an antiphoton are one and the same thing).

According to the quantum theory, no position can be specified exactly. This applies equally well to the position of the event horizon around a black hole. Because of the intrinsic quantum mechanical nature of things, you cannot say definitely whether something is inside or outside the event horizon, only whether something is probably inside or outside the event horizon. The location of the event horizon is then fuzzy. When two photons are created in the vicinity of the event horizon, there is a probability – purely quantum mechanical in nature – that one photon will be inside the event horizon and will disappear down toward the singularity, and the other will be outside the event horizon and fly off to great distances where it can be detected. Hawking's great discovery was that black holes are not truly black. They shine with their own radiance generated from pure gravitational curvature!

6.2. A Two-Way Street

The physical implications of this discovery were immense and caused a wrenching turnabout in our view of black holes. The energy to create the radiation came from the gravitational field, but the gravitational field came from the mass of the matter that had collapsed to make the black hole. When the photons carry off energy, the energy of the black hole must decline. This can only happen if the mass of the black hole declines as well. As black holes emit Hawking radiation, they are shining away their very mass! Black holes are not completely one-way affairs after all. Even though it is still true that tidal forces will tear an object beyond recognition as it falls into the singularity, the mass is not gone forever. It will emerge later in the form of the Hawking radiation to permeate the Universe. A black hole is just nature's way of turning all that bothersome matter into pure random radiation. We will see that nature has yet other tricks with the same fate in mind. Gather ye rosebuds while ye may, a photon yet ye'll be!

Hawking discovered that the black hole radiation does not come out in an arbitrary fashion. The spectrum of the radiation corresponds exactly to a single temperature, when it might have been some odd, nonthermal shape. The temperature is determined in turn by the mass of the black hole. The variation with mass is inverse so that a massive black hole has a low temperature, and a low-mass black hole has a higher temperature. For a black hole of stellar mass, the temperature is very low. Little radiation could be emitted in a time as short as the age of the Universe, and so the radiation is of little practical importance. Our standard picture of black holes as gaping one-way maws holds true.

6.3. Mini-Black Holes

If the mass of the black hole should be less than that of an average asteroid, however, the situation is markedly different. Such small black holes would be very hot and would radiate prodigious amounts of radiation. As these small black holes radiate, their mass shrinks so they get hotter and radiate even faster. The process runs away faster and faster. In less than the age of the Universe, such small black holes could evaporate completely! The final stages of this process are so accelerated that the last energy would emerge in an explosion of high-energy gamma rays.

These so-called *mini-black holes* could not be created in the collapse of an ordinary star. They might have arisen in the turbulence that may have marked the original state of the big bang. If this were the case, there could be swarms of mini-black holes in the Universe, some of which would be explosively evaporating at any time. The properties of such explosions have been worked out theoretically, and the radiation has been sought, but so far unsuccessfully. The notion that such tiny black holes could exist persists, however, and we will touch on a modern view of the role they could play at the deepest levels of physics and cosmology in Chapter 14 (Section 5).

6.4. White Holes

One can imagine (mathematically) the reverse of a black hole, or a *white hole.* A white hole is obtained by running time backward compared to the flow of events for a black hole. For a black hole, one starts with ordinary space. A star collapses to make a black hole, and then you have a black hole forever, gobbling up matter, but releasing nothing (forgetting for the moment Hawking radiation). Now run the movie backward in time. One must start with a white hole that has existed since the beginning of the Universe, spewing forth matter but swallowing nothing. At some time, the "last stuff" pours forth, and one is left with empty, flat space.

Black holes are regarded seriously because we can predict that they might well occur in the course of stellar evolution and because we think we have found them, as Chapter 10 will show. From the properties of known stars, the properties of the resulting black holes can be predicted. White holes are not regarded on the same footing because they must exist since the beginning of time. Their properties cannot be predicted because we cannot predict the beginning of the Universe. White holes could have any property – large mass or small. Because we cannot predict their properties, white holes have no firm place in the realm of ordinary pragmatic physics.

Hawking's discoveries may have been a first step toward putting the notion of white holes on a firmer basis. Hawking has blurred the distinction between white holes and black holes by introducing quantum mechanical properties to the event horizon. Now we see that a black hole can emit radiation, a property previously reserved for white holes. Likewise, a white hole should be able to swallow radiation. Hawking has argued that for very small objects the distinction between white holes and black holes may disappear.

7. Fundamental Properties of Black Holes

For all their exotic nature and the complexity of the theory that treats them, black holes can have only three fundamental intrinsic properties. These properties are their mass, their spin or angular momentum, and their electrical charge. These properties are distinguished because they can be measured from outside the black hole and, therefore, determined by ordinary techniques. The mass can be determined by putting an object in orbit around the black hole and seeing how fast it moves. The charge can be determined by holding a test charge and detecting the force of attraction or repulsion from the hole. In practice, one expects real black holes to be electrically neutral because they should rapidly attract enough opposite charge from their surroundings to neutralize any charge that might build up. Measurement of the spin of a black hole is a more subtle process. As the black hole rotates, it drags the nearby space around with it. This dragging can be measured in principle, like the currents in the ocean. Once the mass, spin, and charge of a black hole are known, all its other intrinsic properties are set. For instance, for a noncharged, nonspinning black hole, the size given by the radius of the event horizon is strictly proportional to the mass. The temperature of the Hawking radiation varies inversely with the mass. Other properties that a black hole might have, but cannot, are mountains like the Earth or sunspots and flares like a star. On a more fundamental level, black holes cannot have the property of a lepton number or a baryon number. The forces associated with leptons and baryons are short range and cannot extend outside the event horizon where they can be measured. Black holes do not so much violate the laws of conservation of lepton and baryon number as transcend them. In the realm of black holes, these fundamental physical laws of ordinary space are irrelevant. John A. Wheeler has coined an aphorism to describe this raw simplicity of black holes – he says "black holes have no hair."

To illustrate the power of this notion, consider two compact stars. Let one be made of neutrons, an ordinary neutron star. Let the other be made of antineutrons, an antineutron star! If these two stars were to collide, the neutrons and antineutrons would annihilate to produce pure energy and an explosion of unprecedented proportions. Suppose, however, we dump a few too many neutrons on the first star and it collapses into a black hole. Then we add some antineutrons to the second star so that it, too, collapses to make a black hole. Do we now have a black hole and an antiblack hole? No, we have two identical black holes because the black holes transcend the law of baryon (neutron and antineutron) number. If the two black holes combine, the result is not an explosion but one larger black hole. The form of mass that originally collapsed to make a black hole becomes irrelevant after it has passed through the event horizon. Then only the total mass counts. While he was warming up, Stephen Hawking presented to the world the laws by which black holes combine to make larger ones, an exercise that alone would have assured his reputation as a brilliant physicist.

8. Inside Black Holes

Just because black holes have only three fundamental properties does not mean that their nature, which derives entirely from specifying the values of those three properties, is not complex. Apart from quantum effects, the exterior of a black hole, the event horizon, is a model of simplicity: smooth, perfect, and unperturbed. The insides, however, as exposed by the powerful techniques of mathematics, are a wonder such as to strain one's credibility to the limits.

8.1. Timelike Space

When we discussed the oddities of the flow of time near black holes (Section 5.2), we omitted the oddest twist of all. This aspect can never be observed directly, but it is the real factor that accounts for the existence of the event horizon that blocks our view. Inside the event horizon, space takes on the aspects of time (cf. Figure 9.1). No matter how rockets are fired or forces applied, any object must move inward toward the singularity (or outward, if we are dealing with a white hole) as it ages. There is no choice in the matter, just as you have no choice in the matter of your aging from eighteen to thirty-one. The same principle that drags you on into old age drags an object within the event horizon ever closer to the singularity. Within the event horizon, space is no longer the entity in which you can move around in three dimensions with impunity. There is only one direction, inward. The one-way nature of this space is intimately related to the one-way nature of time. Inside a black hole space is timelike! The timelike nature of space is the reason that everything goes inward inside a black hole, and nothing can get out. It is the reason black holes are black.

8.2. Schwarzschild Black Holes

The simplest black hole is one with mass, but no charge or spin. This kind is called a *Schwarzschild black hole* after the physicist who first gave a mathematical description of such a beast, shortly after Einstein presented his general theory of relativity. There is a poetry to this name that is rendered as black shield from the German. This was the type of black hole illustrated schematically in Figure 9.1.

For a Schwarzschild black hole, the event horizon coincides exactly with what is called the *surface of infinite red shift*. A photon emitted from this surface will have an infinitely long wavelength by the time it escapes to great distances. The event horizon is round for a Schwarzschild black hole, and the singularity is a point at the center of the black hole.

Mathematical investigations have shown that even the lowly Schwarzschild black hole is not so simple. In the idealized case, where one assumes that all the mass is confined to the singularity and that a vacuum exists everywhere else, a black hole is really twain, two equal geometries sharing the same singularity. Each black hole has its own Universe of empty flat space. These two Universes exist at the same instant but in different places. When moving at less than the speed of light, one cannot travel from one to the other but will instead fall into the singularity if passage between them is attempted. This idealized mathematical description does not apply to a black hole that has formed from the collapse of a star. Then the matter of the star introduces other changes in the geometry and curvature of space that are, as yet, too complicated for anyone to have been able to calculate. The "other Universe" is undoubtedly just a mathematical fiction, but it gives a portent of the richness to come.

8.3. Kerr Black Holes

One has only to introduce some rotation to the black hole to complicate affairs in the most interesting fashion. The first basic mathematical solution corresponding to rotating black holes was discovered by the New Zealand physicist Roy Kerr in 1963. Subsequently, the complete solution of the interior of a rotating black hole was worked out by others, but these black holes are still referred to as *Kerr black holes* to distinguish them from Schwarzschild black holes.

If a black hole rotates rapidly enough, the event horizon disappears completely. In this case, one could look directly into the fearsome maw of the singularity. Such a beast is known as a *naked singularity*, a singularity unclothed by an event horizon. There is no formal proof as yet, but there is a strong belief that no black hole can rotate fast enough to create a naked singularity. Certainly any star that rotated so fast would fling itself apart before it could collapse to make a black hole. Firing matter into a black hole tangentially would spin it up. Calculations show, however, that as the black hole nears the limit where the last veil might be dropped, gravitational radiation will become so intense as to carry away any increment in rotational energy. Perhaps there is some way to create a naked singularity, but it seems very difficult. Many researchers have adopted the as yet unproven doctrine that naked singularities cannot exist in the real world of astrophysics. This doctrine that nature denies freedom of expression to unclothed singularities is known informally as "cosmic censorship." Stephen Hawking, a firm believer in cosmic censorship, bet Kip Thorne of Caltech that naked singularities cannot exist. He paid off on the bet when the carefully designed computer models of Matt Choptuik yielded naked singularities. No one has yet found one in their backyard.

Real rotating black holes may have matter swarming around inside the event horizon that will substantially alter the geometry of the inner reaches. The best we can do is to follow the mathematician's description of the idealized case where, once again, the assumption is made that all mass is confined to the singularity, and that all the rest of space is pure vacuum. Welcome to Wonderland, Alice!

The first thing one discovers in the study of rotating black holes is that the singularity is not a point but a ring! One can imagine an intrepid explorer plunging through the center of the ring, avoiding the infinite tidal forces of the singularity itself. Retreating now to the outside, we find that for a rotating black hole the surface of infinite red shift separates from the event horizon. Both surfaces are oblate, flung out around the equator by centrifugal forces, but the surface of infinite red shift is more extended. There is a finite distance between the surface of infinite red shift and the event horizon at the equator. At the poles of the rotation axis, the two surfaces are still contiguous.

The surface of infinite red shift has another property. It is also the *stationary limit* with respect to sideways motion. The rotation of the black hole drags the local space around in the same sense as the hole rotates. The effect is stronger the closer one is to the black hole. At a moderate distance, one could fire rockets and overcome the effect in order to hover in one place. This requires some effort like swimming upstream or walking up the down escalator. At the stationary limit, all efforts to remain still are fruitless. To resist moving around in the same sense as the black hole spins, one would have to fly backward in the local space faster than the speed of light. Inside the stationary limit, all material objects, including photons of light, are forced to rotate with the hole.

On the other hand, because the surface of infinite red shift is removed from the event horizon at the equator, one can, in principle (ignoring the huge tidal forces), fly inside the surface of infinite red shift and return. This can be done by moving with the rotation of the black hole, the path of least resistance. Some paths lead into the event horizon, and there will be no return; however, with a rotating black hole, the option exists to emerge from within the surface of infinite red shift.

The region between the surface of infinite red shift and the event horizon is called the *ergosphere*. This phrase was coined by Roger Penrose (of the singularity theorem) who investigated its properties. It derives from the Greek word ergo, meaning work or energy. Penrose found that, under proper circumstances, energy could be extracted from the black hole. If one of a pair of particles is fired down the hole in a counterrotating sense from within the ergosphere, the recoil will throw the other particle out with more energy than both particles had originally, including their mass energy, $E = mc^2$. You do not get something for nothing. In this case, the excess energy in the ejected particle comes from the rotational energy of the black hole. After the particle is ejected, the black hole will be rotating less rapidly.

There is some question as to whether this *Penrose process* for tapping the energy of a rotating black hole can be of real astrophysical interest. The problem is that a considerable investment of energy must be made in firing the first particle into the event horizon in the proper fashion. A puny nuclear explosion would be far from sufficient; the particle must be moving at nearly the speed of light. Such reactions with massive particles may not occur spontaneously in nature with any reasonable probability. On the other hand, photons are already moving at the speed of light. There have been discussions of Penrose processes operating to swallow some photons and eject others at high energy. This process is also driven by the rotational energy of the black hole and is termed *superradiance*. There is some speculation that the gamma rays seen from quasars could be produced in this way, starting with photons in the more conventional X-ray or ultraviolet range that are produced in the inner edges of a hot accretion disk.

Let us now journey into the event horizon. As we pass within, we come to a region of timelike space in which we must move inward as we age. There is a crucial difference in the rotating case, however, for there is an inner boundary to this timelike region. At this inner boundary is another event horizon, which prevents a return to the space beyond. Within this second event horizon is a region of normal, if highly curved, space. This event horizon prevents a return to the timelike space, rather than preventing a return to normal space.

Within this inner volume of normal space is another surface of infinite red shift, but because one can move in and out of such a surface if appropriate moves are taken, it has no direct consequence. Around the equator of this inner surface of infinite red shift is the line we devoutly wish to avoid. That equatorial line is the location of the ring-shaped singularity. If we stumble against that, we are doomed by the infinite tidal forces.

The special property of this inner region of normal space is that we could elect to stay here forever. By careful choice of movement, we can orbit around and never strike the singularity itself. This is very different from the case for a nonrotating black hole. There the timelike space leads inexorably to the singularity.

Other options await if we continue our imaginary journey within the spinning black hole. At the same place, but in the future, there is a similar space-time structure. Here, however, the sense of the event horizons and timelike space are reversed. As one flies about, one could in principle elect to head outward, passing through an event horizon into a region of outgoing timelike space. This would be bounded by an outer event horizon, and beyond that would be an ergosphere, a surface of infinite red shift, and finally free space. Formally, mathematically, this is not the space from which we entered, but another, separate Universe. The mathematical solution shows that in this new Universe there will be another in-going black hole like the original one we entered, so one can plunge down again and come out in yet a third Universe. The idealized mathematical solution we are exploring has an infinite number of Universes, all connected by rotating black holes!

Let us return to the central regions of the rotating black hole. We found there a more or less spherical region of normal space inside of which lay the ring singularity. Watch carefully now, Alice! The plane of the ring singularity divides the volume into two halves. You can maneuver from the top half, out through the inner surface of infinite red shift, and back in, to come to the bottom half. Alternatively, you could elect to plunge straight through the hole in the middle of the ring. In so doing, you would come to *a* bottom half, but not the one accessed by going out and around the ring. If from this new lower half you went out and around, you would be in a top half, but again not the one from which you started. The space through the ring is not the space you get to by going around the ring. If this is not passing through the looking glass, what is? You can imagine looking down through the ring and seeing another creature, perhaps a puce-colored eight-legged cat. If you go out around the singularity and look, you will not see the creature. Its space is only through the ring, not behind it.

If you join the creature through the ring, you can seek, in the future, a set of outgoing event horizons. These will again lead to an outer, flat Universe, which is none of the ones we have discussed previously. As you leave this black hole, you will feel it pushing you. Unlike the others we have explored, this out-going solution that exists through the ring antigravitates!

Having entertained ourselves thus, we must return to more sober reality. We do not diminish the wonder of the tale to point out again that what has just been described is an idealized mathematical solution. It is a marvelous, exact solution to the full set of equations describing general relativity. Nevertheless, a crucial assumption has been made in order to solve the equations at all. The assumption is that there is no mass anywhere except in the singularity. The presence of any matter or energy within the first set of event horizons would cause a change in the curvature and geometry, and the wonderful world of multiple Universes would probably vanish. The solution to the equations with even a little matter present throughout the volume would not contain any of the extra spaces, in the future or through the ring. Even the presence of an explorer such as we imagined ourselves to be could change the whole situation.

Some research has been done to see what happens to the mathematical solution if the tiniest bit of extra matter is added inside the black hole. There is a strong suggestion that the whole geometry would begin to rattle and shake with the resultant generation of an intense flux of gravitational radiation. This radiation alone would alter the physical and mathematical situation, to eliminate the reality of the extra spaces and Universes. At the very least, in the real Universe, photons of light will continue to flood down the black hole. As they plummet in, they are blue-shifted and attain incredible energies. This energy will build up at the event horizon in what has been termed a *blue sheet*. This sheet of energy would warp the geometry and wipe out any of the multiply-connected interior geometry.

The mathematical "vacuum" solution to the Kerr black hole is a marvelous, mind-stretching exercise. It probably has nothing to do with the guts of a real star-born black hole, rotating or not. The reality is fantastic enough as we shall see in Chapters 10 and 11, and the mystery of the singularity remains. Black holes may form from stars, but they are vastly different from stars. One way to see this is to examine the intellectual frontiers to which research on black holes has led. There one finds mind-bending concepts of worm holes, time machines, multidimensional space, self-reproducing universes, and radical new notions of how to think of time and space under conditions where neither can exist. Those are the topics of Chapters 13 and 14.