Cosmological Parameters

- Composition of the universe
  - What fraction is in the form of matter? Ω_m
    - Always positive.
  - What fraction is in the form of curvature? Ω_k
    - Can be positive (hyperbolic) or negative (spherical).
  - What fraction is in the form of cosmological constant? Ω_Λ
    - Can be positive (anti-gravity) or negative (extra gravity).
- Expansion of the universe
  - How fast is the universe expanding? H
    - Can be positive (expanding) or negative (collapsing)
  - Is the universe decelerating, or accelerating? q
    - Can be positive (decelerating) or negative (accelerating)

Defining Cosmological Parameters

- We start from the three fundamental cosmological equations
  - $H^2 = (8\pi G/3)\rho + C/L^2 + \Lambda/3$ **: Friedmann-Lemaitre eq.**
  - $a = -(4\pi G/3)\rho L + \Lambda L/3$ **: Acceleration eq.**
  - $L=Rl$ **: Expansion of length**
- Using the third equation, the first two equations may be re-written as
  - $H^2 = (8\pi G/3)\rho - k/R^2 + \Lambda/3$ (note that $k=-C/l^2$)
  - $-a/(Rl) = (4\pi G/3)\rho - \Lambda/3$
- Dividing both sides by H^2, we get
  - $1 = (8\pi G/3H^2)\rho - k/(R^2H^2) + \Lambda/(3H^2)$
  - $q = -a/(RlH^2) = (4\pi G/3)(\rho H^2) - \Lambda/(3H^2)$
- These equations define "Cosmological Parameters" as:
  - $\mathbf{1 = \Omega_m + \Omega_k + \Omega_\Lambda}$
  - $\mathbf{q = \Omega_m/2 - \Omega_\Lambda}$

Many Universes (Fig 18.13 on pp.367)

- Friedmann universes (No cosmological constant $\Omega_\Lambda = 0$)
  - $\mathbf{1 = \Omega_m + \Omega_k}$
    - $\Omega_k > 0$: "Open" universe
    - $\Omega_k = 0$: "Flat" universe (a.k.a. Einstein-de Sitter universe: $\Omega_m=1$)
    - $\Omega_k < 0$: "Closed" universe
  - $\mathbf{q = \Omega_m/2}$
    - $q > 0$: Expansion always decelerates in Friedmann universes
    - $q=1/2$ for the Einstein-de Sitter universe
- Friedmann-Lemaitre universes (W/ cosmological constant)
  - $\mathbf{1 = \Omega_m + \Omega_k + \Omega_\Lambda}$
    - Fate of the universe depends on Ω_Λ as well as Ω_k
  - $\mathbf{q = \Omega_m/2 - \Omega_\Lambda}$
    - q can be negative: acceleration is possible
- http://www.as.utexas.edu/~cosmo/expansion.php
  - Cosmology web created by two undergraduate students
    - Farhan Amanullar & Galen Carter-Jeffery

Expansion History

- Cosmological parameters evolve with time.
  - Different terms are dominant at different times.
  - $H^2 = (8\pi G/3)\rho - k/R^2 + \Lambda/3$
    - Matter density, ρ, decreases as $1/R^3$
    - k/R^2 decreases as $1/R^2$
    - Λ is constant
- All universes would look like the Einstein-de Sitter universe in the past.
  - But, the present and future behavior can be very different depending on cosmological parameters.
  - Values of the **present-day** cosmological parameters:
    - $\mathbf{\Omega_m = 0.3}$
    - $\mathbf{\Omega_\Lambda = 0.7}$
    - $\mathbf{\Omega_k = 0}$
- Think about what radiation would do…

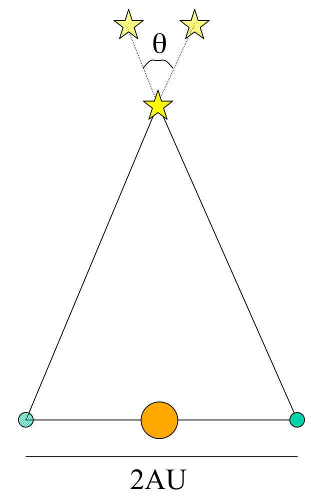
Cosmic Distance Ladders

- To determine the velocity-distance law, one needs to measure distances.
  - Determination of distances is one of the most difficult tasks in astronomy.
- In astronomy, we always measure *relative* distances.
  - E.g., distance to Jupiter = 5.2 AU (relative to Sun-Earth distance)
- Cosmic distance ladders (one is relative to the others)
  - Parallax (<10^3 light years)
  - Main-sequence fitting (<10^5 light years)
  - Cepheid stars (<10^7 light years)
  - Type Ia Supernovae (<10^9 light years)

Accuracy

Parallax

- *Direct method*: the most accurate of all methods.



Distance

= 2AU/θ

When θ=2 arcsec,

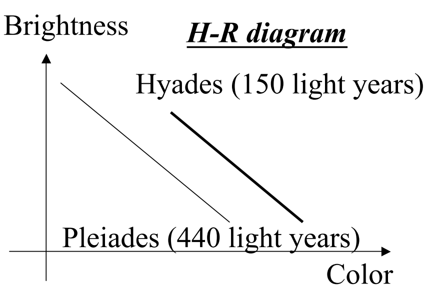
Distance

= 3.26 light years

= 1 parsec (pc)

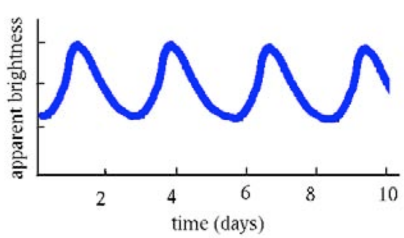
Main-sequence Fitting

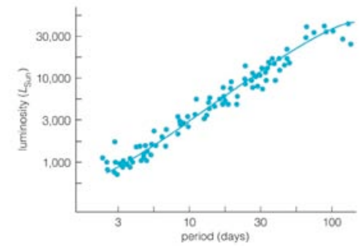
- Measure distances to open clusters relative to the distance to the Hyades open cluster.
  - The distance to the Hyades open cluster (150 light years away) is determined by the direct parallax method.
- Inverse square law
  - Brightness is proportional to 1/(distance)^2



Cepheid Stars

- Pulsating variable stars
  - Very bright stars which can be seen at **millions** of light years away
  - A simple luminosity-period relation
    - The longer the time period between peaks in brightness, the greater the luminosity of Cepheid variable star.
  - Used by Edwin Hubble to determine distances to galaxies.
- Cepheid stars in the Small Magellanic Cloud are used for determining this relation.
  - The distance to the SMC (210,000 light years) is determined by the main-sequence fitting.



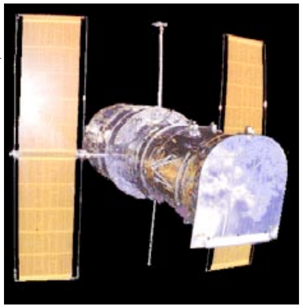


Type Ia Supernovae

- Explosion of a white dwarf with a companion star.
- A single type Ia supernova is as bright as the entire galaxy.
  - Supernovae can be seen at **billions** of light years away!
- Luminosity is approximately the same for all type Ia supernovae.
  - Type Ia supernova is a *standard candle.*
- How do we know that they are standard candles?
  - Search for type Ia supernovae at distant galaxies with known distances (distances measured by the Cepheid variable stars)
  - Compute and compare luminosities – they are approximately the same.
- Using Type Ia supernovae, we can measure distances up to about redshift of unity, or 10 billion light years away.

Hubble Key Project

- One of the key projects of the *Hubble Space Telescope* was to accurately determine the velocity-distance relation and the Hubble's parameter **at present**.
  - The HST observed many Cepheid variable stars and obtained distances.
  - Comparing the distances to the measured redshifts, the Hubble's parameter is obtained.
- Disputes before the HST
  - Alan Sandage and Gustav Tammann
    - H=50 kilometers/s/megaparsec
  - Gerard de Vaucouleurs
    - H=100 kilometers/s/megaparsec
- The HST Key Project
  - H=70 kilometers/s/megaparsec
  - Right in the middle.

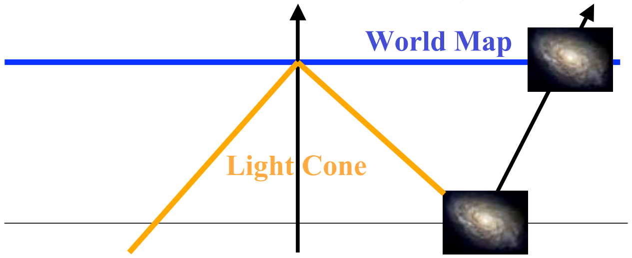


Velocity-distance Law: Subtleties

- ***V=HL***
  - The velocity-distance law is valid throughout the entire universe.
  - But, subtlety occurs when we try to **measure** this relation at great distances (say, at more than 10 billion years away, or z>1).
- Subtlety 1: What do we mean by *L*?
  - We cannot measure the **current** distance to a galaxy at $z>1$, because we can only see its **past** figure.
  - We can only measure *L* in the past!
- Subtlety 2: What do we mean by *H*?
  - The expansion rate changes with time.
  - By measuring the distance to a galaxy at $z>1$, we are measuring *H* in the past!
- Subtlety 3: What do we mean by *V*?
  - $V=cz$ can no longer be used.
  - In fact, *V* cannot be measured observationally anymore!

World Map & World Picture

- World map – "Theorist's view"
  - The entire universe at any instant can be viewed.
  - $V=HL$ can be defined unambiguously.
- World picture – "Observer's view"
  - Observations are limited to within the "light cone".
  - $V=HL$ cannot be measured directly.



Alternative Methods

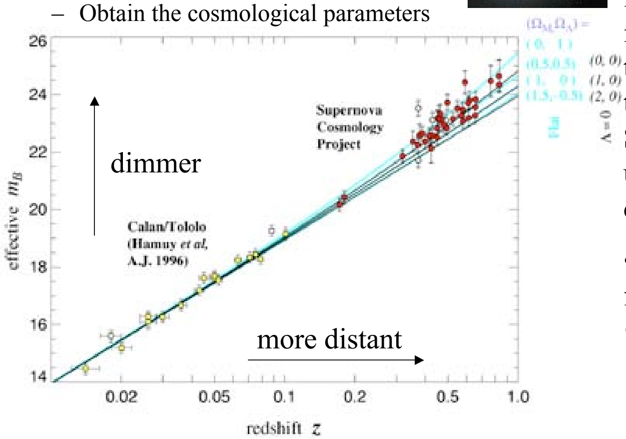
- Let's forget about using *V=HL*.
  - What else can we use to measure cosmological parameters?
- What can we measure? "Observables"
  - Redshift
  - Brightness
  - Size
- Candidate 1: *Brightness-Redshift relation*
  - Supernovae
- Candidate 2: *Size-Redshift relation*
  - Cosmic Microwave Background

Brightness-Redshift Relation

- **Use the "Standard Candle".**
- The key relation: the inverse square law
  - Brightness = Luminosity/(4π x Distance^2)
  - We measure brightness; we know luminosity (SNIa)
    - Distance^2 = Luminosity/(4π x Brightness)
    - This distance, determined from the luminosity, is called the "*luminosity distance*"
  - The luminosity distance is related to the world-map distance, but not the same.
  - The luminosity distance depends on z as well as cosmological parameters such as H, Ω_m, Ω_k, Ω_Λ
- Therefore, by comparing the inferred luminosity distance with the measured redshifts, one can derive the cosmological parameters.

Application: Type Ia Supernovae

- Program
  - Search for many Type Ia Supernovae
  - Measure their brightness and redshifts
  - Plot them and compare with theory
  - Obtain the cosmological parameters

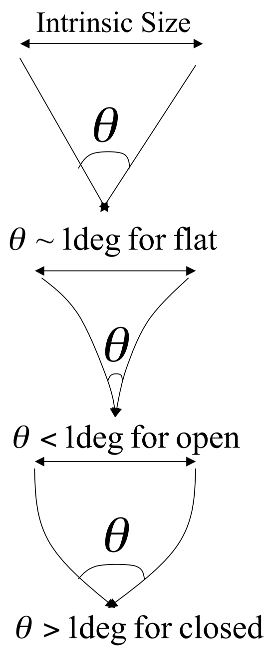


•Supernovae appear to be **dimmer** (i.e., the inferred luminosity distance is much too large) than expected from the Einstein-de Sitter universe or universe without dark energy

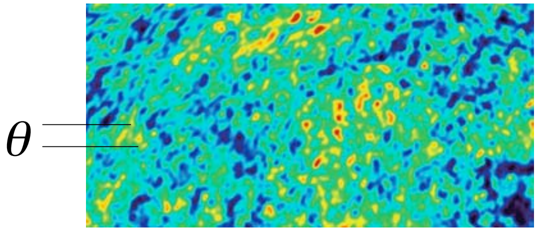
•Expansion was faster than expected → Dark energy

Size-Redshift Relation

- **Use the "Standard Ruler".**
- The key relation: the size-angle relation
  - Angle = Size/Distance
  - We measure angle; we know size
    - Distance = Size/Angle
    - This distance, determined from the luminosity, is called the "angular diameter *distance*"
  - The angular diameter distance is related to the world-map distance, but not the same.
    - Angular diameter distance = Luminosity distance / (1+z)^2
  - The angular diameter distance depends on z as well as cosmological parameters such as H, Ω_m, Ω_k, Ω_Λ
- Therefore, by comparing the inferred angular diameter distance with the measured redshifts, one can derive the cosmological parameters



Application: CMB



- What to Do:
  - Map of fluctuations in CMB
  - Measure the angular size of hot and cold spots
  - Compare the angular size with the intrinsic size (known) to get the angular diameter distance
  - CMB comes from z=1100
  - Obtain the cosmological parameters

Results

- Supernovae primarily determine the deceleration parameter
  - $q = \Omega_m/2 - \Omega_\Lambda \sim -0.55$ (acc.)
- CMB primarily determines the geometry of the universe
  - $\Omega_k = 1 - \Omega_m - \Omega_\Lambda = 0$ (flat)
- Combining the two, we can determine the composition of the universe as…