Black Hole

- One might ask: what would happen when the escape velocity equals the speed of light?
  - Even the light cannot escape: **Black Hole**
  - *Escape Velocity:* $v^2 = 2GM/R$
  - Black holes should be either very massive or very small.
- Schwarzchild radius
  - $R_S=2GM/c^2$
    - ~3 km for the solar mass
    - ~9 mm for the Earth mass
  - A characteristic size of black holes
- Spacetime of BH is positively curved

Spatial Distance in Curved Space

- In flat space (Euclidean space), a distance between two points on a surface is given by
  - **(Distance)^2 = (x-int.)^2 + (y-int.)^2**
  - $ds^2 = dx^2 + dy^2$
- In curved space (non-Euclidean), Euclidean formula no longer applies:
  - **(Distance)^2**
    = F(x-int.)^2 + 2G(x-int.)(y-int.) + H(y-int.)^2
  - $ds^2 = \mathrm{F}dx^2 + 2\mathrm{G}dxdy + \mathrm{H}dy^2$
  - F, G, H : *Metric Coefficients*
    - Metric coefficients generally depend on x and y

Space-time Dist. in Curved Spacetime

- In flat spacetime (Minkowski spacetime), a spacetime distance between two events is given by
  - **(Spacetime Distance)^2 = (t-int.)^2 - (x-int.)^2**
  - $ds^2 = (cdt)^2 - dx^2$
- In curved space (Riemannian spacetime),
  - **(Spacetime Distance)^2**
    = F(t-int.)^2 - 2G(t-int.)(x-int.) - H(x-int.)^2
  - $ds^2 = \mathrm{F}(cdt)^2 - 2\mathrm{G}cdtdx - \mathrm{H}dx^2$
  - ds^2 is still invariant – it does not depend on observers.

Spacetime Metric of BH

$$ds^2 = \left(1 - \frac{R_S}{R}\right)c^2dt^2 - \left[\frac{dR^2}{1 - \frac{R_S}{R}} + R^2 d\Omega^2\right]$$

- Schwarzschild metric
  - This formula describes spacetime around a spherical, non-rotating object.
- Not only space is curved, but also time is curved.

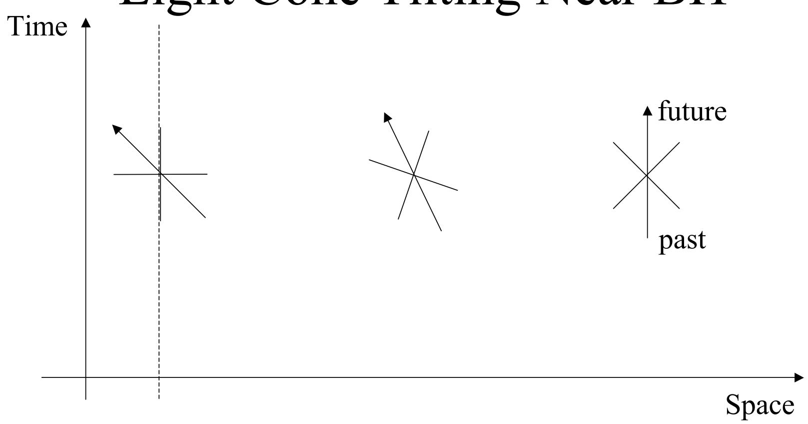
Time Dilation and Light Deflection

- Two observers, A and B.
  - A is a "distant observer", very far away from BH
  - B is falling into BH
- A sees B's time flowing very slowly:
  - $(\Delta t_A) = (\Delta t_B) / \sqrt{(1 - R_S/R)}$
- A sees the wavelength of light emitted by B being streched by
  - $(\lambda_A) = (\lambda_B) / \sqrt{(1 - R_S/R)}$
- A sees the direction of light emitted from behind BH being deflected.
  - $R \sim 1.5\ R_S$: photon sphere
  - $R \sim \sqrt{3}$ x (photon sphere) : capture radius

B's View

- An observer B, who is freely falling into BH, would not notice he would be falling into BH (Equivalence Principle) except for tidal force.
  - His feet are pulled more strongly than his head
- The sign of redshift and time dilation is reversed:
  - B sees the wavelength of light coming from the outside world being blueshifted.
  - B sees time of the outside world flowing very fast.

Light Cone Tilting Near BH



- Schwarzschild radius = Event Horizon
  - No information can get out of the event horizon

Is Black Hole Truly "Black"?

- Hawking Radiation
  - The effect that appears when GR and QM are both considered.
  - The result: **a black hole actually emits black-body (thermal) radiation.**
- In QM world, a pair of particles are constantly created; however, these particles must annihilate away in a very short period of time.
  - Uncertainty Relation between energy and time is given by $\Delta E \Delta t = h$
  - A pair of two electrons can survive for 10^{-21} seconds
  - A pair of two protons can survive for 10^{-24} seconds
- What if these particles are created near the event horizon?

Evaporation of Black Holes

- Particles created near the event horizon carry energy away from BH.
- Mass of BH decreases!
  - Temperature of BH = 10^{-7} K (solar mass/M)
    - BH of Earth's mass: 0.01 K
    - BH of 1/10 Lunar mass: 10 K
  - Smaller BHs emit more energy.
- Evaporation time = 10^{62}(M/solar mass)^3 years
  - Black holes would evaporate on this time scale.
- "BH is a heat engine"
  - It swallows stuff and converts it to energy.