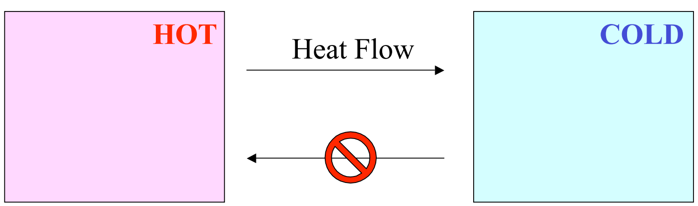
"Perfect" Cosmological Principle?

- Perfect *symmetry* in space **<u>and time</u>**
  - No special locations exist in space and time
  - No special directions exist in space and time
- If the perfect cosmological principle is valid, the physical state of the universe should not change in time. This idea leads to:
  - Static Universe: the universe does not expand.
  - Steady-state Universe: the universe expands at a constant rate. Matter continuously created.
- Now we know that the universe is not symmetric in time – it's homogeneous and isotropic only in space.
  - Example 1: There is the beginning of time.
  - Example 2: The universe cools down as it expands.
  - Example 3: There were more quasars in the past.

Arrow of Time

- Symmetry is broken!
  - Space: Reversible
  - Time: Irreversible
- Why should **time** be so special in four dimension?
  - Relativistic theory (which unifies space and time and treats "spacetime" as the fundamental object) does not tell us that time must be special.
  - In fact, almost all fundamental theories of physics posses time reversibility.
  - However, "everyday" (macroscopic) phenomena, such as thermodynamics, posses time irreversibility.
    - **E.g., 2nd law of thermodynamics – entropy always either increases or remains the same.**

The 2nd Law of Thermodynamics



- The 2nd law of thermodynamics states that:
  - Heat always flows from hot to cold, when no extra work is done to the system.
  - How do we know it? We know it from experiences.
- This law results in the increase of **entropy**, which is given by **the amount of heat given to the system per unit temperature**.

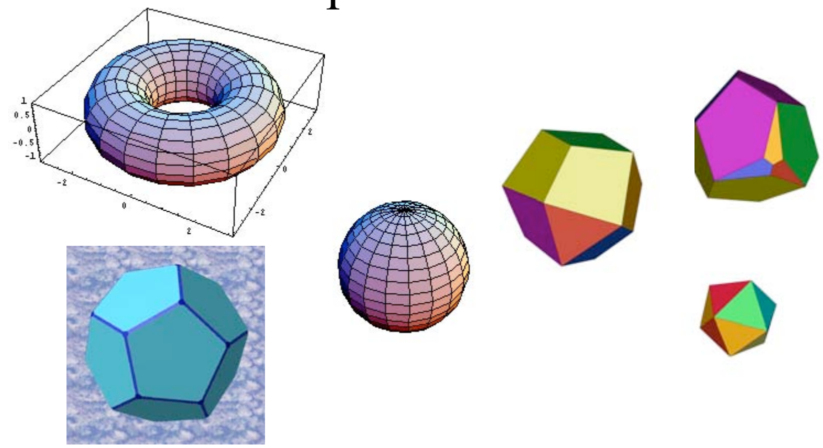
Entropy

- Thermodynamic Entropy, S
  - $S = Q/T$ [joules/Kelvin]
  - Q: the amount of heat given to the system
  - T: temperature of the system
- Example: add a cup of boiled water to either (a) boiled water, or (b) cold water
  - The change caused by adding a cup of boiled water is more dramatic for the case (b)
    - A larger increase of entropy for colder system.
  - In this example, entropy measures the "**degree of disturbance**", or "**complexity**".
- Entropy is closely related to the amount of information:
  - S can also be written as $S = N k_B \log(W)$
  - N: the number of particles in the system
  - k_B: the Boltzmann constant
  - W: the number of possible states in the system

The Edge of the Universe?

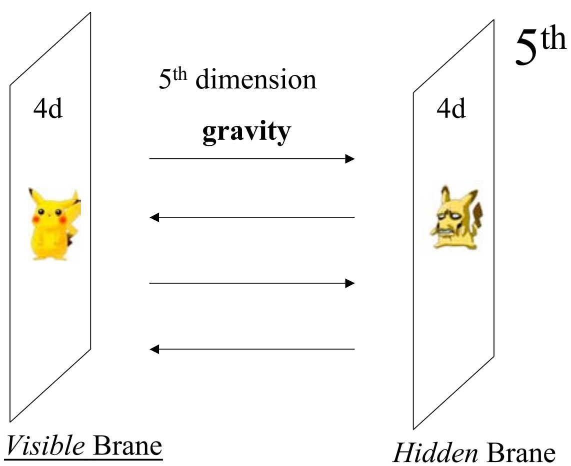
- FAQ
  - Is there the edge of the Universe?
  - What's there outside of the Universe?
  - What is the Universe expanding into?
  - What was there before the Big Bang?
  - Where did the Big Bang occur?

The Shape of the Universe?



- CAUTION: we are living on the **3-dimensional** surface. Since it is not possible to visualize the 3-d surface, here are shown the 2-d ones.

5^{th} Dimension?



- There may be another world, and there may exist the 5^{th} dimension…
  - Only gravity can communicate between the two "branes"
  - ("brane" came from "membrane".)

Curved Space

- Euclidean geometry is "flat"
  - Imagine that you have a piece of paper and a ball.
  - A piece of paper has no curvature
  - The surface of a ball is "curved" – there is curvature
- Curved space cannot be described by the Euclidean geometry; therefore, it is called **non-Euclidean**.
- In curved space, there is a characteristic length scale, *R*.
  - Example: the surface of the Earth
  - How do we know that the surface of the Earth is curved?
- Homogeneous and isotropic non-Euclidean geometry
  - Spherical geometry
  - Hyperbolic geometry
- Is our universe flat, spherical, or hyperbolic?

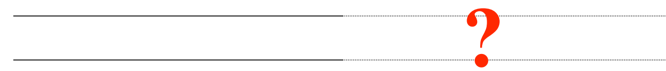
Euclidean Axioms (Postulates)

Euclid (325-270 B.C.)

1. A straight line can be drawn between any two points
2. A finite line can be extended infinitely in both directions
3. A circle can be drawn with any center and any radius
4. All right angles are equal to each other
5. **Given a line and a point not on the line, only one line can be drawn through the point parallel to the line**
   - **Euclidean parallel postulate**

Parallel Postulate

- Parallel lines = Lines that do not intersect each other
- How do we know that two lines that appear to be parallel **continue to be parallel when extended to large distances?**

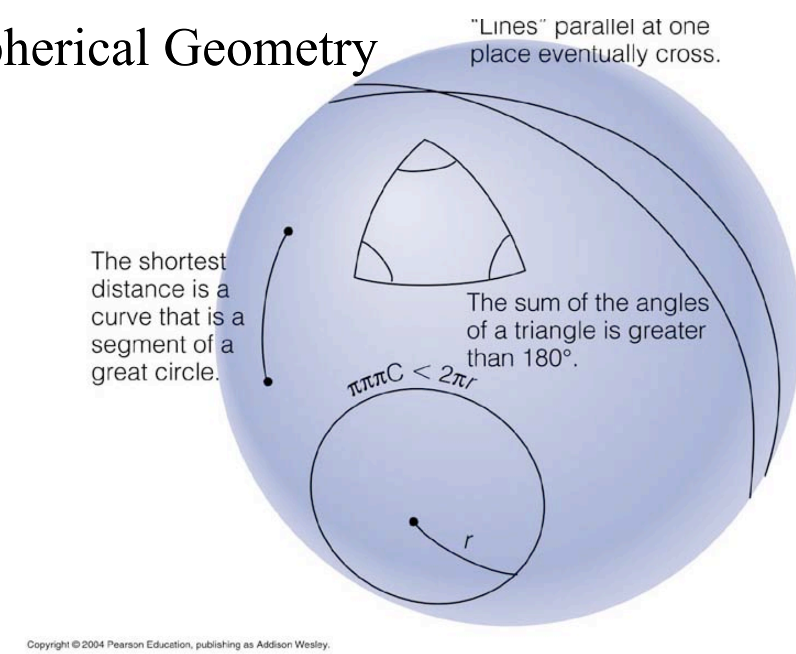


- "Parallel postulate" is valid only for the Euclidean geometry – there are many other geometries, **non-Euclidean geometries**, for which the parallel postulate is invalid.

Spherical Geometry

- It's basically the surface of a sphere.
- All lines will eventually intersect: *no parallel lines exist!*
  - Euclid had to extend his "parallel" lines to very large distances on the Earth before he noticed this fact.
- In spherical geometry, the sum of the interior angles of a triangle is **greater** than two right angles (π=180 degrees)
  - In flat geometry, the sum of the angles of a triangle must always be 180 degrees.
- The circumference of a circle is **less** than π times its diameter.
  - In flat geometry, the circumference of a circle must always be π times its diameter.

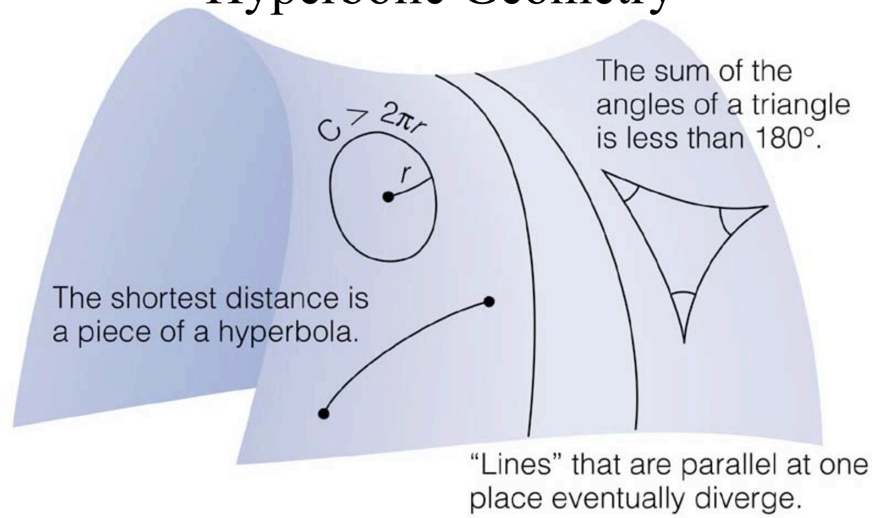
Spherical Geometry



Hyperbolic Geometry

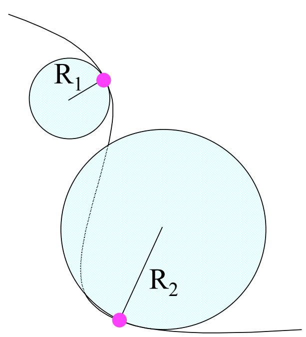
- It's similar to the surface of a horse's saddle.
  - But it is not possible to draw a *real* hyperbolic geometry, where space is homogeneous and isotropic
- Not only one, but many other lines do not intersect: *many parallel lines exist!*
- In hyperbolic geometry, the sum of the interior angles of a triangle is **less** than two right angles (π=180 degrees)
- The circumference of a circle is **greater** than π times its diameter.

Hyperbolic Geometry



Curvature

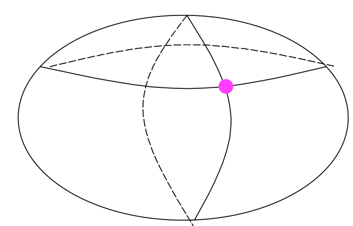
- How curved is it?
  - The radius of an *osculating circle* can be used to measure curvature of a line at a given point.
  - Curvature = 1/(curvature radius)
    - Curvature is in units of 1/length
  - The signs posted on the road saying "R=300ft" or "R=500ft"
    - Which one is more curved?
- A straight line (zero curvature) has R=infinity



Gaussian Curvature

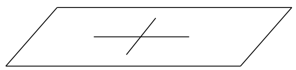
- Curvature of a surface
  - Draw **two** principal osculating circles at a given point on the surface
  - Obtain two principal curvature radii, R_1 and R_2
  - Gaussian curvature is given by $1/(R_1 R_2)$, **up to the overall sign.**
- K=Gaussian curvature
  - K is in units of 1/area

Johann Carl Friedrich Gauss (1777-1855)

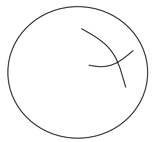


Flat, Spherical, Hyperbolic

- Homogeneous and Isotropic space can be either flat, spherical, or hyperbolic.
  - ***K* is the same everywhere**
- Flat
  - Zero *K*
  - $K=0$
- Spherical
  - Positive *K*
  - $K=1/R^2 > 0$
- Hyperbolic
  - Negative *K*
  - $K= -1/R^2 < 0$

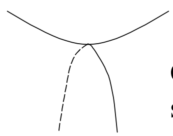


$R_1 = R_2 =$ Infinity



$R_1 = R_2 = R$

Osculating circles on the same side



$R_1 = R_2 = R$

Osculating circles on opposite sides

Measuring Curvature

- θ=Sum of the angles of a triangle minus π
  - **$\theta = K$ x (area of triangle)**
  - $\theta=0$ (flat)
  - $\theta>0$ (spherical)
  - $\theta<0$ (hyperbolic)

- θ=Change in direction of an arrow through a closed circuit of the "vector transport"
  - **$\theta = K$ x (area enclosed by circuit)**
  - This is neat – try it yourself!