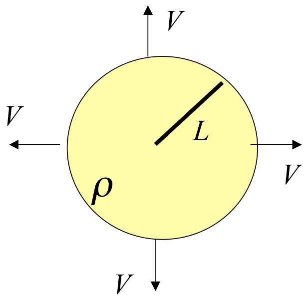
Expanding cosmic sphere

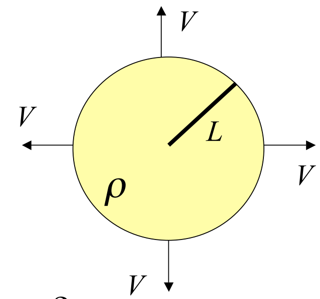
- Let's pick a small spherical region (filled with matter) in the universe.
  - Radius: L
  - Mass: M
  - Density: ρ
  - Expansion Velocity: V



- Will this region expand forever?
  - It depends on the expansion velocity, V.

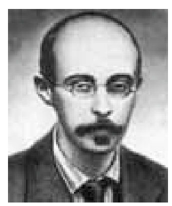
Escape Velocity

- Escape velocity is given by
  - $(V_{escape})^2=2GM/L$
    - The mass is $M=(4\pi G/3)\rho L^3$
  - $(V_{escape})^2= (8\pi G/3)\rho L^2$



- Is V larger or smaller than V_{escape}?
  - $V< V_{escape}$: the region will recollapse in the future
  - $V= V_{escape}$: the region will expand forever
  - $V> V_{escape}$: the region will expand forever

Friedmann Equation



- Let's write the expansion velocity using the escape velocity as follows:
  - $V^2 = (V_{escape})^2 + C$
  - The constant, "C", determines the fate of the region:
    - $C<0$: recollapse
    - $C=0$, $C>0$: expand forever
- Recalling the form of the escape velocity, one obtains the following "Friedmann equation":
  - $V^2 = (8\pi G/3)\rho L^2 + C$
  - The velocity-distance relation, $V=HL$, gives the most popular form of the Friedmann equation:
  - $H^2 = (8\pi G/3)\rho + C/L^2$
    - **This equation determines how the universe expands and what the fate of the universe is --- extremely important in cosmology!**

Acceleration

- In addition to the Friedmann equation,
  - $H^2 = (8\pi G/3)\rho + C/L^2$

  which describes the expansion velocity, there is the second equation which describes the acceleration of the expansion:
  - $a = -(4\pi G/3)\rho L$
    - Notice the negative sign in front: the presence of matter always ***decelerates*** (**a<0**) the expansion.
- Einstein did not like this!
  - Einstein wanted the universe to be static – neither expanding or contracting – however, this equation does not permit it.
  - He added the "cosmological constant", Λ, to this equation in order to cancel the effect of matter:
    - $a = -(4\pi G/3)\rho L + \Lambda L/3 = 0$
  - **This Λ is what Einstein called later "the biggest blunder".**
- More importantly, Λ can *accelerate* the expansion, unlike the ordinary matter.

The fate of the universe

- We have now two fundamental cosmological equations:
  - $H^2 = (8\pi G/3)\rho + C/L^2 + \Lambda/3$
  - $a = -(4\pi G/3)\rho L + \Lambda L/3$
- These equations determine how the scale factor, R, changes with time. (remember $L=Rl$)
  - We have three quantities to specify
    - ρ, C, Λ
  - Here is the bottom line: we need to determine these quantities by observations.
    - These are called the "cosmological parameters" and determination of the cosmological parameters has been the most important task in cosmology as they determine the evolution of R in the past and in the future.

Where is relativity?

- The derivations of the Friemann equation so far did not use relativity – it was totally Newtonian.
- Where is relativity?
  - In Newtonian picture, it is not clear what C really means or what determines C.
  - In relativistic picture, C is actually related to the geometry of space:
    - $C<0$: spherical geometry
    - $C=0$: flat geometry
    - $C>0$: hyperbolic geometry
  - In other words, the geometry of the universe determines the fate of the universe!!
    - Spherical geometry: recollapse
    - Flat or hyperbolic geometry: expand forever