General Relativity (1916)

- General theory of relativity
  - An extension of "special theory of relativity", which did not include gravity but dealt only with "inertial motion" (i.e., motion with constant velocity)
  - How do we deal with gravitational force?
- Equivalence Principle
  - Equivalence principle states that "**inertial force** and **gravitational force** are the same thing."
  - Or, "**acceleration** and **gravity** are the same thing."
- Minkowski space to Riemannian space
  - Gravity causes geometry of spacetime to be **curved**.

Equivalence Principle

- Let's imagine that you are in an elevator.
  - What happens when the elevator begins to go up?
  - What happens when the elevator begins to go down?
  - What happens when the elevator moves at constant speed?
  - What would happen when the wire hanging the elevator is cut?
- Let's imagine that you are an astronaut on board a space station.
  - Why do you think you are a weightless?
  - What would you do to make "gravity" in flight?

Acceleration = Gravity

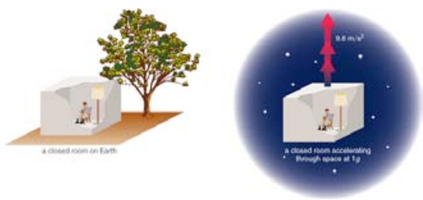
- Newton's 2nd law of motion
  - $F = m\,a$
- Gravitational force = $m\,g$
  - $m\,a = m\,g$
  - Therefore, $a = g$, for a freely falling body
- But, wait. What do we mean by *mass*?
  - Mass could be measured by its acceleration, given the *inertial force*
  - Mass could be measured by its weight (or gravitational force)
  - But, who said these two "masses" are the same thing?

Inertial Mass = Gravitational Mass

- Maybe there are two distinctive definitions of "mass".
- Inertial mass, m_{inertial}, may be defined by
  - Inertial force = $m_{\text{inertial}}\,a$
- Gravitational mass, m_{gravity}, may be defined by
  - Gravitational force = $m_{\text{gravity}}\,g$
- Then, Newton's equation becomes
  - $m_{\text{inertial}}\,a = m_{\text{gravity}}\,g$
- Now, let's drop two balls with different weights (different m_{gravity}) from the roof of RLM and see which one reaches the ground first.
  - (Ignoring friction by air) both balls will reach the ground at the same time. This must imply that $m_{\text{inertial}} = m_{\text{gravity}}$
  - Acceleration would be different for two balls, otherwise!

Is it surprising?

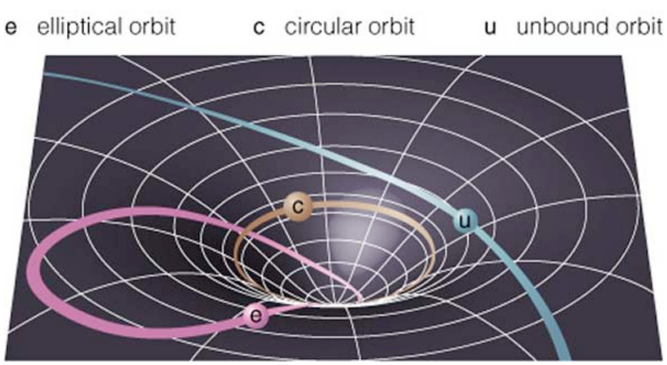
- It's surprising because equivalence principle then states that gravity can be canceled, or mimicked, by acceleration.
  - Case 1: Free-fall (gravity canceled)
  - Case 2: Constant acceleration (gravity mimicked)
- When gravity is canceled by acceleration, one can still use special relativity.
  - Free-falling frame and inertial frame are totally equivalent.



A way to sense "gravity"

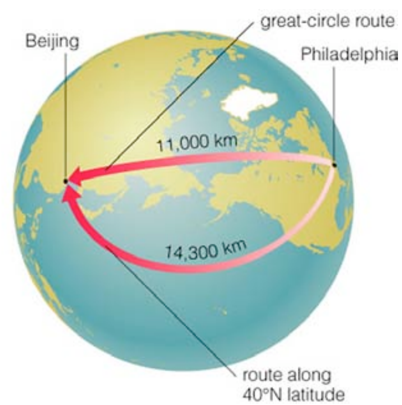
- When gravity is uniform (g is constant everywhere), its effect can be canceled by a uniform acceleration precisely.
- However, if gravity is non-uniform (which is always true), then there will be a **tidal force**
  - Case 1: g on Earth depends on altitude
  - Case 2: g in the Solar System depends on distance from the Sun
  - Case 3: g on Earth by Moon depends on locations on Earth (cause of tides)
- Let's imagine two balls falling into the center of Earth. What happens?

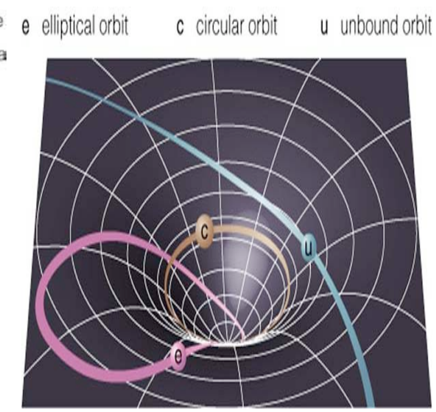
Spacetime curvature and Gravity



- Gravity is now described by curvature of spacetime
- Spacetime is curved by the presence of energy
- Minkowski (flat) spacetime is equivalent to "zero gravity" = "zero curvature"

Shortest Paths (Geodesics)



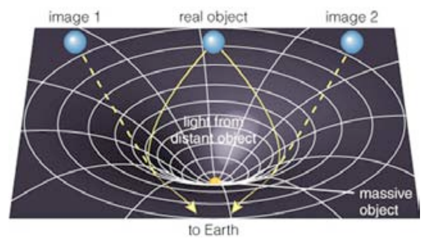


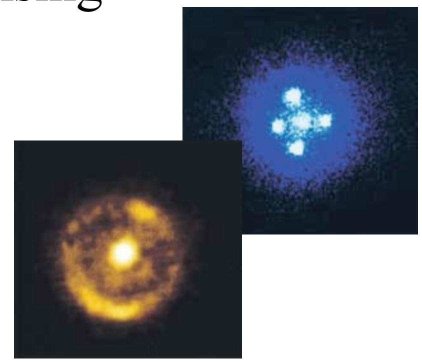
- The shortest paths, geodesics, may look "curved", but these are the "straight" lines in curved geometry.

Strength of Gravity

- How do we quantify the strength of gravity of an object?
- How much is the spacetime around an object curved?
  - Gravity is stronger when mass is larger
  - Gravity is stronger when distance is shorter
- Escape velocity
  - Kinetic energy = Gravitational potential energy
  - $(1/2)m\,v^2 = G\,M\,m/R \rightarrow v^2 = 2GM/R$
  - E.g., Sun's gravity: 619 km/s, Earth's gravity: 11 km/s

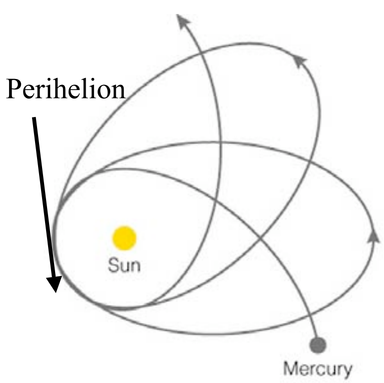
Gravitational Lensing





- Deflection of light = 2 $(v/c)^2$ radians
- The Sun will deflect light by 1.75 arc-seconds
  - Measured by Authur Eddington during solar eclipse in 1919.

Perihelion Shift



Note: The amount of precession with each orbit is highly exaggerated in this picture.

- Mercury's perihelion was measured to be shifting over years
  - Subtracting precession of perihelion due to gravity from other planets remains the shift of **43 arc-second per century**
- General Relativity accounts for this naturally: perihelion shift = $(v/c)^2$ radians per revolution
  - v is the escape velocity from the Sun's gravity at Mercury's orbit.

Gravitational Redshift

- As light escapes from gravitational potential, it loses energy.
- Light with smaller energy has a longer wavelength → Color of light gets redder: **Redshift**
- **1+Redshift**

  = Wavelength received/Original Wavelength
- When gravity is weak ($v/c < 1$), gravitational redshift is
  - Gravitational Redshift ~ $(1/2)(v/c)^2$
- This result can also be interpreted as "**gravitational time dilation**"
  - 1+Redshift = Time dilation
  - When redshift ~ 0.1 (rather strong gravity!), clock ticks more slowly by 10%