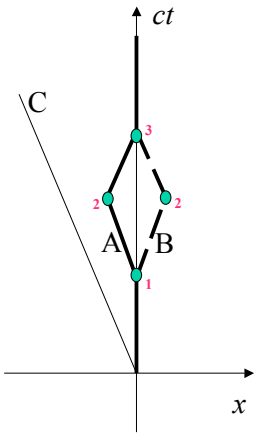
Twin Paradox

- There are twins, A and B
- B moves relative to A
  - A's point of view
    - B is moving at speed v
    - B's clock ticks more slowly by γ.
    - Therefore, B appears to be aging more slowly.
  - B's point of view
    - A is moving at speed v
    - A's clock ticks more slowly by γ.
    - Therefore, A appears to be aging more slowly.
- So, which one is older, when they meet?
  - **Twin Paradox**

Case 1

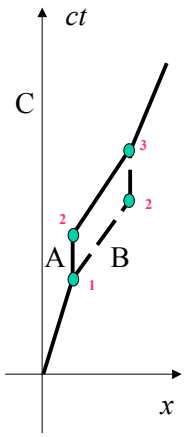
Original Frame



- A and B are at rest at the same place until event 1.
- Then A and B go on a trip on opposite directions.
- A and B turn around and come back at events 2.
- A and B finally meet at event 3.
- In this case, A's and B's worldlines are **symmetric**. (mirror symmetry)
  - A and B have traveled the **same spacetime distance**.
  - Therefore, A and B have aged the same years.

Case 1 (a different point of view)

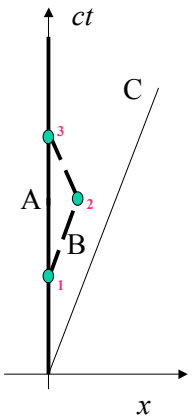
C's Frame



- C's point of view
  - C is moving to the left with respect to the original frame
- A and B are moving to the right together until event 1.
- Then A is at rest but B speeds up.
- A turns around earlier than B.
  - Then B is at rest but A moves to the right faster than before
- A and B finally meet at event 3.
- In this case, A's and B's worldlines are still **symmetric**. (point symmetry)
  - A and B have traveled the same spacetime distance; thus, A and B have aged the same years

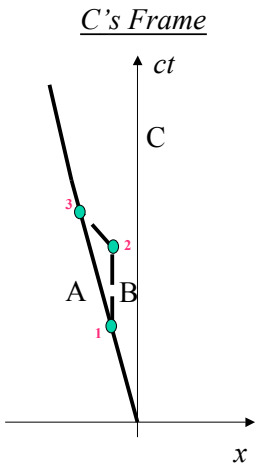
Case 2

Original Frame



- A remains at rest at all times.
- B leaves home at event 1, turns around at event 2, and finally meets A at event 3.
- In this case, A's and B's worldlines are **not symmetric**!
- What happens?
  - The answer is that A has aged more than B.
- Why?
  - B's spacetime distance is **shorter** than A's
  - Remember, $ds^2=c^2dt^2-dx^2$

Case 2 (a different point of view)



- In C's frame, A is moving to the left at all times. B is initially moving to the left, together with A.
- B becomes at rest at event 1, and then moves to the left faster than before.
- A and B meet at event 3.
- In this case, A's and B's worldlines are still **not symmetric**!
  - A has aged more than B.
  - B's spacetime distance is still shorter.

So, what was it?

- Motion of A and B remains completely relative **only when** both are moving at constant velocity.
  - Motion has to be inertial for a "perfect relativity" to be valid
- However, for two people to know their initial ages and then meet later again, the motion cannot stay inertial → motion is no longer perfectly relative.