Entropy

- Thermodynamic Entropy, S
  - $S = Q/T$ [joules/Kelvin]
  - Q: the amount of heat given to the system
  - T: temperature of the system
- Example: add a cup of boiled water to (a) boiled water, or (b) cold water
  - The change caused by adding a cup of boiled water is more dramatic for the case (b)
    - A larger increase of entropy for colder system.
  - In this example, entropy measures the "**degree of disturbance**", or "**complexity**".
- Entropy is closely related to the amount of information:
  - S can also be written as $S = N k_B \log(W)$
  - N: the number of particles in the system
  - k_B: the Boltzmann constant
  - W: the number of possible states in the system

Euclidean Axioms (Postulates)

Euclid (325-270 B.C.)

1. A straight line can be drawn between any two points
2. A finite line can be extended infinitely in both directions
3. A circle can be drawn with any center and any radius
4. All right angles are equal to each other
5. Given a line and a point not on the line, only one line can be drawn through the point parallel to the line
   - **Euclidean parallel postulate**

Parallel Postulate

- Parallel lines = Lines that do not intersect each other
- How do we know that two lines that appear to be parallel **continue to be parallel when extended to large distances?**

- "Parallel postulate" is valid only for the Euclidean geometry – there are many other geometries, **non-Euclidean geometries**, for which the parallel postulate is invalid.

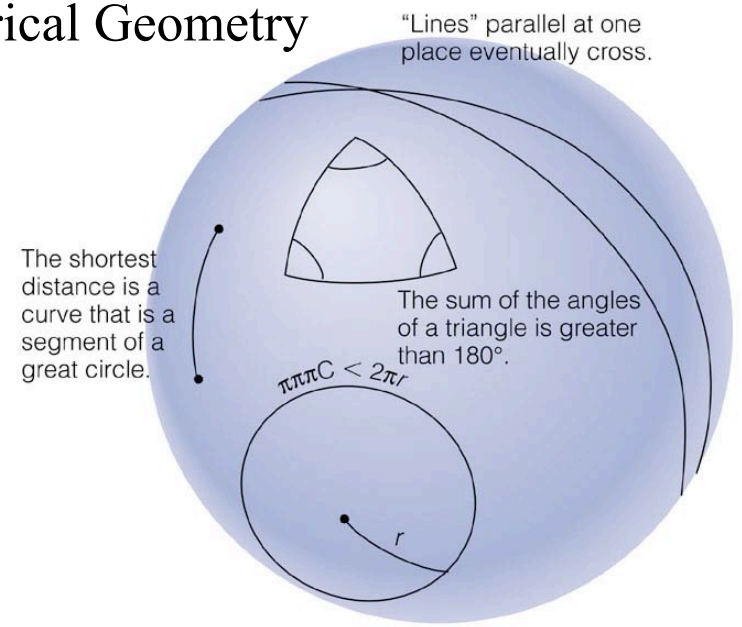
Curved Space

- Euclidean geometry is "flat"
  - Imagine that you have a piece of paper and a ball.
  - A piece of paper has no curvature
  - The surface of a ball is "curved" – there is curvature
- Curved space cannot be described by the Euclidean geometry; therefore, it is called **non-Euclidean**.
- In curved space, there is a characteristic length scale, R.
  - Example: the surface of the Earth
  - How do we know that the surface of the Earth is curved?
- Homogeneous and isotropic non-Euclidean geometry
  - Spherical geometry
  - Hyperbolic geometry
- Is our universe flat, spherical, or hyperbolic?

Spherical Geometry

- It's basically the surface of a sphere.
- All lines will eventually intersect: *no parallel lines exist!*
  - Euclid had to extend his "parallel" lines to very large distances on the Earth before he noticed this fact.
- In spherical geometry, the sum of the interior angles of a triangle is **greater** than two right angles (π=180 degrees)
  - In flat geometry, the sum of the angles of a triangle must always be 180 degrees.
- The circumference of a circle is **less** than π times its diameter.
  - In flat geometry, the circumference of a circle must always be π times its diameter.

Spherical Geometry



Hyperbolic Geometry

- It's basically the surface of a saddle.
  - But it is not possible to draw a *real* hyperbolic geometry, where space is homogeneous and isotropic
- Not only one, but many other lines can intersect: *many parallel lines exist!*
- In hyperbolic geometry, the sum of the interior angles of a triangle is **less** than two right angles (π=180 degrees)
- The circumference of a circle is **greater** than π times its diameter.

Hyperbolic Geometry