the opposition to Copernicanism into a hopeless rear-guard action. The rest of this chapter examines that new evidence drawn from the heavens by three of Copernicus' immediate successors.

Tycho Brahe

(reading from Thomas S. Kuhn)

If Copernicus was the greatest European astronomer in the first half of the sixteenth century, Tycho Brahe (1546–1601) was the preëminent astronomical authority of the second. And, judged purely by technical proficiency, Brahe was the greater man. But comparison is largely meaningless, because the two have different strengths and weaknesses which would not readily have merged in a single personality, and both sorts of strength were essential to the Copernican Revolution. As a cosmological and astronomical theorist, Brahe displayed a relatively traditional frame of mind. His work shows little of that Neoplatonic concern with mathematical harmonies that had been instrumental in Copernicus' break with the Ptolemaic tradition and that at the start provided the only real evidence of the earth's motion. He propounded no enduring innovations in astronomical theory. He was, in fact, a lifelong opponent of Copernicanism, and his immense prestige helped to postpone the conversion of astronomers to the new theory.

But though Brahe was no innovator of astronomical concepts, he was responsible for immense changes in the techniques of astronomical observation and in the standards of accuracy demanded from astronomical data. He was the greatest of all naked-eye observers. He designed and built many new instruments, larger, stabler, and better calibrated than those in use before. With great ingenuity he investigated and corrected many errors that developed in using these instruments, establishing a whole series of new techniques for the collection of accurate information about the position of planets and stars. Most important of all, he began the practice of making regular observations of planets as they moved through the heavens rather than observing them only when in some particularly favorable configuration. Modern telescopic observation indicates that when Brahe took particular care in determining the position of a fixed star his data were consistently accurate to 1′ of arc or better, a phenomenal achievement with the naked eye. His observations of planetary position seem normally to have been reliable to about 4′ of arc, more than twice the accuracy achieved by the best observers of antiquity. But even more important than the accuracy of Brahe's individual observations was the reliability and the scope of the entire body of data he collected. In his own lifetime he and the observers he trained freed European astronomy from its dependence on ancient data and eliminated a whole series of apparent astronomical problems which had derived from bad data. His observations provided a new statement of the problem of the planets, and that new statement was a prerequisite to the problem's solution. No planetary theory could have reconciled the data employed by Copernicus.

Trustworthy, extensive, and up-to-date data are Brahe's primary contribution to the solution of the problem of the planets. But he has another and a larger role in the Copernican Revolution as the author of an astronomical system that rapidly replaced the Ptolemaic system as the rallying point for those proficient astronomers who, like Brahe himself, could not accept the earth's motion. Most of Brahe's reasons for rejecting Copernicus' proposal are the usual ones, though he developed them in more detail than most of his contemporaries. But Brahe gave particular emphasis to the immense waste space that the Copernican theory opened between the sphere of Saturn and the stars merely to account for the absence of observable parallactic motion. He himself had looked for parallax with his great new instruments. Since he found none, he felt forced to reject the earth's motion. The only alternative compatible with his observations would have required a distance between the stellar sphere and Saturn seven hundred times the distance between Saturn and the sun.

But Brahe was nothing if not a proficient astronomer. Though he rejected the earth's motion, he could not ignore the mathematical harmonies which the *De Revolutionibus* had introduced into astronomy. Those new harmonies did not convert him – they were not, for him, sufficiently strong evidence to counterbalance the difficulties inherent in the earth's motion – but they must at least have increased his discontent with the Ptolemaic system, and he rejected it, too, in favor of a third system of his own invention. Brahe's system, the "Tychonic," is shown in Figure 37. Once again the earth lies stationary at the geometric center of a stellar sphere whose daily rotation accounts for the diurnal circles of the stars. As in the Ptolemaic system, the sun, moon, and planets are carried westward daily with the stars by the outer sphere, and they have additional eastward orbital motions of their own. In the diagram these orbital motions are represented by circles, though in the full Tychonic system minor epicycles, eccentrics, and equants are also required. The circles of the moon and sun are centered on the earth; to this point the system is still Ptolemaic. But the centers of the five remaining planetary orbits are transferred from the center of the earth to the sun. Brahe's system is an extension, though perhaps not a conscious one, of Heraclides' system, which attributed sun-centered orbits to Mercury and Venus.

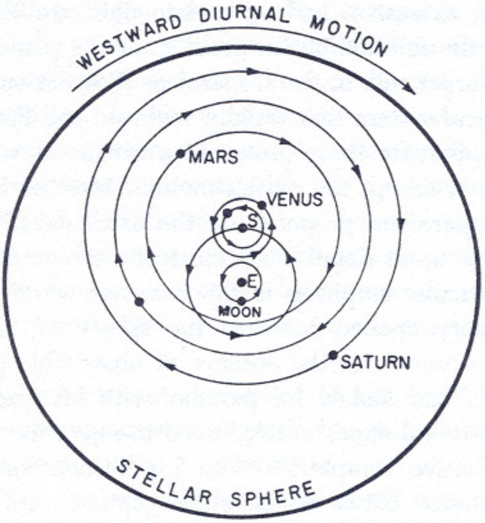

Figure 37. The Tychonic system. The earth is once again at the center of a rotating stellar sphere, and the moon and sun move in their old Ptolemaic orbits. The other planets are, however, fixed on epicycles whose common center is the sun.

The remarkable and historically significant feature of the Tychonic system is its adequacy as a compromise solution of the problems raised by the *De Revolutionibus.* Since the earth is stationary and at the center, all the main arguments against Copernicus' proposal vanish. Scripture, the laws of motion, and the absence of stellar parallax, all are reconciled by Brahe's proposal, and this reconciliation is effected without sacrificing any of Copernicus' major mathematical harmonies. The Tychonic system is, in fact, precisely equivalent mathematically to Copernicus' system. Distance determination, the apparent anomalies in the behavior of the inferior planets, these and the other new harmonies that convinced Copernicus of the earth's motion are all preserved.

The harmonies of the Tychonic system may be developed individually and in detail by the same techniques employed in discussing Copernicus' system, but for present purposes the following abbreviated demonstration of the mathematical equivalence of the Copernican and Tychonic systems should be sufficient. Imagine the sphere of the stars in Figure 37 immensely expanded until an observer on the moving sun could no longer observe any stellar parallax from opposite sides of the sun's orbit. This expansion does not affect the system's mathematical account of any of the planetary motions. Now imagine that within this expanded stellar sphere the various planets are driven about their orbits by a clockwork mechanism like that indicated schematically in Figure 38*a* for the earth, the sun, and Mars. In the diagram the sun is attached to the central earth by an arm of fixed length which carries it counterclockwise about the earth, and Mars is attached to the sun by another arm of fixed length which moves it clockwise about the moving sun. Since the lengths of both arms are fixed throughout the motion, the clockwork mechanism will produce just the circular orbits indicated in Figure 37.

Now imagine that, without interfering with the gears that drive the arms in Figure 38*a*, the whole mechanism is picked up and, with the arms turning as before, put down again with the sun fixed at the central position formerly held by the earth. This is the situation indicated in Figure 38*b*. The arms have the same lengths as before; they are driven at the same rates by the same mechanism; and they therefore retain the same *relative* positions at each instant of time. All of the geometric spatial relations of the earth, sun, and Mars in the diagram of Figure 38*a* are preserved by the arrangement of Figure 38*b*, and since only the fixed point of the mechanism has been changed, all the relative motions must be identical.

But the motions produced by the mechanism of Figure 38*b* are Copernican motions. That is, the fixed arms shown in the second diagram move both the earth and Mars in circular orbits about the sun, and those orbits are just the basic ones described by Copernicus. Carrying out the same argument with the hypothetical mechanism of Figure 38 elaborated to include all the planets, demonstrates that the equivalence is general. Omitting minor epicycles and eccentrics, which have no bearing on the harmonies of Copernicus' system, the Tychonic system is transformed to the Copernican system simply by holding the sun fixed instead of the earth. The relative motions of the planets are the same in both systems, and the harmonies are therefore preserved. Mathematically the only possible difference between the motions in the two systems is a parallactic motion of the stars, and that motion was eliminated at the start by expanding the stellar sphere until parallax was imperceptible.

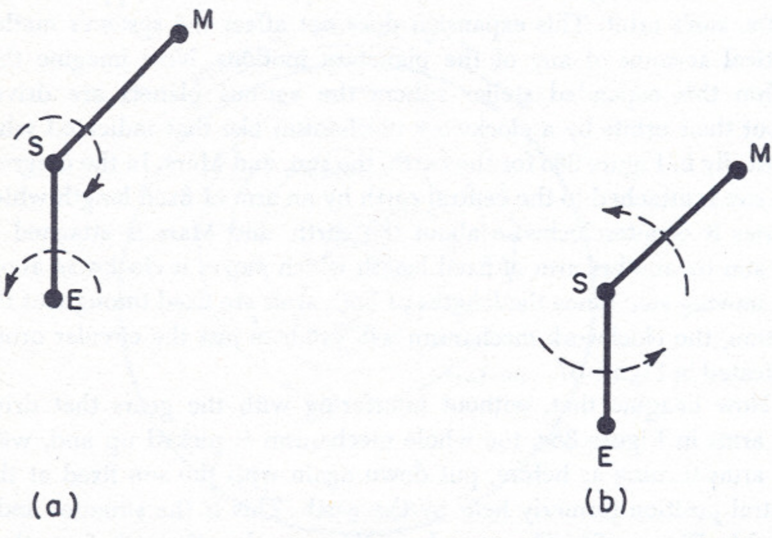

Figure 38. The geometrical equivalence of (*a*) the Tychonic and (*b*) the Copernican systems. In (*a*) the sun S is carried eastward about the stationary earth *E* by the the rigid arm *ES*. Simultaneously, the planet Mars, *M*, is carried westward about S by the steady rotation of the arm *SM*. Since *ES* rotates more rapidly than *SM*, the net motion of Mars is eastward except during the brief period when *SM* crosses over *ES*. In the second diagram (*b*) the same arms are shown rotating about the fixed sun S. The *relative positions* of *E*, S, and *M* are the same as those in (*a*), and they will stay the same while the arms in the two diagrams rotate. Notice particularly that in (*b*) the angle *ESM* must decrease as it does in (*a*) because *ES* rotates about the sun more rapidly than *SM*.

The Tychonic system has incongruities all its own: most of the planets are badly off center; the geometric center of the universe is no longer the center for most of the celestial motions; and it is difficult to imagine any physical mechanism that could produce planetary motions even approximately like Brahe's. Therefore the Tychonic system did not convert those few Neoplatonic astronomers, like Kepler, who had been attracted to Copernicus' system by its great symmetry. But it did convert most technically proficient non-Copernican astronomers of the day, because it provided an escape from a widely felt dilemma: it retained the mathematical advantages of Copernicus' system without the physical, cosmological, and theological drawbacks. That is the real importance of the Tychonic system. It was an almost perfect compromise, and in retrospect the system seems to owe its existence to the felt need for such a compromise. The Tychonic system, to which almost all the more erudite seventeenth-century Ptolemaic astronomers retreated, appears to be an immediate by-product of the *De Revolutionibus*.

Brahe himself would have denied this. He proclaimed that he had taken nothing in his system from Copernicus. But he can scarcely have been conscious of the pressures at work on him and his contemporaries. Certainly he knew both Ptolemaic and Copernican astronomy thoroughly before he thought of his own system, and he was clearly aware in advance of the predicament that his own system was to resolve. The immediate success of the system is one index of the strength and prevalence of the need. That two other astronomers disputed Brahe's priority and claimed to have worked out similar compromise solutions for themselves provides additional evidence for the role of the *De Revolutionibus* and the resulting climate of astronomical opinion in the genesis of the Tychonic system. Brahe and his system provide the first illustration of one of the major generalizations that closed the last chapter: the *De Revolutionibus* changed the state of astronomy by posing new problems for all astronomers.

Brahe's criticisms of Copernicus and his compromise solution of the problem of the planets show that, like most astronomers of his day, he was unable to break with traditional patterns of thought about the earth's motion. Among Copernicus' successors Brahe is one of the immense body of conservatives. But the effect of his work was not conservative. On the contrary, both his system and his observations forced his successors to repudiate important aspects of the Aristotelian-Ptolemaic universe and thus drove them gradually toward the Copernican camp. In the first place, Brahe's system helped to familiarize astronomers with the mathematical problems of Copernican astronomy, for geometrically the Tychonic and Copernican systems were identical. More important, Brahe's system, abetted by his observations of comets, to be discussed below, forced his followers to abandon the crystalline spheres which, in the past, had carried the planets about their orbits. In the Tychonic system, as indicated by Figure 37, the orbit of Mars intersects the orbit of the sun. Both Mars and the sun cannot, therefore, be embedded in spheres that carry them about, for the two spheres would have to penetrate and move through each other at all times. Similarly, the sun's sphere passes through the spheres of Mercury and Venus. Abandoning the crystalline spheres does not make a man a Copernican; Copernicus himself had utilized spheres to account for the planetary motions. But the spheres had, in one of a number of modifications, been an essential ingredient of the Aristotelian cosmological tradition which was the principal barrier to the success of Copernicanism. Any break with the tradition worked for the Copernicans, and the Tychonic system, for all its traditional elements, was an important break.

Brahe's skillful observations were even more important than his system in leading his contemporaries toward a new cosmology. They provided the essential basis for the work of Kepler, who converted Copernicus' innovation into the first really adequate solution of the problem of the planets. And even before they were used to revise Copernicus' system, the new data collected by Brahe suggested the necessity of another major departure from classical cosmology – they raised questions about the immutability of the heavens. Late in 1572, when Brahe was at the beginning of his career in astronomy, a new celestial body appeared in the constellation Cassiopeia, directly across the pole from the Big Dipper. When first observed it was very brilliant, as clear as Venus at its greatest brightness; during the next eighteen months the new occupant of the heavens grew gradually dimmer; and finally it vanished altogether early in 1574. From the start the new visitor drew the interest of scientists and nonscientists throughout Europe. It could not be a comet, the only sort of celestial apparition widely recognized by astronomers and astrologers, for it had no tail, and it always appeared in the same position against the sphere of the stars. Clearly it was a portent; astrological activity multiplied; and astronomers everywhere devoted their observations and their writings to the "new star" in the heavens.

The word "star" is the key to the astronomical and cosmological significance of the new phenomenon. If it were a star, then the immutable heavens had changed, and the basic contrast between the superlunary region and the corruptible earth was in question. If it were a star, the earth might more easily be conceived as a planet, for the transitory character of terrestrial affairs would now have been discovered in the heavens as well. Brahe and the best of his contemporaries did conclude that the visitor was a star. Observations like the one illustrated in Figure 39 indicated that it could not be located

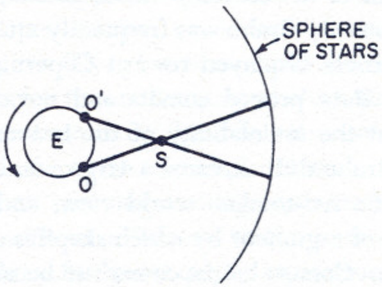

Figure 39. Diurnal parallax of a body below the stars. If S is between the earth and the sphere of the stars, then it should appear at different positions against the background of stars when observed by terrestrial observers at *O* and *O'*. Two observers are not required. The eastward rotation of the earth (or the equivalent westward rotation of the observed body and the stellar sphere) carries an observer from *O* to *O'* in six hours; as a result of the rotation the body S appears to change its position continually, returning to its starting point among the stars after twenty-four hours. If S were as close as the moon, its apparent displacement during six hours would be very nearly 1°. Bodies farther from the earth show less displacement.

With modern instruments the technique illustrated above is useful in determining the distances to the moon and planets, but naked-eye observations are not accurate enough for this application. The large size of the moon and its rapid orbital motion disguise the parallactic effect. The planets are too far away.

below the sphere of the moon or even close to the sublunary region. Probably it was among the stars, for it was observed to move with them. Another cause for cosmological upheaval had been discovered.

The sixteenth-century discovery of the mutability of the heavens might have been relatively ineffectual if the only evidence of superlunary change had been drawn from the new star, or nova, of 1572. It was a transient phenomenon; those who chose to reject Brahe's data could not be refuted; by the time the data were published the star had disappeared; and some less careful observers could always be discovered who had observed a parallax sufficient to place the nova below the moon. But fortunately additional and continuing evidence of superlunary change was provided by comets which Brahe observed carefully in 1577, 1580, 1585, 1590, 1593, and 1596. Once again no measurable parallax was observed, and the comets too were therefore located beyond the moon's sphere where they moved through the region formerly filled by the crystalline spheres.

Like the observations of the nova, Brahe's discussions of comets failed to convince all of his contemporaries. During the first decades of the seventeenth century Brahe was frequently attacked, occasionally with the same bitterness displayed toward Copernicus, by those who believed that other data proved comets and novas to be sublunary phenomena and that the inviolability of the heavens could therefore be preserved. But Brahe did convince a large number of astronomers of a basic flaw in the Aristotelian world view, and, more important, he provided a mode of argument by which skeptics could continuously check his conclusions. Comets bright enough to be seen with the naked eye appear every few years. After their superlunary character had been deduced from observation and then widely debated, the evidence that comets provided for the mutability of the heavens could not indefinitely be ignored or distorted. Once again the Copernicans were the gainers.

Somehow, in the century after Copernicus' death, all novelties of astronomical observation and theory, whether or not provided by Copernicans, turned themselves into evidence for the Copernican theory. That theory, we should say, was proving its fruitfulness. But, at least in the case of comets and novas, the proof is very strange, for the observations of comets and novas have nothing whatsoever to do with the earth's motion. They could have been made and interpreted by a Ptolemaic astronomer just as readily as by a Copernican. They are not, in any direct sense, by-products of the *De Revolutionibus*, as the Tychonic system was.

But neither can they be quite independent of the *De Revolutionibus* or at least of the climate of opinion within which it was created. Comets had been seen frequently before the last decades of the sixteenth century. New stars, though they appear less frequently to the naked eye than comets, must also have been occasionally accessible to observers before Brahe's time; one more appeared in the year before his death and a third in 1604. Even Brahe's fine instruments were not required to discover the superlunary character of novas and comets; a parallactic shift of 1° could have been measured without those instruments, and a number of Brahe's contemporaries did independently conclude that comets were superlunary using instruments that had been known for centuries. The Copernican Maestlin needed only a piece of thread to decide that the nova of 1572 was beyond the moon. In short, the observations with which Brahe and his contemporaries speeded the downfall of traditional cosmology and the rise of Copernicanism could have been made at any time since remote antiquity. The phenomena and the requisite instruments had been available for two millenniums before Brahe's birth, but the observations were not made or, if made, were not widely interpreted. During the last half of the sixteenth century age-old phenomena rapidly changed their meaning and significance. Those changes seem incomprehensible without reference to the new climate of scientific thought, one of whose first outstanding representatives is Copernicus. As suggested at the end of the last chapter, the *De Revolutionibus* marked a turning point, and there was to be no turning back.

Johannes Kepler

Brahe's work indicates that after 1543 even the opponents of Copernicanism, at least the ablest and most honest ones, could scarcely help promoting major reforms in astronomy and cosmology. Whether or not they agreed with Copernicus, he had changed their field. But the work of an anti-Copernican like Brahe does not show the extent of those changes. A better index of the novel problems that accrued to astronomy after Copernicus' death is provided by the research of Brahe's most famous colleague, Johannes Kepler (1571–1630). Kepler was a lifelong Copernican. He seems first to have been converted to the system by Maestlin when he was a student at the Protestant university of Tübingen, and his faith in it never wavered after his student days. Throughout his life he referred in the rhapsodic tones characteristic of Renaissance Neoplatonism to the suitability of the role that Copernicus had attributed to the sun. His first important book, the *Cosmographical Mystery*, published in 1596, opened with a lengthy defense of the Copernican system, emphasizing all those arguments from harmony that we discussed in Chapter 5 and adding many new ones besides: Copernicus' proposal explains why

Mars's epicycle had been so much larger than Jupiter's and Jupiter's than Saturn's; sun-centered astronomy shows why, of all the celestial wanderers, only the sun and moon fail to retrogress; and so on and on. Kepler's arguments are the same as Copernicus', though more numerous, but Kepler, in contrast to Copernicus, develops the arguments at length and with detailed diagrams. For the first time the full force of the mathematical arguments for the new astronomy was demonstrated.

But though Kepler was full of praise for the conception of a sun-centered planetary system, he was quite critical of the particular mathematical system that Copernicus had developed. Again and again Kepler's writings emphasized that Copernicus had never recognized his own riches and that after the first bold step, the transposition of the sun and earth, he had stayed too close to Ptolemy in developing the details of his system. Kepler was acutely and uncomfortably aware of the incongruous archaic residues in the *De Revolutionibus,* and he took it upon himself to eliminate them by exploiting fully the earth's new status as a planet governed, like the other planets, by the sun.

Copernicus had not quite succeeded in treating the earth as just another planet in a sun-centered system. Unlike the qualitative sketch in the First Book of the *De Revolutionibus*, the mathematical account of the planetary system developed in the later books attributed several special functions to the earth. For example, in the Ptolemaic system the planes of all planetary orbits had been constructed so that they intersected at the center of the earth, and Copernicus preserved this terrestrial function in a new form by drawing all orbital planes so that they intersected at the center of the earth's orbit. Kepler insisted that, since the sun governed the planets and the earth had no unique status, the planes of the orbits must intersect in the sun. By redesigning the Copernican system accordingly he made the first significant progress since Ptolemy in accounting for the north and south deviations of the planets from the ecliptic. Kepler had improved Copernicus' mathematical system by applying strict Copernicanism to it.

A similar insistence upon the parity of the planets enabled Kepler to eliminate a number of pseudo problems that had distorted Copernicus' work. Copernicus had, for example, believed that the eccentricities of Mercury and Venus were slowly changing, and he had added circles to his system to account for the variation. Kepler showed that the apparent change was due only to an inconsistency in Copernicus' definition of eccentricity. In the *De Revolutionibus* the eccentricity of the earth's orbit was measured from the sun (it is the distance SO_E in Figure 34*a*, p. 169) while the eccentricities of all other orbits were measured from the center of the earth's orbit (Mars's eccentricity is O_EO_M in Figure 34*b*). Kepler insisted that all planetary eccentricities must, in a Copernican universe, be computed in the same way and from the sun. When the new method was incorporated in his system, several of the apparent variations of eccentricity vanished, and the number of circles required in computation was reduced.

Each of these examples shows Kepler striving to adapt Copernicus' overly Ptolemaic mathematical techniques to the Copernican vision of a sun-dominated universe, and it was by continuing this effort that Kepler finally resolved the problem of the planets, transforming Copernicus' cumbersome system into a supremely simple and accurate technique for computing planetary position. His most essential discoveries were made while studying the motion of Mars, a planet whose eccentric orbit and proximity to the earth produce irregularities that had always challenged the ingenuity of mathematical astronomers. Ptolemy had been unable to account for its motion as satisfactorily as for that of the other planets, and Copernicus had not improved on Ptolemy. Brahe had attempted a new solution, undertaking a long series of observations specially for the purpose, but surrendering the problem as he encountered its full difficulties. Kepler, who had worked with Brahe during the last years of Brahe's life, inherited the new observations and, in the years after Brahe's death, took up the problem himself.

It was an immense labor which occupied much of Kepler's time for almost ten years. Two orbits had to be worked out: the orbit of Mars itself and the orbit of the earth from which Mars is observed. Again and again Kepler was forced to change the combination of circles used in computing these orbits. System after system was tried and rejected because it failed to conform to Brahe's brilliant observations. All of the intermediate solutions were better than the systems of Ptolemy and of Copernicus; some gave errors no larger than 8′ of arc, well within the limits of ancient observation. Most of the systems that Kepler discarded would have satisfied all earlier mathematical astronomers. But they had lived before Brahe, whose data were accurate to 4′ of arc. To us, Kepler said, Divine goodness has given a most diligent observer in Tycho Brahe, and it is therefore right that we should with a grateful mind make use of this gift to find the true celestial motions.

A long series of unsuccessful trials forced Kepler to conclude that no system based upon compounded circles would solve the problem. Some other geometric figure must, he thought, contain the key. He tried various sorts of ovals, but none eliminated the discrepancies between his tentative theory and observation. Then, by chance, he noticed that the discrepancies themselves varied in a familiar mathematical fashion, and investigating this regularity he discovered that theory and observation could be reconciled if the planets moved in elliptical orbits with variable speeds governed by a simple law which he also specified. These are the results that Kepler announced in *On the Motion of Mars*, first published at Prague in 1609. A mathematical technique simpler than any employed since Apollonius and Hipparchus yielded predictions far more accurate than any that had ever been made before. The problem of the planets had at last been solved, and it was solved in a Copernican universe.

The two laws that constitute Kepler's (and our) final solution of the problem of the planets are described in detail in Figure 40. The planets move in simple elliptical paths, and the sun occupies one of the two foci of each elliptical orbit – that is Kepler's First Law. His Second Law follows immediately, completing the description embodied in the First – the orbital speed of each planet varies in such a way that a line joining the planet to the sun sweeps through equal areas of the ellipse in equal intervals of time. When ellipses are substituted for the basic circular orbits common to Ptolemy's and Copernicus' astronomy and when the law of equal areas is substituted for the law of uniform motion about a point at or near the center, all need for eccentrics, epicycles, equants, and other *ad hoc* devices vanishes. For the first time a single uncompounded geometric curve and a single speed law are sufficient for predictions of planetary position, and for the first time the predictions are as accurate as the observations.

The Copernican astronomical system inherited by modern science is, therefore, a joint product of Kepler and Copernicus. Kepler's system of six ellipses made sun-centered astronomy work, displaying simultaneously the economy and the fruitfulness implicit in Copernicus' innovation. We must try to discover what was required for this transition of the Copernican system to its modern, Keplerian, form. Two of the prerequisites of Kepler's work are already apparent. He had to be a convinced Copernican, a man who would begin his search for more adequate orbits by treating the earth as a mere planet and who would construct the planes of all planetary orbits through the center of the sun. In addition, he needed Brahe's data. The data used by Copernicus and his European predecessors were too infected with errors to be

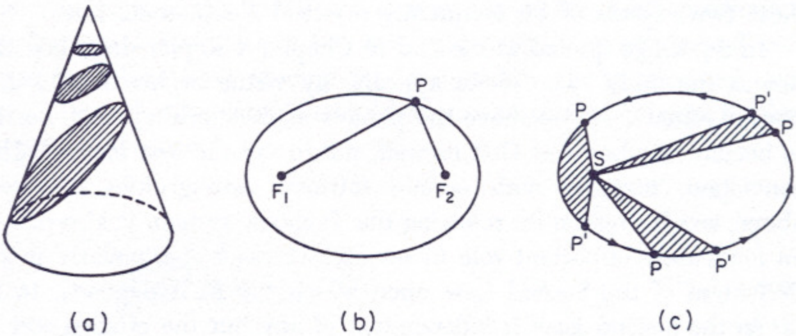

Figure 40. Kepler's first two Laws. Diagrams (*a*) and (*b*) define the ellipse, the geometric curve in which all planets that obey Kepler's First Law must move. In (*a*) the ellipse is shown as the closed curve in which a plane intersects a circular cone. When the plane is perpendicular to the axis of the cone, the intersection is a circle, a special case of the ellipse. As the plane is tilted, the curve of intersection is elongated into more typically elliptical patterns.

A more modern and somewhat more useful definition of the ellipse is given in diagram (*b*). If two ends of a slack string are attached to two points F_1 and F_2 in a plane, and if a pencil P is inserted into the slack and then moved so that it just keeps the string taut at all times, the point of the pencil will generate an ellipse. Changing the length of the string or moving the foci F_1 and F_2 together or apart alters the shape of the ellipse in the same way as a change in the tilt of the plane in diagram (*a*). Most planetary orbits are very nearly circular, and the foci of the corresponding ellipses are therefore quite close together.

Diagram (*c*) illustrates Kepler's Second Law, which governs orbital speed. The sun is at one focus of the ellipse, as required by the First Law, and its center is joined by straight lines to a number of planetary positions P and P', arranged so that each of the three shaded sectors SPP' has the same area. The Second Law states that, since each of these areas is the same, the planet must move through each of the corresponding arcs PP' in equal times. When near the sun, the planet must move relatively quickly so that the short line SP will sweep out the same area per unit time as is swept out by the longer line SP when the planet is moving more slowly farther from the sun.

explained by any set of simple orbits, and even if freed from error they would not have sufficed. Observations less precise than Brahe's could have been explained, as Kepler himself showed, by a classical system of compounded circles. The process by which Kepler arrived at his famous Laws depends, however, upon more than the availability of accurate data and a prior commitment to the planetary earth. Kepler was an ardent Neoplatonist. He believed that mathematically simple laws are the basis of all natural phenomena and that the sun is the physical cause of all celestial motions. Both his most lasting and his most evanescent contributions to astronomy display these two aspects of his frequently mystical Neoplatonic faith.

In a passage quoted at the end of Chapter 4 Kepler described the sun as the body "who alone appears, by virtue of his dignity and power, suited . . . [to move the planets in their orbits], and worthy to become the home of God himself, not to say the first mover." This conviction, together with certain intrinsic incongruities discussed above, was his reason for rejecting the Tychonic system. It also played an immensely important role in his own research, particularly in his derivation of the Second Law upon which the First depends. In its origin the Second Law is independent of any but the crudest sort of observation. It arises rather from Kepler's physical intuition that the planets are pushed around their orbits by rays of a moving force, the *anima motrix*, which emanates from the sun. These rays must, Kepler believed, be restricted to the plane of the ecliptic, in or near which all the planets moved. Therefore the number of rays that impinged on a planet and the corresponding force that drove the planet around the sun would decrease as the distance between the planet and the sun increased. At twice the distance from the sun half as many rays of the *anima motrix* would fall on a planet (Figure 41*a*), and the velocity of the planet in its orbit would, in consequence, be half of its orbital velocity at its original distance from the sun. A planet, P, moving about the sun, S, on an eccentric circle (Figure 41*b*) or some other closed curve must move at a speed inversely proportional to SP. The speed will be greatest when the planet is at the perihelion, p, closest to the sun, and least at the aphelion, a, where the planet is farthest from the sun. As the planet moves around the orbit, its speed will vary continually between these extremes.

Long before he began to work on elliptical orbits or stated the law of areas in its familiar modern form, Kepler had worked out this inverse-distance speed law to replace both the ancient law of uniform circular motion and the Ptolemaic variant which permitted uniform motion with respect to an equant point. This early speed law was very much "pulled from a hat" by a strange intuition – one that was rapidly discarded by his successors – of the forces that must govern a sun-dominated universe. Furthermore, this early form is not quite correct. The later law of areas, Kepler's so-called Second Law, is not quite equivalent to the inverse-distance law, and the law of areas gives somewhat better results. But when used to compute planetary position the two forms of the speed law lead to almost the same predictions. Kepler mistakenly thought the two equivalent in principle and used them interchangeably throughout his life. For all its visionary overtones the early Neoplatonic speed law proved fundamental in Kepler's most fruitful research.

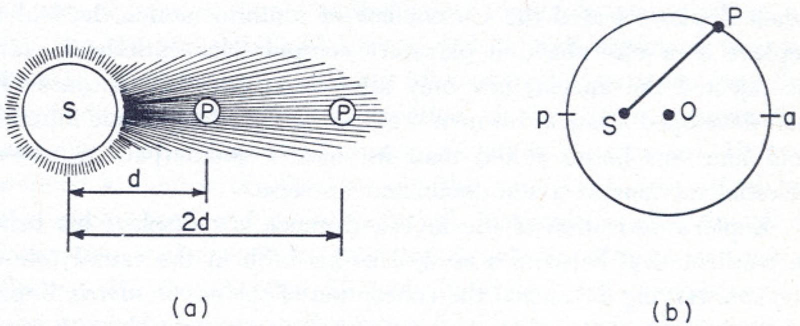

Figure 41. Kepler's earliest speed law. Diagram (*a*), which shows typical rays of the *anima motrix* radiating from the sun, illustrates the physical theory from which Kepler derived the law. Diagram (*b*) shows how the law could be applied to a planet moving on an eccentric circle.

Unlike his derivation of the speed law, Kepler's work on elliptical orbits was completely dependent upon the most painstaking and exhaustive study of the best available astronomical observations. Trial orbit after trial orbit had to be abandoned because, after laborious computation, it did not quite match Brahe's data. Kepler's scrupulous attempt to fit his orbits to objective data is often cited as an early example of the scientific method at its best. Yet even the law of elliptical orbits, Kepler's First Law, was not derived from observation and computation alone. Unless the planetary orbits are assumed to be precisely reëntrant (as they were after Kepler's work but not before), a speed law is required to compute orbital shape from naked-eye data. When analyzing Brahe's observations, Kepler made constant use of his earlier Neoplatonic guess.

The interrelation of orbit, speed law, and observation was obscured in our earlier discussions of astronomical theory, because ancient and medieval astronomers chose a simple speed law in advance. Before Kepler astronomers assumed that each of the compounded circles which moved a planet around its orbit must rotate uniformly with respect to a point at or near its center. Without some such assumption they could not have begun the elaboration of orbits to fit observations, for in the absence of a speed law the specification of an orbit tells little or nothing about where a planet will appear among the stars at a particular time. Neither speed law nor orbit can be independently derived from or checked against observation. Therefore, when Kepler rejected the ancient law of uniform motion, he had to replace it or else abandon planetary computations entirely. In fact, he rejected the ancient law only after (and probably because) he had developed a law of his own – a law that his Neoplatonic intuition told him was better suited than its ancient counterpart to govern celestial motions in a sun-dominated universe.

Kepler's derivation of the inverse-distance law displays his belief in mathematical harmonies as well as his faith in the causal role of the sun. Having developed the conception of the *anima motrix* Kepler insisted that it must operate in the simplest way compatible with crude observation. He knew, for example, that planets move fastest at perihelion, but he had few other data, none of them quantitative, on which to base an inverse-distance law. But Kepler's belief in number harmonies and the role of this belief in his work is more forcefully exhibited in another one of the laws that modern astronomy inherits from him. This is Kepler's so-called Third Law, announced during 1619 in the *Harmonies of the World.*

The Third Law was a new sort of astronomical law. Like their ancient and medieval counterparts the First and Second Laws govern only the motions of individual planets in their individual orbits. The Third Law, in contrast, established a relation between the speeds of planets in different orbits. It states that if T_1 and T_2 are the periods that two planets require to complete their respective orbits once, and if R_1 and R_2 are the average distances between the corresponding planets and the sun, then the ratio of the squares of the orbital periods is equal to the ratio of the cubes of the average distances from the sun, or $(T_1 / T_2)^2 = (R_1 / R_2)^3$. This is a fascinating law, for it points to a regularity never before perceived in the planetary system. But, at least in Kepler's day, that was all it did. The Third Law did not, in itself, change the theory of the planets, and it did not permit astronomers to compute any quantities that were previously unknown. The sizes and the periods associated with each planetary orbit were available in advance.

But though it had little immediate practical use, the Third Law is just the sort of law that most fascinated Kepler throughout his career. He was a mathematical Neoplatonist or Neopythagorean who believed that all of nature exemplified simple mathematical regularities which it was the scientists' task to discover. To Kepler and others of his turn of mind a simple mathematical regularity was itself an explanation. To him the Third Law in and of itself explained why the planetary orbits had been laid out by God in the particular way that they had, and that sort of explanation, derived from mathematical harmony, is what Kepler continually sought in the heavens. He propounded a number of other laws of the same kind, laws which we have since abandoned because, though harmonious, they do not fit observation well enough to seem significant. But Kepler was not so selective. He thought that he had discovered and demonstrated a large number of these mathematical regularities, and they were his favorite astronomical laws.

In Kepler's first major work, the *Cosmographical Mystery*, he argued that both the number of the planets and the size of their orbits could be understood in terms of the relation between the planetary spheres and the five regular or "cosmic" solids. These are the solids shown in Figure 42*a*, and they have the unique characteristic that all of the faces of each solid are identical and that only equilateral figures are used for faces. It had been shown in antiquity that there could be only five such solids: cube, tetrahedron, dodecahedron, icosahedron, and octahedron. Kepler proclaimed that if the sphere of Saturn were circumscribed about the cube within which Jupiter's sphere was inscribed, and if the tetrahedron were placed just inside Jupiter's sphere with Mars's sphere inscribed in it, and so on for the three other solids and three other spheres, then the relative dimensions of all the spheres would be just those that Copernicus had determined by measurement. The construction is shown in Figure 42*b*. If it is to be used, there can be only six planets, corresponding to the five regular solids, and when it is used the permissible relative dimensions of the

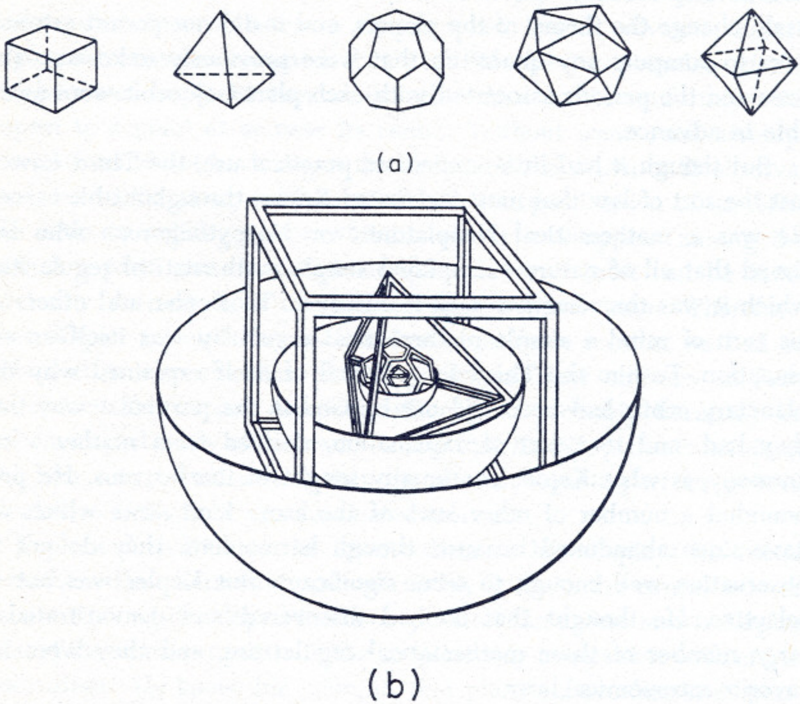

Figure 42. Kepler's application of the five regular solids. Diagram (*a*) shows the solids themselves. From left to right they are: cube, tetrahedron, dodecahedron, icosahedron, and octahedron. Their order is the one that Kepler developed to account for the sizes of the planetary spheres. Diagram (*b*) shows the solids in this application. Saturn's sphere is circumscribed about the cube, and Jupiter's sphere is inscribed in it. The tetrahedron is inscribed in Jupiter's sphere, and so on.

planetary spheres are determined. That, said Kepler, is why there are only six planets and why they are arranged as they are. God's nature is mathematical.

Kepler's use of the regular solids was not simply a youthful extravagance, or if it was, he never grew up. A modified form of the same law appeared twenty years later in his *Harmonies of the World*, the same book that propounded the Third Law. Also in that book Kepler elaborated a new set of Neoplatonic regularities which related the maximum and minimum orbital speeds of the planets to the concordant intervals of the musical scale. Today this intense faith in number harmonies seems strange, but that is at least partly because today scientists are prepared to find their harmonies more abstruse. Kepler's application of the faith in harmonies may seem naïve, but the faith itself is not essentially different from that motivating bits of the best contemporary research. Certainly the scientific attitude demonstrated in those of Kepler's "laws" which we have now discarded is not distinguishable from the attitude which drove him to the three Laws that we now retain. Both sets, the "laws" and the Laws, arise from the same renewed faith in the existence of mathematical harmony that had so large a role in driving Copernicus to break with the astronomical tradition and in persuading him that the earth was, indeed, in motion. But in Kepler's work, and particularly in the parts of it that we have now discarded, the Neoplatonic drive to discover the hidden mathematical harmonies embedded in nature by the Divine Spirit are illustrated in a purer and more distinct form.

Galileo Galilei

Kepler solved the problem of the planets. Ultimately his version of Copernicus' proposal would almost certainly have converted all astronomers to Copernicanism, particularly after 1627 when Kepler issued the *Rudolphine Tables*, derived from his new theory and clearly superior to all the astronomical tables in use before. The story of the astronomical components of the Copernican Revolution might therefore end with the gradual acceptance of Kepler's work because that work contains all the elements required to make the Revolution in astronomy endure. But, in fact, the astronomical components of the story do not end there. In 1609 the Italian scientist Galileo Galilei (1564–1642) viewed the heavens through a telescope for the first time, and as a result contributed to astronomy the first qualitatively new sort of data that it had acquired since antiquity. Galileo's telescope changed the terms of the riddle that the heavens presented to astronomers, and it made the riddle vastly easier to solve, for in Galileo's hands the telescope disclosed countless evidences for Copernicanism. But Galileo's new statement of the riddle was not formulated until