# 4 Newton and the law of gravity

## 4.1 Introduction

Richard Westphal's monumental biography *Never at Rest* was the product of a lifetime's study of Isaac Newton's life and work. In the preface, he writes:

> The more I have studied him, the more Newton has receded from me. It has been my privilege at various times to know a number of brilliant men, men whom I acknowledge without hesitation to be my intellectual superiors. I have never, however, met one against whom I was unwilling to measure myself so that it seemed reasonable to say that I was half as able as the person in question, or a third or a fourth, but in every case a finite fraction. The end result of my study of Newton has served to convince me that with him there is no measure. He has become for me wholly other, one of the tiny handful of supreme geniuses who have shaped the categories of human intellect, a man not finally reducible to the criteria by which we comprehend our fellow beings.[1]

In the next paragraph, he writes:

> Had I known, when in youthful self-confidence I committed myself to the task, that I would end up in similar self-doubt, surely I would never have set out.[1]

Newton's impact upon science is so all pervasive that it is worthwhile filling in some of the background to his character and extraordinary achievements. The chronology which follows is that adopted in the Introduction of the volume *Let Newton Be*.[2]

## 4.2 Lincolnshire 1642–61

Newton was born in the hamlet of Woolsthorpe near Grantham on Christmas day 1642 (according to the old style Julian calendar). Newton's father was a successful farmer, who died three months before Newton's birth. When Isaac Newton was three years old, his mother married the Reverend Barnabas Smith and moved into his house, leaving Isaac behind to be brought up by his grandmother. Isaac hated his step-father, as is revealed by this entry in the list of sins he had committed up to the age of 19:

> Threatening my father and mother Smith to burn them and the house over them.[3]

It has been argued that this separation from his mother at an early age was a cause of his 'suspicious, neurotic, tortured personality'.[4] He was sent to the Free Grammar School at Grantham where he lodged with an apothecary. It was probably in these lodgings that

he was introduced to chemistry and alchemy, which remained a lifelong passion. Newton himself claimed that he invented toys and did experiments as a teenager. He is reported to have constructed a model of a mill powered by a mouse, clocks, 'lanthorns' and fiery kites, which he flew to frighten the neighbours. He was already a loner who did not get on particularly well with his schoolmates. Following his school education, it was decided that he should return to the Free Grammar School at Grantham to prepare for entry to his uncle's old college, Trinity College, Cambridge.

## 4.3 Cambridge 1661–5

To begin with, Newton was a 'subsidiser', meaning that he earned his keep by serving the fellows and rich students at Trinity College. He took courses in Aristotelian philosophy, logic, ethics and rhetoric. His notebooks record that he was reading other subjects privately, the works of Thomas Hobbes, Henry More and René Descartes, as well as those of Kepler and Galileo. It became apparent to him that he was weak in mathematics and so he started to work 'ferociously' at the subject. By the end of this period, he had mastered whole areas of mathematics. He also continued to carry out experiments in a wide range of different topics. In 1664, he was elected a scholar of Trinity College, ensuring him a further four years of study without domestic duties. In 1665, he took his B.A. degree, and then the effects of the Great Plague began to spread north to Cambridge. The University was closed and Newton returned to his home at Woolsthorpe.

## 4.4 Lincolnshire 1665–7

During the next two years, Newton's burst of creative scientific activity must be one of the most remarkable ever recorded. The only comparable feat of which I am aware is the achievement of Albert Einstein in 1905. Over 50 years later, Newton wrote:

> In the beginning of 1665, I found the method of approximating series and the rule for reducing any dignity (power) of any binomial into such a series. The same year in May, I found the method of tangents of Gregory and Slusius and in November had the direct method of fluxions and the next year in January had the theory of colours and in May following I had entrance into the inverse method of fluxions. And in the same year I began to think of gravity extending to the orbit of the Moon and (having found out how to estimate the force with which a globe revolving within a sphere presses the surface of the sphere) from Kepler's rule of the periodical times of the planets being in sesquialternate proportion* of their distance from the centres of their Orbs, I deduced that the forces which keep the planets in their Orbs must [be] reciprocally as the squares of their distances from the centres about which they revolve: and thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the Earth, and found them answer pretty nearly. All this was in the two plague years 1665–6. For in those days I was in the prime of my age for invention and minded mathematics and philosophy more than at any time since.[5]

* *Sesquialternate proportion* means 'to the power 3/2'.

It is a quite astonishing list – Newton had laid the foundations of three quite separate areas. In mathematics, he discovered the *binomial theorem* and the *differential* and *integral calculus*. In *optics*, he discovered the decomposition of light into its primary colours. He began his unification of *celestial mechanics* with the *theory of gravity*, which was to lead ultimately to his laws of motion and the theory of universal gravitation. Let us look into his achievements in the fields of optics and of the law of gravity in more detail.

### 4.4.1 Optics

Newton was a skilled experimenter and undertook a number of optical experiments using lenses and prisms while he was at Trinity College and at Woolsthorpe. Many investigators had noted that when white light is passed through a prism all the colours of the rainbow are produced. Descartes was the proponent of the generally accepted view that, when light passes through a prism, it is modified by the material of the prism and so becomes coloured.

In 1666, Newton, in his own words, 'applied myself to the grinding of Optick glasses of other figures than spherical' and among these he produced a triangular glass prism 'to try therewith the celebrated *Phaenomena of Colours*'. Sunlight was passed through a tiny hole onto the prism and, to Newton's surprise, the coloured light produced by the prism was of oblong form. This experiment led him to carry out his famous *experimentum crucis*, illustrated in Figs. 4.1(*a*), (*b*), the second picture showing Newton's own drawing of the experiment.

In this experiment, the coloured spectrum produced by the first prism was projected onto a plane board and a small hole was cut in the board to allow only one of the colours to pass through. A second prism was placed in the path of this second light beam, and Newton discovered that there was no further decomposition of this beam into more colours. The experiment was repeated for all the colours of the spectrum and consistently he found that there was no further splitting up of the colours, contrary to the expectations of Déscartes' theory. Newton concluded that:

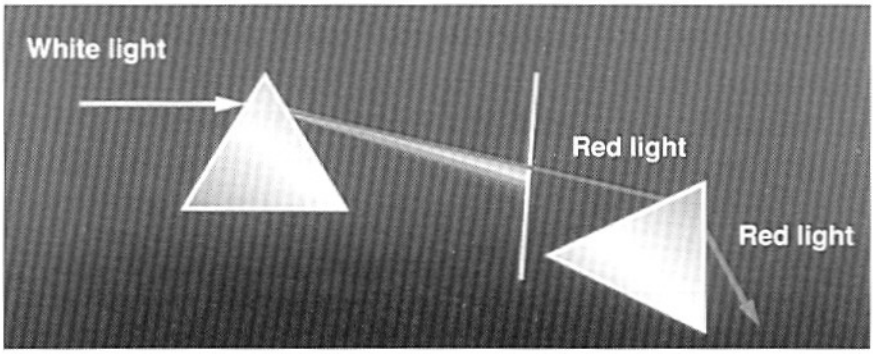
> Light itself is a Heterogeneous mixture of differently refrangible rays.[6]

The word *refrangible* is what we would now call *refracted*. Newton had established that white light is a superposition of all the colours of the rainbow and that, when white light is passed through a prism, rays of different colour are deflected by different amounts.

This work was not presented in public until 1672, in a paper to the Royal Society, and Newton immediately found himself in the thick of a hot dispute. In addition to describing his new results, he regarded them as evidence in favour of his view that light can be considered to be made up of 'corpuscles', that is, of particles which travel from the source of light to the eye. Newton did not have a satisfactory theory of 'light corpuscles', or why they should be refracted in different ways by material media. His problems were exacerbated by the fact that the *experimentum crucis* was difficult to reproduce. The ensuing unpleasant debate with Huyghens and Hooke obscured the key result that white light is composed of all the colours of the spectrum.

This work led him to another important conclusion – that it is not possible to build a large refracting telescope of the type pioneeered by Galileo, because white light is split

(*a*)

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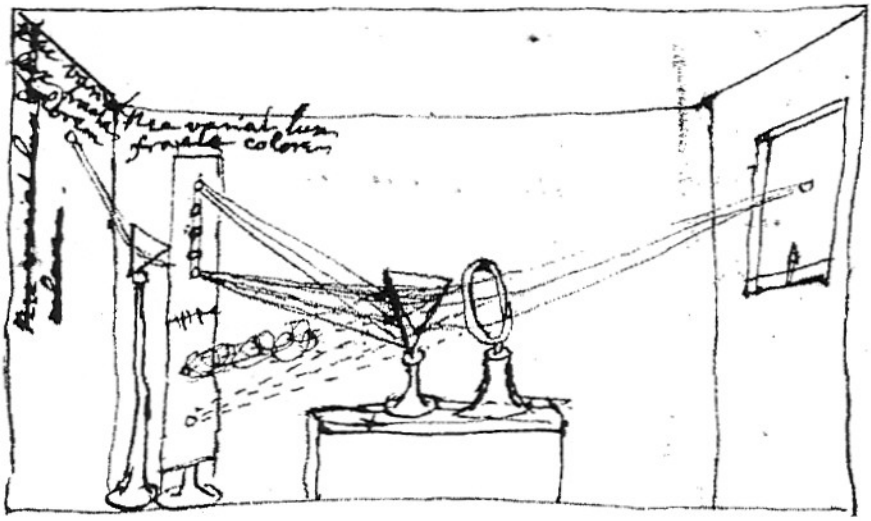
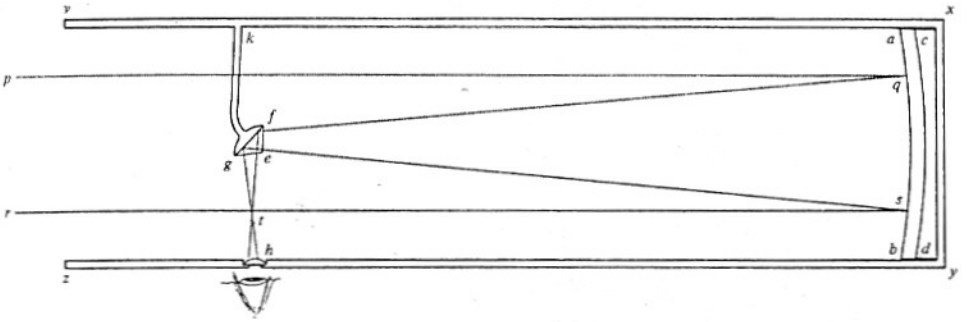


Figure 4.1: (*a*) A schematic diagram showing the decomposition of white light into the colours of the spectrum. On passing through the second prism, the colours are not split up into any further colours. (*b*) Newton's sketch of his *experimentum crucis*. (From C. Hakfoort, 1988, *Let Newton Be: a New Perspective on his Life and Works*, eds. J. Fauvel, R. Flood, M. Shortland, and R. Wilson, p. 87, Oxford: Oxford University Press. By kind permission of the Warden and Fellows, New College Oxford.)

up into its primary colours by a refracting lens and so different colours are focussed at different positions on the optical axis of the telescope, the phenomenon known as *chromatic aberration*. Because of this problem, Newton designed and built a new type of telescope, an *all-reflecting telescope*, which would not suffer from chromatic aberration. He ground the mirrors himself and constructed the mount for the telescope and mirrors. This configuration is now called a 'Newtonian' design and is shown in Fig. 4.2(*a*). Figure 4.2(*b*) shows a contemporary drawing of Newton's telescope as well as a comparison of the magnifying power of Newton's telescope with that of a refracting telescope, indicating the superiority of Newton's design. Nowadays, all large optical astronomical telescopes are reflectors, the descendants of Newton's invention.

(*a*)

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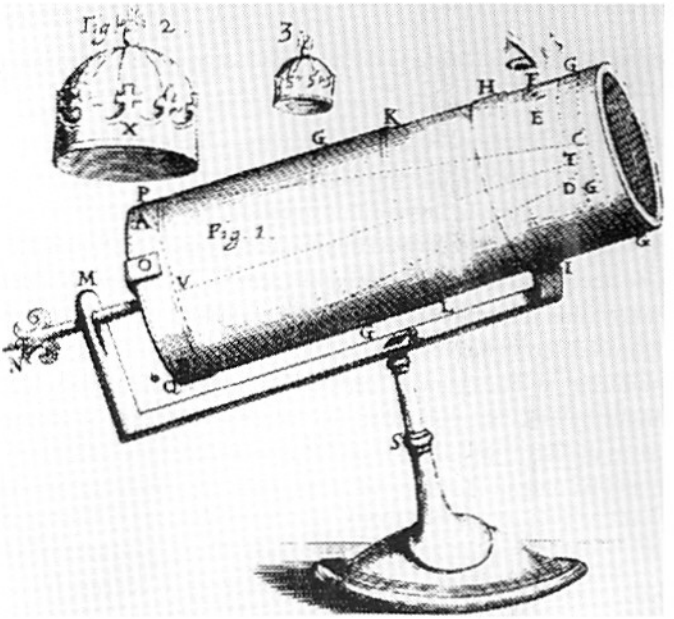


Figure 4.2: (*a*) The optical layout of Newton's reflecting telescope; *abcd* is the mirror made of speculum metal, *efg* is a prism and *h* is a lens. (From I.B. Cohen, 1970, *Dictionary of Scientific Biography*, Vol. 11, p. 57, New York: Charles Scribner's Sons © 1970–80. Reprinted by permission of the Gale Group.) (*b*) Newton's reflecting telescope, as illustrated in the *Philosophical Transactions of the Royal Society*, 1672. The two crowns show the improvement in magnification of Newton's telescope over the traditional refracting telescope, such as that used by Galileo. (From J. Fauvel, R. Flood, M. Shortland and R. Wilson eds., 1988, *Let Newton Be: a New Perspective on his Life and Works*, p. 16, Oxford: Oxford University Press.)

These were great experimental achievements, but Newton had an almost pathological dislike of writing up his work for publication. The systematic presentation of his optical work in his book *Opticks* was only published in 1704, long after his discoveries and inventions had been made.

### 4.4.2 The law of gravity

Newton's most famous achievement of his years at Woolsthorpe was the discovery of the law of gravity. The calculations carried out in 1665–6 were only the beginning of the story, but they contained in essence the theory which was to appear in its full glory in his *Principia Mathematica* of 1687. As Newton himself recounted, he was aware of Kepler's third law of planetary motion, which was deeply buried in Kepler's *Harmony of the World*. The volume was in the Trinity College library and it is likely that Isaac Barrow, the Lucasian Professor of Mathematics, drew Newton's attention to it.

In Newton's own words,

> the notion of gravitation [came to my mind] as I sat in contemplative mood [and] was occasioned by the fall of an apple.[7]

Newton asked whether the force of gravity, which makes the apple fall to the ground, is the same force which holds the Moon in its orbit about the Earth and the planets in their orbits about the Sun. To answer this question, he needed to know how the force of gravity varies with distance. He derived this relation from Kepler's third law by a simple argument. First, he needed an expression for *centripetal acceleration*, which he rederived for himself. Nowadays, this formula can be derived by a simple geometrical argument using vectors.

Figure 4.3 shows the velocity vectors of a particle moving at constant speed $|\boldsymbol{v}|$ in a circle with radius $|\boldsymbol{r}|$ at times $t$ and $t + \Delta t$. In the time $\Delta t$, the angle subtended by the radius vector $\boldsymbol{r}$ changes by $\Delta\theta$. The change in the velocity vector in $\Delta t$ is given by the vector triangle shown on the right. As $\Delta t \to 0$, the vector $\Delta\boldsymbol{v}$ continues to point towards the centre of the circle and its magnitude is $|\boldsymbol{v}|\Delta\theta$. Therefore, the centripetal acceleration is of magnitude

$$|\boldsymbol{a}| = \frac{|\Delta\boldsymbol{v}|}{\Delta t} = \frac{|\boldsymbol{v}|\Delta\theta}{\Delta t} = |\boldsymbol{v}|\omega = \frac{|\boldsymbol{v}|^2}{|\boldsymbol{r}|} \tag{4.1}$$

where $\omega$ is magnitude of the (constant) angular velocity of the particle. This is the famous expression for the *centripetal acceleration* $a$ of an object moving in a circle of radius $r$ at speed $v$.

In 1665, Newton was well aware of the fact that the orbits of the planets are actually ellipses, and not circles, but he assumed that it would be adequate to apply Kepler's third law to circular orbits, since the ellipticities of the orbits of the planets are generally very small.

He also knew that accelerations are proportional to the forces which cause them and so the force $f$ which keeps a planet in its circular orbit must be proportional to its centripetal acceleration $v^2/r$, that is,

$$f \propto \frac{v^2}{r}; \tag{4.2}$$

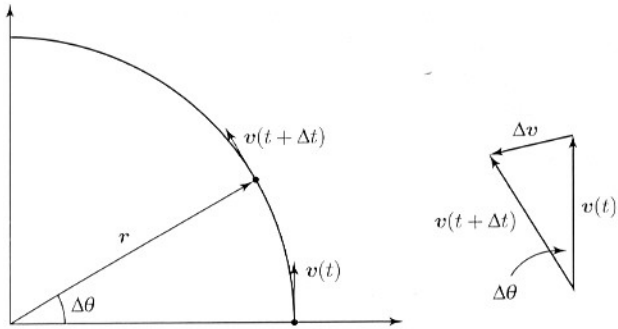


Figure 4.3: A vector diagram illustrating the origin of centripetal acceleration.

but Kepler's third law states that the period $T$ of the planet's orbit is proportional to $r^{3/2}$. The speed of a planet in its orbit is $v = 2\pi r/T$ and therefore

$$v \propto \frac{1}{r^{1/2}}. \tag{4.3}$$

Now, we can substitute for $v$ from (4.3) into (4.2) and so find

$$f \propto \frac{1}{r^2}. \tag{4.4}$$

This is the primitive form of Newton's *inverse square law of gravity* – the force of gravity decreases as the inverse square of the distance from the body.

This was the key result he was looking for – if gravity were truly universal then the force which causes apples to fall to the ground should be of exactly the same nature as the force which keeps the Moon in its orbit about the Earth. The only difference is that the acceleration of the apple should be greater than that of the Moon, because the Moon is 60 times further from the centre of the Earth than the apple. Thus, according to the universal theory of gravity, the centripetal acceleration of the Moon should be only $1/60^2 = 1/3600$ times the acceleration due to gravity on the surface of the Earth. Newton had all the data he needed to make this comparison. Using modern numbers, the acceleration due to gravity on the surface of the Earth is on average 9.80665 m s$^{-2}$. The period of revolution of the Moon about the Earth is 27.32 days and its mean distance from the Earth is $r = 384\,408\,000$ m. Putting these together, the mean speed of the Moon $v$ is 1023 m s$^{-1}$ and its centripetal acceleration $v^2/r$ is $2.72 \times 10^{-3}$ m s$^{-2}$. The ratio of this acceleration to the local acceleration due to gravity is $9.80665/2.72 \times 10^{-3} = 3600$, exactly as predicted.

Newton's calculations did not give quite such good agreement, but were sufficiently close to persuade him that the force which holds the Moon in its orbit about the Earth and the planets in their orbits about the Sun, is of exactly the same type as the force which causes the acceleration due to gravity on Earth. From this result, the general formula for the gravitational attraction between any two bodies of masses $M_1$ and $M_2$ follows:

$$f = \frac{GM_1M_2}{r^2}, \tag{4.5}$$

where $G$ is the gravitational constant and $r$ is the distance between the bodies. The force acts along the line joining the two bodies and is always an attractive force.

This work was not published, because there were a number of steps in the calculation which needed further elaboration.

(i) Kepler had shown that the orbits of the planets are ellipses and not circles – how did that affect the calculation?
(ii) Newton was uncertain about the influence of the other bodies in the Solar System upon each others' orbits.
(iii) He was unable to explain the details of the Moon's motion about the Earth, which, as we now know, is influenced by the fact that the Earth is not spherical.
(iv) Probably most important of all, there is a key assumption in the calculation, that all the mass of the Earth can be located at its centre in working out the acceleration due to gravity at its surface and its influence upon the Moon. The same assumption was made for all the bodies in the solar system. In his calculations of 1665–6, Newton regarded this step as an approximation. He was uncertain about its validity for objects close to the surface of the Earth.

Newton laid this work aside until 1679.

## 4.5 Cambridge 1667–96

The University reopened in 1667 and Newton returned to Trinity College in the summer of that year. He became a fellow of the college in the autumn of 1667 and two years later, at the age of 26, he was elected Lucasian Professor of Mathematics, a position which he held for the next 32 years. As Lucasian Professor, Newton's duties were not heavy. He was required to give at least one lecture per week in every term and to deposit his lectures, properly written up, in the University Library. In the period 1670 to 1672, he deposited lectures on optics, during 1673–83 lectures on arithmetic and algebra and in 1684–5 most of book I of what was to become the *Principia Mathematica*. In 1687, the lectures were entitled 'The system of the world' and this became part III of the *Principia*. There seems to be no record of his lectures for 1686, nor from 1688 until he left Cambridge in 1696.

His lectures were not particularly successful. His assistant Humphrey Newton, who was no relation, wrote of Newton during the years he was preparing the *Principia*:

> He seldom left his chamber except at term time, when he read [his lectures] in the [Old S]chools as being Lucasian Professor, where so few went to hear him, and fewer that understood him, that of times he did, in a manner, for want of hearers, read to the walls . . . [when he lectured he] usually stayed about half an hour; when he had no auditors, he commonly returned in a fourth part of that time or less.[8]

The first of Newton's publications appeared during this period. In late 1668, Nicholas Mercator published his book *Logarithmotechnica*, in which he described some of the techniques for the analysis of infinite series which Newton had already worked out in greater generality. Newton set about writing up his mathematical work so that his priority

could be established. He wrote hastily the work known as *De Analysi* (*On Analysis*), which Isaac Barrow was allowed to show to a London mathematician, Mr Collins. Newton insisted, however, that the work remain anonymous. The results were communicated by letter among British and continental mathematicians and so the work became widely known.

Newton incorporated the most important parts of *De Analysi* into another manuscript, *Methodus Fluxionum et Serierum Infinitarum*, the *Method of Fluxions and Infinite Series*, which was not published until long afterwards in 1711 by William Jones. The manuscript was, however, read by Leibniz in October 1676 when he was on a visit to London. Although Leibniz scholars have established that he only copied out the sections on infinite series, this incident was the source of the later bitter accusations that Leibniz had plagiarised Newton's discovery of the differential and integral calculus.

Newton was prompted to return to his researches on the laws of motion and gravity in 1679 by an interchange of letters with Robert Hooke. Hooke challenged Newton to work out the curve followed by a particle falling in an inverse-square field of force, the law Newton had derived 14 years earlier. This stimulated Newton to derive two crucial results. As noted by Chandrasekhar in his remarkable commentary on the *Principia*,[9]

> Newton's interest in dynamics was revived sufficiently for him to realise for the first time the real meaning of Kepler's law of areas. And as he wrote, 'I found now that whatsoever was the law of force which kept the Planets in their Orbs, the area described by a radius drawn from them to the Sun would be proportional to the times in which they were described'; and he proved the two propositions that
>
> > all bodies circulating about a centre sweep out areas proportional to the time
>
> and that
>
> > a body revolving in an ellipse ... the law of attraction directed to a focus of the ellipse ... is inversely as the square of the distance.

The first remarkable discovery, that Kepler's second law, the law of areas, is correct whatever the nature of the force law, so long as it is a central force, is now recognised as a consequence of the law of conservation of angular momentum. Chandrasekhar noted that: 'The resurrection of Kepler's law of areas in 1679 was a triumphant breakthrough from which the *Principia* was later to flow'.[9] At this point, another dispute broke out with Hooke, who claimed that he was the originator of the inverse square law of gravity. Newton violently rejected this claim and would not communicate any of the results of his calculations to Hooke, or to anyone else.

In 1684, Edmund Halley travelled to Cambridge to ask Newton precisely the same question which had been posed by Hooke. Newton's immediate response was that the orbit is an ellipse, but the proof could not be found among his papers. In November 1684, Newton sent the proof to Halley. Halley returned to Cambridge where he saw an incomplete manuscript of Newton's entitled *De Motu Corporum in Gyrum*, or *On the Motion of Revolving Bodies*, which was eventually to be transformed into the first part of the *Principia*. With a great deal of persuasion, Newton agreed to set about systematising all his researches on motion, mechanics, dynamics and gravity.

Only in 1685, when he was working at white heat on the preparation of what was to become his great treatise, the *Philosophiae Naturalis Principia Mathematica*, or the

*Principia* for short, did he demonstrate that, for a spherically symmetric body, the gravitational attraction can be found exactly by placing all its mass at its centre. In the words of J.W.L. Glaisher, on the occasion of the bicentenary of the publication of the *Principia*,

> No sooner had Newton proved this superb theorem – and we know from his own words that he had no expectation of so beautiful a result till it emerged from his mathematical investigation – then all the mechanism of the universe at once lay spread before him. . . . We can imagine the effect of this sudden transition from approximation to exactitude in stimulating Newton's mind to still greater effort.[10]

Nowadays, this result is proved in a few lines using Gauss's theorem in vector calculus as applied to the inverse square law of gravity, but these techniques were not available to Newton. It is not generally appreciated what a crucial step this calculation was in the development of the *Principia*.

Humphrey Newton gives us a picture of Newton during the writing of the *Principia*.

> (He) ate sparing (and often) forgot to eat at all. (He rarely dined) in the hall, except on some public days (when he would appear) with shoes down at the heel, stockings untied, surplice on, and his head scarcely combed. (He) seldom went to the Chapel (but very often) went to St. Mary's Church, especially in the forenoon.[11]

The *Principia* is one of the greatest intellectual achievements of all time. The theory of statics, mechanics and dynamics, as well as the law of gravity, are developed entirely through mathematical relations and mechanistic interpretations of the physical origins of the forces are excluded. In the very beginning, what we now call *Newton's laws of motion* are set out in their definitive form – we will return to these in more detail in Chapter 7. Despite the fact that Newton had developed his own version of the integral and differential calculus, the *Principia* is written entirely in terms of geometrical arguments. Often these are difficult to follow for the modern reader, largely because the geometrical arguments used are no longer familiar to physicists. Chandrasekhar's remarkable reconstruction of the arguments in modern geometrical terms gives some impression of the methods which Newton must have used. As an example of the economy of expression used by Newton in the *Principia*, Fig. 4.4 shows a translation of the proof of the elliptical orbits under the influence of an inverse square field of force. Comparison with Chandrasekhar's much lengthier derivation shows how terse Newton's geometric proofs were.

## 4.6 Newton the alchemist

Throughout his time at Cambridge, Newton maintained a profound interest in alchemy and in the interpretation of ancient and biblical texts. These aspects of his work tend to be regarded, at best, as unfortunate aberrations, and yet they were of the greatest significance for Newton.

Newton studied alchemy with the same seriousness he devoted to his mathematics and physics. Whereas his great contributions to the latter disciplines became public knowledge, his alchemy remained very private, his papers containing over a million words on alchemical topics. From the late 1660s to the mid 1690s, he carried out extensive chemical experiments

PROPOSITION XI. PROBLEM VI

*If a body revolves in an ellipse; it is required to find the law of the centripetal force tending to the focus of the ellipse.*

Let S be the focus of the ellipse. Draw SP cutting the diameter DK of the ellipse in E, and the ordinate Q$v$ in $x$; and complete the parallelogram Q$x$PR. It is evident that EP is equal to the greater semiaxis AC: for drawing HI from the other focus H of the ellipse parallel to EC, because CS, CH are equal, ES, EI will be also equal; so that EP is the half-sum of PS, PI that is (because of the parallels HI, PR, and the equal angles IPR, HPZ), of PS, PH, which taken together are equal to the whole axis 2AC. Draw QT perpendicular to SP, and putting L for the principal latus rectum of the ellipse (or for $\frac{2BC^2}{AC}$), we shall have

$$L \cdot QR : L \cdot Pv = QR : Pv = PE : PC = AC : PC,$$

$$\text{also, } L \cdot Pv : Gv \cdot Pv = L : Gv, \text{ and, } Gv \cdot Pv : Qv^2 = PC^2 : CD^2.$$

By Cor. II, Lem. VII, when the points $P$ and $Q$ coincide, $Qv^2 = Qx^2$, and $Qx^2$ or $Qv^2 : QT^2 = EP^2 : PF^2 = CA^2 : PF^2$, and (by Lem. XII) $= CD^2 : CB^2$. Multiplying together corresponding terms of the four proportions, and simplifying, we shall have

$$L \cdot QR : QT^2 = AC \cdot L \cdot PC^2 \cdot CD^2 : PC \cdot Gv\, CD^2 \cdot CB^2 = 2PC : Gv,$$

since $AC \cdot L = 2BC^2$. But the points Q and P coinciding, 2PC and G$v$ are equal. And therefore the quantities $L \cdot QR$ and $QT^2$, proportional to these, will be also equal. Let those equals be multiplied by $\frac{SP^2}{QR}$, and $L \cdot SP^2$ will become equal to $\frac{SP^2 \cdot QT^2}{QR}$. And therefore (by Cor. I and V, Prop. VI) the centripetal force is inversely as $L \cdot SP^2$, that is, inversely as the square of the distance SP.Q.E.I.

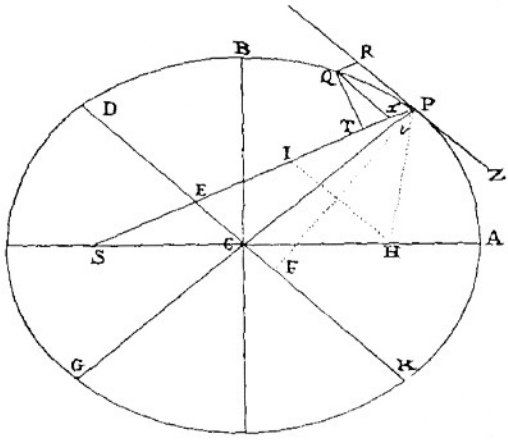


Figure 4.4: A translation from the Latin of Newton's proof that the orbits of the planets are ellipses under the influence of an inverse square law field of gravity. (From J. Roche, 1988, *Let Newton Be: a New Perspective on his Life and Works*, eds. J. Fauvel, R. Flood, M. Shortland and R. Wilson, p. 55, Oxford: Oxford University Press. Reprinted by permission of Oxford University Press.)

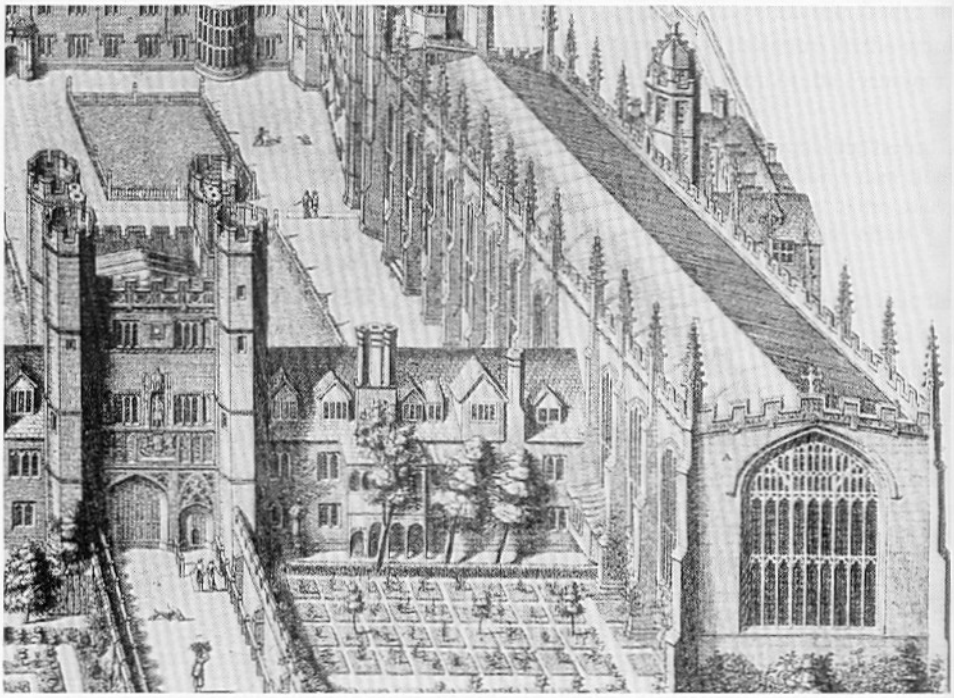

Figure 4.5: A seventeenth century engraving of the Great Gate and Chapel of Trinity College. The small shed seen in the bay of the chapel closest to the wall of Great Court may have been Newton's chemical laboratory. (Engraving by David Loggan, from his *Cantabrigia illustrated*, Cambridge 1690. Reprinted in J. Golinski, 1988, *Let Newton Be: a New Perspective on his Life and Works*, eds. J. Fauvel, R. Flood, M. Shortland, and R. Wilson, p. 152, Oxford: Oxford University Press. Courtesy of the Society of Antiquaries of London.)

in his laboratory at Trinity College. Figure 4.5 is a contemporary engraving of Trinity College; Newton's rooms were on the first floor to the right of the Gatehouse. It has been suggested that the shed which can be seen leaning against the Chapel outside the College was Newton's laboratory. He taught himself all the basic alchemical operations, including the construction of his own furnaces. Humphrey Newton, his assistant from 1685 to 1690, wrote:

> About six weeks at spring and six at the fall, the fire in the laboratory scarcely went out . . . what his aim might be, I was not able to penetrate into . . . Nothing extraordinary, as I can remember, happened in making his experiments, which if there did . . . I could not in the least discern it.[12]

The two Newtons kept the furnaces going continuously for six weeks at a time, taking turns to tend them overnight.

From the late 1660s to the mid 1690s, Newton devoted a huge effort to systematising everything he had read on chemistry and alchemy (Fig. 4.6). His first attempt to put some order into his understanding was included in his *Index Chemicus*, or *Chemical Dictionary*, of the late 1660s; it was followed over the next 25 years by successive versions. According to Golinski,[13] the final index cites more than 100 authors and 150 works. There are over 5000 page references under 900 separate headings. In addition, Newton devoted a huge effort to deciphering obscure allegorical descriptions of alchemical processes. It was part

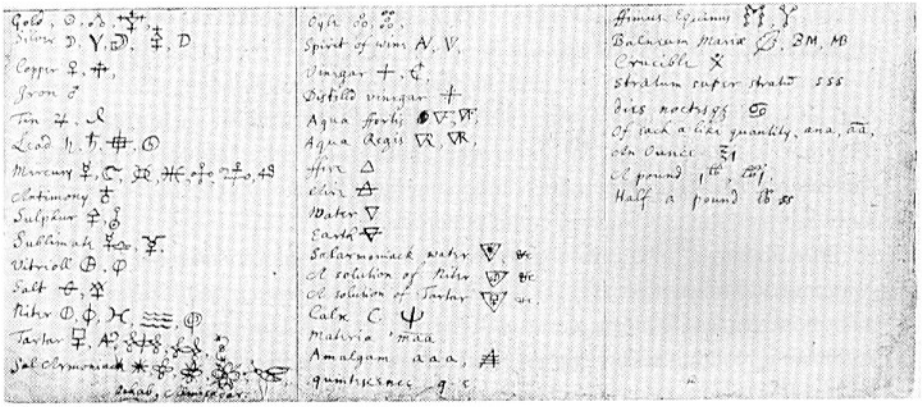


Figure 4.6: Newton's list of alchemical symbols. (From J. Golinski, 1988, *Let Newton Be: a New Perspective on his Life and Works*, eds. J. Fauvel, R. Flood, M. Shortland, and R. Wilson), p. 155, Oxford: Oxford University Press. With kind permission of the Provost, Fellows and Scholars of King's College, Cambridge.)

of the mystique of the alchemists that the fundamental truths should not be made known to the unworthy or the vulgar. The ultimate example of this was Newton's belief that the biblical account of the creation was actually an allegorical description of alchemical processes. In a manuscript note of the 1680s, Newton noted that

> Just as the world was created from dark chaos through the bringing forth of the light and through the separation of the airy firmament and of the waters from the Earth, so our work brings forth the beginning out of black chaos and its first matter through the separation of the elements and the illumination of matter.

What Newton was trying to achieve is illuminated by a manuscript dating from the 1670s entitled *Of Nature's Obvious Laws and Processes in Vegetation*. To quote Golinski,

> Newton distinguished 'vulgar chemistry' from a more sublime interest in the processes of vegetation . . . 'Nature's actions are either vegetable or purely mechanical' he wrote. The imitation of mechanical changes in nature would be common, or vulgar, chemistry, whereas the art of inducing vegetation was 'a more subtle, secret and noble way of working.'

Newton's objective was no less than to isolate the essence of what gave rise to growth and life itself. Such a discovery would make man like God and so had to be kept a secret from the vulgar masses.

## 4.7 The Interpretation of ancient texts and the scriptures

Newton devoted almost as much effort to interpreting the works of the ancients and biblical texts as he did to his alchemical studies. Newton convinced himself that all his great discoveries had in fact been known to the ancient Greek philosophers. In 1692, Nicholas Fatio de Duillier, Newton's protegé during the period 1689–93, wrote to Christiaan Huyghens that Newton had discovered that all the chief propositions of the *Principia* had been

known to Pythagoras and Plato but that they had turned these discoveries into a 'great mystery'.

Newton believed that the ancients had kept this knowledge secret for the same reason he kept his alchemical work secret. These truths were of such great significance that they had to be preserved for those who could truly appreciate them. For this reason, the Greeks had to disguise their deep understandings in a coded language of symbolism, which only initiates such as Newton could penetrate.

Piyo Rattansi[16] suggests that Newton's objective was to give his great discoveries an ancient and honourable lineage. The *Principia Mathematica* had been well received on the continent as a text in mathematics, but not as physics. The continental physicists disliked the introduction of the 'unintelligible' force of attraction, which had to be introduced to account for the law of gravity. To them Newton's 'force of gravity' was a mystic force because they could not envisage any physical mechanism for causing gravitational attraction – all other physical forces could be explained by actions involving concrete bits of matter but what was it that caused gravity to act across the Solar System? The continental scientists interpreted the law of gravity as being the reintroduction of 'occult causes', which the whole of the revolution in science had been bent upon eliminating. Newton was claiming as ancient an authority for his discoveries as any branch of knowledge.

In addition, throughout his lifetime, he had a deep interest in the scriptures and their interpretation. His prime interest centred upon the near East and how a proper interpretation of the texts could be shown to predict retrospectively all the significant events of history. His aim was to show that the seat of all knowledge began in Israel and from there it travelled to Mesopotamia and Egypt. The results of these researches were published in two books, *The Chronology of the Ancient Kingdoms Amended* and *Observations upon the Prophecies of Daniel and the Apocalypse of St John*. The latter work runs for 323 pages and was published in twelve editions between 1733 and 1922.

Rattansi puts Newton's studies of the biblical texts in another revealing light:

> His biblical work was intended to vindicate the authority of the Bible against those Catholics who had tried to show that only by supplementing it with the tradition of the universal church could it be made authoritative. It served at the same time as a weapon against the free-thinkers who appealed to a purely 'natural' religion and thereby did away with the unique revelation enshrined in Christian religion. It did so by demonstrating that history was continually shaped according to providential design which could be shown, only after its fulfilment, to have been prefigured in the divine word.[17]

Later, he writes:

> Newton followed the Protestant interpretation of Daniel's visions and made the Roman church of later times the kingdom of the Antichrist which would be overthrown before the final victory of Christ's kingdom.[18]

Newton held the view that the scriptures had been deliberately corrupted in the fourth and fifth centuries. This view was at odds with the tenets of the Church of England and, as a fellow of Trinity College, he was expected to be a member of the Church. A major crisis was narrowly avoided by the arrival of a royal decree, exempting the Lucasian Professor from the necessity of taking holy orders.

In 1693, Newton had a nervous breakdown. After his recovery, he lost interest in academic studies and, in 1696, left Cambridge to take up the post of Warden of the Mint in London.

## 4.8 London 1696–1727

By the time Newton took up his position as Warden of the Mint, he was already recognised as the greatest living English scientist. Although the position was normally recognised as a sinecure, he devoted all his energies to the recoinage needed at that time to stabilise the currency. He was also responsible for prosecuting forgers of the coin of the realm, an offence which carried the death penalty. Apparently, Newton carried out this unpleasant aspect of his responsibility 'with grisly assiduity'. He was an effective manager and administrator and in 1700 was appointed Master of the Mint.

In 1703, following the death of Hooke, Newton was elected President of the Royal Society. He now held positions of great power, which he used to further his own interests. Halley entered the service of the Mint in 1696 and in 1707 David Brewster was appointed as general supervisor of the conversion of the Scottish coinage into British. Newton had secured the Savillian chair of Astronomy at Oxford for David Gregory in 1692 as well as the Savillian Chair of Geometry for Edmond Halley in the early 1700s. He also ensured that William Whiston was elected to his own Lucasian Chair in Cambridge from which he had eventually resigned in 1703.

In his old age, he did not mellow. An acrimonious quarrel arose with Flamsteed, the first Astronomer Royal, who had made very accurate observations of the Moon, which Newton wished to use in his analysis of its motion. Newton became impatient to use these observations before they were completed to Flamsteed's satisfaction. Newton and Halley took the point of view that, since Flamsteed was a public servant, the observations were national property and they eventually succeeded not only in obtaining the incomplete set of observations but also in publishing them in an unauthorised version without Flamsteed's consent in 1712. Flamsteed managed to recover about 300 of the unauthorised copies and had the pleasure of burning this spurious edition of his monumental work. He later published his own definitive set of observations in his *Historia Coelestis Brittanica*.

The infamous dispute with Leibniz was even nastier. It was fomented by Fatio de Duillier who brought the initial charge of plagiarism against Leibniz. Leibniz appealed to the Royal Society to set up an independent panel to pass judgement on the question of priority. Newton appointed a committee to look into the matter but then wrote the committee's report himself in his *Commercium Epistolicum*. He wrote it as if its conclusions were impartial findings which came down in Newton's favour. But he did not stop there. A review of the report was published, in the *Philosophical Transactions of the Royal Society*, which was also written by Newton, anonymously. The story is very unpleasant, to say the least, particularly since Leibniz had unquestionably made original and lasting contributions to the development of the differential and integral calculus. Indeed, the system of notation which is now used universally is that of Leibniz, not Newton.

Newton died on the 20 March 1727 at the age of eighty-five and was buried in Westminster Abbey.

## 4.9 References


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## Appendix to Chapter 4: Notes on conic sections and central orbits

It may be useful to recall here some basic aspects of the geometry and algebra associated with conic sections and central forces.

### *A4.1 Equations for conic sections*

The conic sections are the shapes obtained by slicing through a double cone at different angles. Their geometrical definition is as follows: they are the curves generated in a plane by the requirement that the ratio of the perpendicular distance of any point on the curve from a fixed straight line lying in the plane and the distance of that point on the curve from