Reading from

"Concepts of Space: The History of Theories of Space in Physics"

by Max Jammer

CHAPTER 4

THE CONCEPT OF ABSOLUTE SPACE

Newton's conceptual scheme, as expounded in his *Philosophiae naturalis principia mathematica,* became the basis of classical physics and as such the subject of much profound analysis. We need mention only Neumann and Mach, who investigated its epistemological implications, and Wolff and Hegel, who explored its metaphysical foundations. So far as the purely physical teachings of the *Principia* are concerned, they are susceptible of different epistemological and metaphysical interpretations; for that work, as the first comprehensive hypothetico-deductive system of mechanics, lends itself, as does every system of the kind, to a variety of philosophical constructions. And so questions arise that allow of no absolute answer. Newton himself appears to have understood the distinction between the purely theoretical-deductive part of a theory and its practical application. In the Scholium to Proposition LXIX in the first book he says: "In mathematics we are to investigate the quantities of forces with their proportions consequent upon any conditions supposed; then, when we enter upon physics, we compare those proportions with the phenomena of Nature, that we may know what conditions of those forces answer to the several kinds of attractive bodies."[1] The comparison to which Newton here alludes (*conferendae sunt*)[2] seems to correspond to an "epistemic correlation"[3] in modern philosophy of science, except for Newton's quite different conception of the character of mathematics (*mathesis*). For to Newton mathematics, particularly geometry, is not a purely hypothetical system of propositions, logically deducible from axioms and definitions; instead geometry is nothing but a special branch of mechanics. "Therefore geometry is founded in mechanical practice, and is nothing but that part of universal mechanics which accurately proposes and demonstrates the art of measuring."[4]

This view of the relation of geometry to mechanics, Newton believes, follows from the impossibility of abstract geometry.

For the description of right lines and circles, upon which geometry is founded, belongs to mechanics. Geometry does not teach us to draw these lines, but requires them to be drawn, for it requires that the

[1] F. Cajori, ed., *Sir Isaac Newton's Mathematical principles of natural philosophy and his System of the world. A revision of Mott's translation* (University of California Press, Berkeley, 1934) [quoted as *Principles*], p. 192.

[2] For the original Latin text, references are given from the Thomson-Blackburn edition of the *Principia* (Glasgow, 1871) [quoted as *Principia*]. On p. 188 we read: ". . . deinde, ubi in physicam descenditur, conferendae sunt hae rationes cum phaenomenis . . ."

[3] Cf. F. S. C. Northrop, *The logic of the sciences and humanities* (New York, Macmillan, 1947), p. 119.

[4] Newton, *Principles,* p. xvii; *Principia,* p. xiii, "Auctoris Praefatio ad Lectorem," reads: "Fundatur igitur geometria in praxi mechanica, & nihil aliud est quam mechanicae universalis pars illa, quae artem mensurandi accurate proponit ac demonstrat."

learner should first be taught to describe these accurately before he enters upon geometry, then it shows how by these operations problems may be solved. To describe right lines and circles are problems, but not geometrical problems. The solution of these problems is required from mechanics, and by geometry the use of them, when so solved, is shown.[5]

Newton's view of the unity of geometry and mechanics (cf. his conception of "fluxions" and his aversion to handling geometric problems algebraically) can be traced back to his teacher Isaac Barrow, for whom geometric curves had essentially a mechanical character. In "De quadratura curvarum" Newton writes: "Quantitates mathematicas, non ut ex partibus quam minimis constantes, sed ut motu continuo descriptas, hic considero . . . Hae geneses in rerum nature locum vere habent et in motu corporum quotidie cernuntur."[6] This realistic conception of mathematics is of the first importance for Newton's notion of absolute space, as we shall soon see. At this point it interests us as being an important feature of Newton's methodology, showing, as it does, that the primary concepts underlying the structure of Newton's system are not hypothetical and unreal, justified only by subsequent experimental verification.

It should be borne in mind, too, that such a remark applies not only to the mathematical apparatus employed in the *Principia,* but to its fundamental laws, as for example the laws of motion. We can see today that these laws are assumptions inaccessible to experimental verification, but to Newton they were facts of immediate experience. For although Newton calls the laws of motion "axioms" (*Axiomata sive leges motus*), the term "axiom" as employed by Newton in this context certainly does not have the modern meaning of an arbitrary assumption; phrases like "lex tertia . . . per theoriam comprobata est"[7] or "certa sit lex tertia motus"[8] show clearly that Newton by his use of the term axiom thought the relevant statement to be the point of

[5] Newton, *Principles,* p. xvii.
[6] *Opuscula Newtoni* (Lausanne and Geneva, 1744), vol. 1, p. 203.
[7] Newton, *Principia,* p. 25.
[8] *Ibid.,* p. 27.

departure for further investigation, and thus in conformity with his general plan, of which he writes: "To derive two or three general principles of motion from phaenomena, and afterwards to tell us how the properties and actions of all things follow from those manifest principles would be a very great step in philosophy."[9] It is in the light of these remarks that the historical treatment of Newton's theory of space must proceed. In other words, as historians we are bound to view Newton's system of mechanics not from the vantage point of a modern textbook on classical mechanics, but from that which Newton himself adopted. Accordingly we shall not confine ourselves to the *Principia* alone, but will take into consideration other writings of his as well, for example, the *Opticks*, the correspondence, and especially the famous exchange of letters between Leibniz and Newton's disciple, Samuel Clarke, who wrote under the guidance of the master.

Although Newton cannot, as we have already remarked, be regarded as a positivist in the modern sense of the word, yet he drew a clear line of demarcation between science on the one hand and metaphysics on the other. The famous "Hypotheses non fingo," although originally expressed only with relation to an explanation of gravitation, became his motto for the exclusion of the occult, metaphysical, or transcendental religious entities. His aim was not to abolish metaphysics, but to keep it distinct from physical investigation. It is well known that Newton, himself a religious man, never denied the existence of beings and entities that transcend human experience; he contended only that their existence had no relevance to scientific explanation: In its *mundus discorsi*, science has no place for them. Intimately acquainted with the problems of religion and metaphysics, Newton managed to keep them in a separate compartment of his mind, but for one exception, namely, his theory of space. Space thus occupies a unique place in his teachings.

In order fully to understand the Newtonian idea of space, it

[9] Newton, *Opticks* (ed. 4, London, 1730; Dover, New York, 1952), p. 401.

is necessary to bear in mind the general conceptual background of his physical system. Apart from space and time, force and mass are the fundamental concepts of the Newtonian physics. In Newton "force" is not the sophisticated notion of modern physics. It is not a mathematical abstraction, but an absolutely given entity, a real physical being. As for "mass," Newton, reverting to the view of Galileo, conceives of it as the most essential attribute of matter and thus places himself in diametrical opposition to Descartes, who identified matter with extension and regarded extension as the chief characteristic of matter. The Newtonian concept of "mass-point," still used in present-day textbooks, marks the chasm that separates Newton's concept of mass from Descartes' concept of spatial extension. A priori, it was perhaps a matter of predilection or preference which of the two, mass or extension, was to be given priority, since every real body has both and is inconceivable apart from either. Newton's abstraction proved to be the more fruitful.

Since mechanics deals with motion, space as the correlate of mass-point — just as the void was the correlate of the atom — has to be introduced at the very beginning of the system. It is therefore no accident that almost at the very beginning of the *Principia* we find the famous Scholium dealing with the concept of space.

I do not define time, space, place, and motion, as being well known to all. Only I must observe, that the common people conceive those quantities under no other notions but from the relation they bear to sensible objects. And thence arise certain prejudices, for the removing of which it will be convenient to distinguish them into absolute and relative, true and apparent, mathematical and common . . .

Absolute space in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces; which our senses determine by its position to bodies; and which is commonly taken for immovable space; such is the dimension of a subterraneous, an aerial, or celestial space, determined by its position in respect to the earth. Absolute and relative space are the same in figure and magnitude; but they do not remain always numerically the same. For if the earth, for instance, moves, a space of our air, which relatively and in respect of the earth remains always the same, will at one time be one part of the absolute space into which the air passes; at another time it will be another part of the same, and so, absolutely understood, it will be continually changed.[10]

In believing that time, space, place, and motion are concepts well known to all, Newton, as we see, does not feel called upon to give a rigorous and precise definition of these terms. Yet, because these notions arise only in connection with sensible objects, certain prejudices cling to them, and to overcome these Newton deemed it necessary to set up the distinctions of absolute and relative, true and apparent, mathematical and common. Since space is homogeneous and undifferentiated, its parts are imperceptible and indistinguishable to our senses, so that sensible measures have to be substituted for them. These coördinate systems, as they are called today, are Newton's relative spaces.

But because the parts of space cannot be seen, or distinguished from one another by our senses, therefore in their stead we use sensible measures of them. For from the positions and distances of things from any body considered as immovable, we define all places; and then with respect to such places, we estimate all motions, considering bodies as transferred from some of those places into others. And so, instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs.[11]

In modern physics, coördinate systems are nothing but a useful fiction. Not so for Newton. Given Newton's realistic conception of mathematical objects, it is easy to understand why these relative spaces form "sensible measures." Not only is the reference body accessible to our senses, but likewise the "relative space" is dependent on it. But this accessibility to sense perception yields a notion that is of temporary validity only and lacking in generality. It is quite possible that there is no body at rest, to which the places and motions of other bodies may be referred; in a word: all these relative spaces may be moving coördinate systems. But moving in what? In order to answer this question, Newton takes flight from the realm of experience

[10] Newton, *Principles*, p. 6.
[11] *Ibid.*, p. 8.

altogether, at least for the time being. In his famous words, "But in philosophical disquisitions, we ought to abstract from our senses,"[12] Newton introduces absolute and immutable space, of which relative space is only a measure. The final degree of accuracy, the ultimate truth, can be achieved only with reference to this absolute space. And it is therefore rightly called "true space."

What, it may be asked at this point, guarantees the final truth of absolute space, the very conception of which appears to contradict Newton's methodological rule: "We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances"?[13] In Newton's time this question became a highly controversial one and remained so until the beginning of the twentieth century. Is the concept of an absolute space a necessity for physics? Or can a consistent conceptual scheme be constructed that explains all physical phenomena without the use of such a concept? As every historian of physics knows, the problem reappeared in the nineteenth century as the problem of the ether and gave rise to an immense amount of discussion and experiment.

To Newton, absolute space is a logical and ontological necessity. For one thing, it is a necessary prerequisite for the validity of the first law of motion: "Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it."[14] Rectilinear uniform motion requires a reference system different from that of any arbitrary relative space. Further, the state of rest presupposes such an absolute space. Newton explains:

> Absolute motion is the translation of a body from one absolute place into another; and relative motion, the translation from one relative place into another. Thus in a ship under sail, the relative place of a body is that part of the ship which the body possesses; or that part of the cavity which the body fills, and which therefore moves together

[12] *Ibid.*
[13] *Ibid.*, p. 398.
[14] *Ibid.*, p. 13.

with the ship: and relative rest is the continuance of the body in the same part of the ship, or of its cavity. But real, absolute rest, is the continuance of the body in the same part of that immovable space, in which the ship itself, its cavity, and all that it contains, is moved. Wherefore, if the earth is really at rest, the body, which relatively rests in the ship, will really and absolutely move with the same velocity which the ship has on the earth. But if the earth also moves, the true and absolute motion of the body will arise, partly from the true motion of the earth, in immovable space, partly from the relative motion of the ship on the earth; and if the body moves also relatively in the ship, its true motion will arise, partly from the true motion of the earth, in immovable space, and partly from the relative motions as well of the ship on the earth, as of the body in the ship; and from these relative motions will arise the relative motion of the body on the earth.[15]

Since the first law of motion, as we have seen, is for Newton a matter of immediate experience, and since the law depends for its validity upon an absolute reference system, absolute space becomes indispensable to Newtonian mechanics. The interesting point, however, is that for Newton the introduction of the concept of absolute space into his system of physics did not result from methodological necessity only. Newton was led by his mathematical realism to endow this concept, as yet merely a mathematical structure, with independent ontological existence. He realized that there was a great difficulty to be overcome: the "inertial system," or, in less modern words, the system in which the first law holds, is not uniquely determined. Newton's mechanics is invariant for a translational transformation with constant velocity, that is, a Galilean transformation. Newton recognizes that a whole class of "spaces" or reference systems comply with this requirement. In Corollary V we read: "The motions of bodies included in a given space are the same among themselves, whether that space is at rest, or moves uniformly forwards in a right line without any circular motion." [16]

If Newton had been a confirmed positivist he would have acknowledged all uniformly moving inertial systems as equivalent

[15] *Ibid.*, p. 7.
[16] Ibid., p. 20.

to each other. As it was, only one absolute space existed for him. How is this space to be distinguished from among the multitude of inertial systems? For the solution of this problem Newton resorts to cosmology. In Hypothesis I of his *The system of the world* [17] he states: "That the centre of the system of the world is immovable. This is acknowledged by all, while some contend that the earth, others that the sun, is fixed in that centre."

To Newton, now, the center of the world is the center of gravity of the system composed of the sun, the earth, and the planets;[18] this center either is at rest or moves uniformly forward in a straight line; the latter alternative, however, is eliminated by Hypothesis I. In this way Newton defines the unique absolute space among all possible inertial frames. It it interesting to note that in the last-mentioned Corollary Newton is concerned to find the astronomical location of this universal center of gravity, which is his reference point for the determination of absolute space. He maintains that the movable centers of the earth, sun, and planets cannot serve as such a center, since they all gravitate toward each other. However, if the body toward which other bodies gravitate most has to be placed in the center, then it is the sun that should be allowed this privilege. Yet, since the sun itself is moving, a fixed point has to be chosen from which the center of the sun recedes least, and from which, if its density and volume were greater, it would recede still less.

All this points to the rather limited scope of Newton's cosmological conceptions. It is also interesting to note that Newton did not take into account the fixed stars when trying to determine the center of gravity of the world. Had he done so, he might have come very near the conception of the body "Alpha," which was introduced by C. Neumann[19] at the end of the last century. The fact that Newton ignored the fixed stars in this respect is the more curious, since for him they were still really "fixed," that is,

[17] *Ibid.*, p. 419.
[18] *Ibid.*, p. 419, corollary to Proposition XII.
[19] C. Neumann, *Ueber die Prinzipien der Galilei–Newton'schen Theorie* (1869).

not moving in space. For although Bruno had already imagined the sun to be in motion, and although Halley confirmed this anticipation in 1718, when he announced [20] that Sirius, Aldebaran, Betelgeuse, and Arcturus had unmistakably shifted their positions in the sky since Ptolemy assigned their places in his catalogue, it was only after the death of Newton that the proper motion of the stars became an accepted truth.

Newton's cosmological assumption that the center of the world is at rest escapes all possibility of experimental or observational verification. The fact was clearly recognized by Berkeley, one of the great opponents of the theory of absolute space. In "De motu" he writes: "Uti vel ex eo patet quod, quum secundam illorum principia qui motum absolutum inducunt, nullo symptomate scire liceat, utrum integra rerum compages quiescat, an moveatur uniformiter in directum, perspicuum sit motum absolutum nullius corporis cognosci posse." [21]

As we shall see in what follows, Newton was convinced that dynamically, though not kinematically, absolute space can be determined through the existence of centrifugal forces in rotational motion. Although Newton does not explicitly draw the conclusion that centrifugal forces determine absolute motion which in its turn determines absolute space, it is clear that this was his intention and it was always recognized as such by his commentators. If space is a physical reality, as Newton undoubtedly assumes, and if accelerated motion furnishes a criterion for its identification, it would appear to be a serious inconsequence to hold that uniform translational motion, since it fails to provide such a criterion, is different from all other kinds of motion; furthermore, space would seem to possess a dual structure, absolute for accelerated motion and relative for uniform translation. Newton's cosmological assumption protects

[20] E. Halley, *Phil. Trans.* 30, 737 (1718).

[21] A. A. Luce and T. E. Jessop, ed., *The works of George Berkeley* (Nelson, London, 1951), vol. 4, p. 28.

him against such an objection, which incidentally was raised by Leibniz in his correspondence with Huygens.

According to Newton, as we have seen, the first law of motion presumes the necessary existence of absolute space but provides no means by which it can be identified experimentally. Hence Newton's next step. Since absolute space and time "do by no means come under the observation of our senses," it becomes necessary to investigate the dynamics of motion. For motion, accelerated motion in particular, is the means and medium through which space can be explored. Inasmuch as they refer to relative or to absolute space, motions are either relative or absolute, so that if it were possible to identify absolute motion, the identification of absolute space would follow. Now absolute motion, according to Newton, can be distinguished from relative motion by its "properties, causes, and effects."

> The causes by which true and relative motions are distinguished one from the other, are the forces impressed upon bodies to generate motion. True motion is neither generated nor altered, but by some force impressed upon the body moved; but relative motion may be generated or altered without any force impressed upon the body. For it is sufficient only to impress some force on other bodies with which the former is compared, that by their giving way, that relation may be changed, in which the relative rest or motion of this other body did consist . . .
>
> The effects which distinguish absolute from relative motion are, the forces of receding from the axis of circular motion. For there are no such forces in a circular motion purely relative, but in a true and absolute circular motion, they are greater or less, according to the quantity of the motion . . .
>
> It is indeed a matter of great difficulty to discover, and effectually to distinguish, the true motions of particular bodies from the apparent; because the parts of that immovable space, in which those motions are performed, do by no means come under the observation of our senses. Yet the thing is not altogether desperate; for we have some arguments to guide us, partly from the apparent motions, which are the differences of the true motions; partly from the forces, which are the causes and effects of the true motions.[22]

[22] Newton, *Principles,* pp. 10, 12.

Thus Newton's first argument with regard to absolute motion is based on the idea that real force creates real motion. To Newton, at least in this context, forces are metaphysical entities conceived anthropomorphically. However, if we leave out of account the import of forces for the determination of absolute space, the notion of force in Newton's mechanics may be interpreted in the modern functional way, as in Heinrich Hertz's *Die Prinzipien der Mechanik:* "Was wir gewohnt sind als Kraft und als Energie zu bezeichnen ist dann fuer uns nichts weiter als eine Wirkung von Masse und Bewegung, nur braucht es nicht immer die Wirkung grobsinnlich nachweisbarer Masse und grobsinnlich nachweisbarer Bewegung zu sein." [23]

But undoubtedly there is no question of this functional conception of force in Newton's discussion of absolute space. It is foreign to the general character of his system. His argument "from causes" is based on the traditional metaphysics, the inclusion of which in the framework of physical explanation is strongly objected to by Newton himself. In order to see the vicious circle inherent in Newton's reasoning, we have only to think for a moment of a world of moving masses in which no living organism existed. For in such a world an absolute force could be determined, according to Newton, solely by the absolute motion of the body on which this force was exerted.

The second argument for the existence of absolute motion proceeds from the effects that such motion produces, in particular, the appearance of centrifugal forces ("vires recedendi ab axe motus circularis"). So we have Newton's famous pail experiment, which he describes as follows:

> If a vessel, hung by a long cord, is so often turned about that the cord is strongly twisted, then filled with water, and held at rest together with the water; thereupon, by the sudden action of another force, it is whirled about the contrary way, and while the cord is untwisting itself, the vessel continues for some time in this motion; the surface of the water will at first be plain, as before the vessel began to move; but after that, the vessel, by gradually communicating its

[23] H. Hertz, *Die Prinzipien der Mechanik* (Leipzig, 1894), p. 31.

motion to the water, will make it begin sensibly to revolve, and recede by little and little from the middle, and ascend to the sides of the vessel, forming itself into a concave figure (as I have experienced), and the swifter the motion becomes, the higher will the water rise, till at last, performing its revolutions in the same times with the vessel, it becomes relatively at rest in it. This ascent of the water shows its endeavor to recede from the axis of its motion; and the true and absolute circular motion of the water, which is here directly contrary to the relative, becomes known, and may be measured by this endeavor. At first, when the relative motion of the water in the vessel was greatest, it produced no endeavor to recede from the axis; the water showed no tendency to the circumference, nor any ascent towards the sides of the vessel, but remained of a plain surface, and therefore its true circular motion had not yet begun. But afterwards, when the relative motion of the water had decreased, the ascent thereof towards the sides of the vessel proved its endeavor to recede from the axis; and this endeavor showed the real circular motion of the water continually increasing, till it had acquired its greatest quantity, when the water rested relatively in the vessel. And therefore this endeavor does not depend upon any translation of the water in respect of the ambient bodies, nor can true circular motion be defined by such translation.[24]

For a clear analysis of this experiment, let us consider also the final phase — not described by Newton — when the rotation of the pail is stopped while the water continues its circular motion (owing to conservation of angular momentum). During this final stage of the experiment, as long as friction can be ignored, the water contained in the vessel maintains its paraboloidal surface.

The gist of this experiment can be summarized in modern terms as follows: Both in the beginning of the experiment (when the pail spins alone) and at the end of the experiment (when the water spins alone) pail and water are moving relative to each other in the same manner. Rigorously considered, the directions of the relative rotations are reversed; but owing to the assumed isotropy of space this reversal can obviously have no effect on the dynamical result. If in the second case the time parameter had been reversed, as is permissible in a purely mechanical phenomenon, exactly the same relative motion would have re-

[24] Newton, *Principles*, p. 10.

sulted. Now, were all motion (rotation) purely relative, no physical difference should become apparent between the two states. However, since the surface of the water contained in the pail is level in the first case and paraboloidal in the second, rotation, thus concludes Newton, must be absolute.

This experiment was the cause of much controversy in the history of modern physics and the situation was clarified only with the appearance of Einstein's principle of equivalence in his general theory of relativity. In Newton's interpretation of the pail experiment he obviously again transcends the realm of experience. His simple assumption that the surface of water in the pail would be as curved, even if it were rotating in empty space, as when rotating in space filled with starry matter, is not susceptible of physical verification. And the same inaccessibility to physical verification characterizes all the other attempts to inforce his argument, as, for example, his experiment with the two cord-connected spheres revolving around their common center of gravity, the tension in the cord being taken by him as an indication of the absolute motion of the spheres. "And thus we might find both the quantity and the determination of this circular motion, even in an immense vacuum, where there was nothing external or sensible with which the globes could be compared."[25] But such conditions can never be realized, any more than in the case of the astronomical effects of centrifugal forces, as for example the spheroidicity of the earth and of Jupiter, as Newton expounds the matter in the third book of his *Principia*.[26]

Berkeley rejects Newton's implicit assumption that the pail experiment, if performed in empty space, would yield the same result. As Berkeley explains in his "De motu," the real motion of the pail is far from being circular, if the diurnal rotation of the earth and its annual revolution are taken into account. For

[25] *Ibid.*, p. 12.
[26] Proposition XVIII, Theorem XVI; also Proposition XIX. Problem III (*Principles*, p. 424); *et alia*.

them should be at rest with respect to absolute space.[28] The weakness of this argument is its indefensible assumption that an absolute reference system is an essential prerequisite for the description of the behavior of these bodies.

So Newton takes over from Patritius, Campanella, and Gassendi the concept of an infinite space, which is homogeneous and isotropic and, in addition, succeeds in convincing himself that he has proved the reality of this concept by physical experiment. He thought he had demonstrated that space has an existence proper to itself and independent of the bodies that it contains. In his view, it makes sense, therefore, to state that any definite body occupies just *this* part of space and not another part of space, and the meaning of such a statement does not presuppose a relation to any other bodies in the universe. He was not aware that his procedure violated the very principles of the methodology he professed. Since he was a younger contemporary of Henry More, whose personal acquaintance he made in his youth and whose teachings, via Isaac Barrow, exerted a great influence upon him, it is no wonder that Newton found support for his theory of space in the doctrine of that thinker. More's important works had been published about seven years before the appearance of the *Principia*. But it was the religious element, originating, as we saw, in Jewish cabalistic and Neoplatonic thought, that gained ascendency over Newton in his later years. So a comparison of the first and later editions of the *Principia* shows that the identification of absolute space with God, or with one of his attributes, came into the foreground of Newton's thought only toward the end of his life, that is, at the beginning of the eighteenth century. However, his interest in Biblical and post-Biblical literature may be traced back to the influence of one of the teachers in Cambridge, Joseph Mede, a fellow of Christ's College. Mede, apart from his studies in apocryphal and other esoteric literature,

[28] This argument in defense of absolute motion reappeared later in Alois Hoefler's *Studien zur gegenwärtigen Philosophie der Mechanik* (Leipzig, 1900), p. 133.

stimulated philological interest among his students in the Hebrew of the Bible by his etymological theory, quite popular at that time, that Hebrew was the mother of all languages.

It also has been established that Durand Hotham's book on Jacob Böhme exerted a strong influence on young Newton. Böhme's *Mysterium magnum*, a commentary on Genesis, shows extraordinary parallels to the *Zohar* and to other sources of Jewish theosophy. The Hebrew *Chokmah*, a body of books ascribed to King Solomon, seemed to have passed over to the Gnostic *Sophia* and by another transition to the "Virgin Wisdom" of Böhme. We know also with certainty that Henry More[29] and Isaac Barrow exerted a very strong influence on Newton at that time. Henry More was the spiritual leader at Christ's College in Cambridge and the chief disseminator of Cabalistic and Neoplatonic ideas, as described in detail in Chapter II. Isaac Barrow, Newton's famous teacher, promulgated More's ideas in mathematized form in his *Mathematical lectures*. In Barrow's geometry, space is the expression of divine omnipresence, just as time is the expression of the eternity of God. Under the influence of these strong forces it seems most probable that Newton, even when writing on purely physical problems, had similar ideas in the back of his mind. In fact, that he had theological and religious ideas in his mind when writing the *Principia* is evident from his letter (December 10, 1692) to Richard Bentley in which he confessed: "When I wrote my treatise about our system, I had an eye upon such principles as might work with considering men for the belief of a Deity; and nothing can rejoice me more than to find it useful for that purpose." It was, however, only in 1713 that Newton prepared the General Scholium of Book III to be published in the second edition (1713). It is in this Scholium, in addition to Queries 19–31 of the *Opticks* (missing in the first edition), that we find explicit statements of Newton's ideas on the relation between his theory of absolute space and

[29] For the facts of personal contact between More and Newton, see L. T. More, *Isaac Newton* (Scribner, New York, 1934), pp. 11, 31, 182.

theology. Undoubtedly, Newton's increasing interest in theological and spiritual questions during his later years was one of the motives for the preparation of the Scholium. Another reason was Cotes's request that he prevent any recurrence of criticisms which pronounced Newton's theory of space as leading to atheism. In a letter (March 18, 1713) to Newton, the editor of the second edition of the *Principia* writes: "I think it will be proper to add something by which your Book may be cleared from some prejudices which have been industriously laid against it . . . That You may not think unnecessary to answer such Objections You may be pleased to consult a Weekly Paper called '*Memoires of Literature*' and sold by Ann Baldwin in Warwick-Lane." The article referred to is Leibniz's letter (February 10, 1711) to the Dutch physician Hartsoeker, in which Leibniz attacks Newton's theory of gravitation. Of greater relevance for our subject, however, is Berkeley's attack on Newton's theory of space, which Cotes certainly had in mind, although he did not mention Berkeley by name. Berkeley published in 1710 his *Principles of human knowledge,* in which he criticizes Newton's concept of absolute space on theological grounds as being a pernicious and absurd notion. Space, according to Berkeley, has to be conceived as relative only, "Or else there is something beside God which is eternal, uncreated, infinite, indivisible, unmutable." [30]

It is therefore not surprising that in the General Scholium Newton gives free reign to his religious enthusiasm:

> It is the dominion of a spiritual being which constitutes a God: a true, supreme, or imaginary dominion makes a true, supreme, or imaginary God. And from true dominion it follows that the true God is a living, intelligent, and powerful Being; and, from his other perfections, that he is supreme, or most perfect. He is eternal and infinite; omnipotent and omniscient; that is, his duration reaches from eternity to eternity; his presence from infinity to infinity; he governs all things, and knows all things that are or can be done. He is not eternity and infinity, but eternal and infinite; he is not duration or space, but he

[30] Berkeley, *Principles of human knowledge,* in *A new theory of vision and other writings* (Dent, London, 1938), p. 173.

endures and is present. He endures for ever, and is everywhere present; and by existing always and everywhere, he constitutes duration and space.[31]

Here, for the time, Newton identifies space and time with God's attributes. God is not eternity and infinity, but he is eternal and infinite. Eternal and omnipresent, God constitutes duration and space. A few lines further on we read, "In ipso continentur & moventur universa," "In him are all things contained and moved," a statement to which Newton adds the marginal remarks that this was the opinion of the Ancients: St. Paul (Acts 17:27, 28), St. John (14:2), Moses (Deut. 4:39), David (Ps. 139:7–9), Solomon (I Kings 8:27), Job (22:12–14), Jeremiah (23:23, 24). Here we have unmistakably an echo of More's *Enchiridion metaphysicum* and his *Divine dialogues,* but with this difference, that Newton's expressions are more reserved and more carefully chosen. He seemed to be aware that he might easily be misunderstood and counted among the pantheistic thinkers of his time, who in orthodox circles were identified with the atheists.

Since every particle of space is always in existence, and every indivisible moment of duration is everywhere, "certainly the Maker and Lord of all things cannot be never and nowhere."[32] Elsewhere Newton speaks of

> the Wisdom and Skill of a powerful ever-living Agent; who, being in all Places, is more able by his Will to move the Bodies . . . within his boundless uniform Sensorium, and thereby to form and reform the Parts of the Universe, than we are by our Will to move the Parts of our own Bodies.[33]

This identification of the omnipresence of space with the omnipresence of God leads to a serious difficulty, and Leibniz with his sharp intellect exploited it remarkably in his controversy with Clarke. For according to Newton's conception, the divisibility of space — relative spaces are parts of the absolute space — would

[31] Newton, *Principles,* p. 544.
[32] Newton, *Principia,* p. 528: "Certe rerum omnium fabricator ac dominus non erit nunquam, nusquam."
[33] Newton, *Opticks* (Dover ed.), p. 403.

appear to involve the divisibility of God. Clarke's response to Leibniz's argument may be summarized as follows: Absolute space is one; it is infinite and essentially indivisible. The assumption that it can be divided leads to a contradiction, since any partition — according to Clarke — would require an intermediary space. Hence divine infinity and omnipresence imply no divisibility of the substance of God. Clarke concludes that it is only because a pictorial and unjustified meaning is attached to the notion of divisibility that the difficulty arose.

Another point of interest in this controversy is the term "sensorium," occurring in the above quotation, and earlier in Query 28:

> . . . does it not appear from Phaenomena that there is a Being incorporeal, living, intelligent, omnipresent, who in infinite Space, as it were in his Sensory, sees the things themselves intimately, and throughly perceives them, and comprehends them wholly by their immediate presence to himself.[34]

In the letter opening the controversy, which was to end only with Leibniz's death in 1716, Leibniz says:

> Sir Isaac Newton says, that Space is an Organ, which God makes use of to perceive Things by. But if God stands in need of any Organ to perceive Things by, it will follow, that they do not depend altogether upon him, nor were produced by him.[35]

But did Newton really identify space with an organ of God? Or was this expression only an unfortunate *lapsus calmi*? Clarke's response to Leibniz gives the answer to this question:

> Sir Isaac Newton doth not say, that Space is the Organ which God makes use of to perceive Things by; nor that he has need of any Medium at all, whereby to perceive Things; But on the contrary, that he, being Omnipresent, perceives all Things by his immediate Presence to them, in all Space whereever they are, without the Intervention or Assistance of any Organ or Medium whatsoever. In order to make this more intelligible, he illustrates it by a Similitude: That as the Mind of Man, by its immediate Presence to the Pictures of Things,

[34] *Ibid.*, p. 370.

[35] *A Collection of Papers which passed between the late learned Mr. Leibnitz and Dr. Clarke* (London, 1717), p. 3.

form'd in the Brain by the means of the Organ of Sensation, sees those Pictures as if they were the Things themselves; so God sees all Things, by his immediate Presence to them: he being actually present to the Things themselves, to all Things in the Universe; as the Mind of Man is present to all the Pictures of Things formed in his Brain . . . And this Similitude is all that he means, when he supposes Infinite Space to be (as it were) the Sensorium of the Omnipresent Being.[36]

Accordingly, it seems to be clear that Newton used the term "Sensorium" merely as a comparison and did not identify space with an organ of God.

With Newton's conception of space now before us we may turn to the question why he thought it needful and appropriate to introduce theological considerations into the very body of his scientific writings. Apart from the reasons dictated by polemic, as we have seen, there are certainly other motives; John Tull Baker, in his monograph entitled *An historical and critical examination of English space and time theories*, discusses some of them. He writes that, in the first place, absolute space and time find a place in the *Principia* because as attributes of God they rendered to the *Principia* a completeness, as a cosmology, which it might have lacked otherwise. Furthermore, their inclusion in the very beginning of the Newtonian system gives to the foundations of mechanics and mathematical physics a theological justification, an idea congenial to Newton:

In the second place, the postulations of absolute time and absolute space suggest the construction of mathematical entities which might be approached as limits of perfection on the description of physical facts. Just as relative time always more nearly approaches absolute time as we refine our measurements and relative motion approximates absolute motion as we examine forces more carefully, so the scheme of things as a whole may be more clearly understood as we progress in more detailed experiment and analysis.[37]

According to this interpretation, the use of absolutes by Newton may be understood as an ideal of perfection, an ideal attain-

[36] *Ibid.*, p. 11.
[37] J. T. Baker, *An historical and critical examination of English space and time theories* (Sarah Lawrence College, Bronxville, New York, 1930), p. 30.

able in matters of space only. In addition it may be rightly claimed that absolute space and absolute time have always had a strong appeal to human emotion. Through their presence clarity and rigor, certainty and definiteness seem to be guaranteed.

One thing is certain: Newton's mechanics, as expounded in the *Principia,* is one great vindication of his theory of absolute space and absolute motion. At the end of the Scholium in the first book he says: "How we are to obtain the true motions from their causes, effects, and apparent differences, and the converse, shall be explained more at large in the following treatise. For to this end it was that I composed it."[38] "Hunc enim in finem tractatum sequentem composui."[39] To demonstrate the existence of true motion and absolute space — such is the program of the *Principia.* All Newton's achievements and discoveries in the realm of physics are in his view subordinate to the philosophical conception of absolute space. The outstanding success of Newtonian mechanics in the physics and astronomy of the last two centuries seemed an indubitable guarantee of the soundness of its philosophical implications. It is not surprising, therefore, that the criticisms leveled by Leibniz and Huygens against the theory of absolute space found no echo in this long period. Today we are in a position to understand the force of these criticisms, which is not to say that the *Principia* ceases to be a landmark in the history of human intellectual achievements. It is this not because of its philosophical conclusions but because of the wealth of its purely physical contents, backed by experimentation and hence verifiable, and further because of the wonderful systematization of this wealth of material.

It is not the purpose of this chapter to provide a comprehensive account of Leibniz's theory of space. In any case it is a task immensely complicated by the fact that Leibniz's theory in the course of its development passed through three differ-

[38] Newton, *Principles,* p. 12.
[39] Newton, *Principia,* p. 12.

ent stages at least. We shall confine our discussion here to his critique of Newton's conception, for the understanding of which it is necessary to bear in mind that in his view space is nothing but a system of relations, devoid of metaphysical or ontological existence. In his fifth letter to Clarke, Leibniz summarizes his conception of space as follows:

> I will here show, how Men come to form to themselves the Notion of Space. They consider that many things exist at once, and they observe in them a certain Order of Co-existence, according to which the relation of one thing to another is more or less simple. This Order is their Situation or Distance. When it happens that one of those Co-existent Things changes its Relation to a Multitude of others, which do not change their Relation among themselves; and that another Thing, newly come, acquires the same Relation to the others, as the former had; we then say it is come into the Place of the former; and this Change we call Motion in That Body, wherein it is the immediate Cause of the Change. And though Many, or even All the Co-existing Things, should change according to certain known Rules of Direction and Swiftness; yet one may always determine the Relation of Situation, which every Co-existent acquires with respect to every other Co-existent; and even That Relation, which any other Co-existent would have to this, or which this would have to any other, if it had not changed or if it had changed any otherwise. And supposing, or feigning, that among those Co-existents, there is a sufficient Number of them, which have undergone no Change; then we may say, that Those which have such a Relation to those fixed Existents, as Others had to them before, have now the same Place which those others had. And That which comprehends all those Places, is called Space.[40]

Leibniz goes on to explain that the relation of situation is a wholly sufficient condition for the idea of space. No absolute reality need be invoked. He makes his point clear by an excellent illustration from genealogy:

> In like manner, as the Mind can fancy to itself an Order made up of Genealogical Lines, whose Bigness would consist only in the Number of Generations, wherein every Person would have his Place: and if to this one should add the Fiction of a Metempsychosis, and bring in the same Human Souls again; the Persons in those Lines might change Place; he who was a Father, or a Grand-father, might become

[40] *A Collection of Papers . . .*, p. 195.

a Son, or a Grand-son &c. And yet those Genealogical Places, Lines, and Spaces, though they should express real Truths, would only be Ideal Things.[41]

The illustration of a tree of genealogy, which shows the mutual relations of kinship between certain persons by attributing to them definite positions within the scheme, serves Leibniz very well. For nobody would hypostatize this system of relations and endow it with ontological existence. Newton's absolute space, in Leibniz's view, is nothing but a similar unjustified hypostatization.

Having thus outlined his concept of space, Leibniz realizes that what he has done is only to define the expression "having the same place," this being enough for the foundation of the concept of physical space. He then proceeds with great ardor to attack More and through him Newton. For the context the following words of Leibniz are worth quoting:

> If the Space (which the Author fancies) void of all Bodies, is not altogether empty; what is it then full of? Is it full of extended Spirits perhaps, or immaterial Substances, capable of extending and contracting themselves; which move therein, and penetrate each other without any Inconveniency, as the Shadows of two Bodies penetrate one another upon the Surface of a Wall? Methinks I see the revival of the odd Imaginations of Dr. Henry More (otherwise a Learned and well-meaning Man), and of some Others, who fancied that those Spirits can make themselves impenetrable whenever they please. Nay, some have fancied, that Man in the State of Innocency, had also the Gift of Penetration; and that he became Solid, Opaque, and Impenetrable by his Fall. Is it not overthrowing our Notions of Things, to make God have Parts, to make Spirits have Extension? [42]

Leibniz's clear conception of space[43] as a system of relations and his well-known "principium identitatis indiscernibilium" are the two solid foundations from which he launches his criticism of Newton's absolute space and absolute motion. On

[41] *Ibid.*, p. 201.
[42] *Ibid.*, p. 205.
[43] For a genetic history of Leibniz's philosophy of space and time, see W. Gent, "Leibnizens Philosophie der Zeit und des Raumes," *Kantstudien* *31*, 61 (1926).

kinematic grounds there can be no doubt that Leibniz is the victor in this dispute. Clarke's refutations of Leibniz's arguments are often not to the point and show a great deal of misunderstanding. However, as soon as Clarke leaves the subject of kinematics and brings forth — no doubt under the briefings of Newton himself — the dynamical arguments in favor of the existence of absolute space and motion, Leibniz faces an insuperable difficulty. With regard to Clarke's reference to the Scholium and Newton's demonstrations therein of the existence of absolute space and absolute motion by means of centrifugal forces, Leibniz feels obliged to admit:

> However, I grant there is a difference between an absolute true motion of a Body, and a mere relative Change of its Situation with respect to another Body. For when the immediate Cause of the Change is in the Body, That Body is truly in Motion; and then the Situation of other Bodies, with respect to it, will be changed consequently, though the Cause of that Change be not in Them.[44]

Having thus bowed to the idea of an "absolute true motion," Leibniz is placed in a dilemma from which he finally sees only one way out: namely, to allow for a double meaning of the concept of motion. On the one hand, it may denote the purely spatial change of situation, which saves his view of the conceptual structure of space; on the other hand, it may signify a dynamical process which is completely unrelated to space as such. But Leibniz is aware that such a stratagem exposes him to the danger of having to fall back on doubtful scholastic concepts like quality, form, substance. It is especially clear from Leibniz's correspondence with Huygens that he tried desperately for years without success to find a dynamical argument for the relativity of motion. Yet it is a curious fact for us today to note that actually he came very near to Mach's solution of the problem. In his "De Causa Gravitatis, et Defensio Sententiae Autoris de veris Naturae Legibus contra Cartesianos" [45] Leibniz tried to demonstrate that gravity is not explicable as a force acting at a distance,

[44] *A Collection of Papers . . .*, p. 213.
[45] *Acta Eruditorum* (1690).

but is reducible to the contiguous action of the surrounding ether. In other words, he tried to reduce gravity to a centrifugal force, saying: "Etsi valde dudum inclinaverim ipse ad gravitatem a vi centrifuga materiae aethereae circulantis repetendam, sunt tamen aliqua quae dubitationes gravissimas injecere." [46] Directly opposite to this was Mach's daring description of centrifugal forces as a disguised gravitational action. So Leibniz, having failed to find the key to dynamical relativity, saw no need to revise what he had written some twenty years before, when he summarized his remarks on Cartesian physics in his "Animadversions on Descartes' *Principles of Philosophy*":[47]

On Art. 25. If motion is nothing but change of contact or immediate vicinity, it follows that it can never be determined which thing is moved. For as in astronomy the same phenomena are presented in different hypotheses, so it is always permissible to ascribe real motion to either one or other of those bodies which change among themselves vicinity or situation; so that one of these bodies being arbitrarily chosen as if at rest, or for a given reason moving in a given line, it may be geometrically determined what motion or rest must be ascribed to the others so that the given phenomena may appear. Hence if there is nothing in motion but this respective change, it follows that no reason is given in nature why motion must be ascribed to one rather than to others. The consequence of this will be that there is no real motion. Therefore in order that a thing can be said to be moved, we require not only its situation in respect to others, but also that the cause of change, the force or action, be in itself.[48]

It is to these lines that Huygens refers in his letter of May 29, 1694 to Leibniz. He objects to the assertion "that it would be absurd, if there exists no real, but only relative motion" ("absonum esse nullum dari motum realem sed tantum relativum"). If Huygens' quotation from Leibniz is verbally inaccurate, it is not so essentially. Huygens declares his intention to stick to his theory — perhaps by way of contrasting his own firmness with Leibniz's wavering — and says that he will not let himself be

[46] G. I. Gerhardt, *Leibnizens mathematische Schriften* (Halle, 1860), part 2, vol. 6, p. 197.
[47] Published in 1692.
[48] G. M. Duncan, *The philosophical works of Leibnitz* (New Haven, 1890), p. 60.

influenced by the experiments in the *Principia*, convinced as he is that Newton is wrong. At the same time he hopes that Newton will retract his theory in the forthcoming second edition of the *Principia*, which he thought would be edited by David Gregory. Huygens' instinct toward his own theory was sound, although he was mistaken about the second edition of the *Principia*, which in fact was prepared by Roger Cotes, as he was mistaken about its possible revision by Newton.

The subject occurs in Huygens' first letter to Leibniz, which reads:

> Je vous diray seulement, que dans vos notes sur des Cartes j'ay remarqué que vous croiez absonum esse nullum dari motum realem, sed tantum relativum. Ce que pourtant je tiens pour tres constant, sans m'arrester au raisonnement et experiences de Newton dans ses Principes de Philosophie, que je scay estre dans l'erreur, et j'ay envie de voir s'il ne se retractera pas dans la nouvelle edition de ce livre, que doit procurer David Gregorius.[49]

Leibniz's reply to this letter (June 22, 1694) is extremely interesting:

> Quant à la difference entre le mouuement absolu et relatif, je croy que si le mouuement ou plus tost la force mouuante des corps est quelque chose de reel comme il semble qu'on doit reconnoistre, il faudra bien qu'elle ait un subjectum. Car a et b allant l'un contre l'autre, j'avoue que tous les phenomenes arriveront tout le meme, quel que soit celuy dans le quel on posera le mouuement ou le repos; et quand il y auroit 1000 corps, je demeure d'accord que les phenomenes ne nous scauroient fournir (ny même aux anges) une raison infallible pour determiner le sujet du mouuement ou de son degré; et que chacun pourroit estre concû à part comme estant en repos, et c'est aussi tout ce que je crois que vous demandes; mais vous ne nieres pas je crois que veritablement chacun a un certain degré de mouuement on, si vous voulés de la force; non-obstant l'equivalence des Hijpotheses. Il est vray que j'en tire cette consequence qu'il y a dans la nature quelque autre chose que ce que la Geometrie y peut determiner. Et parmy plusieurs raisons dont je me sers pour prouuer qu'outre l'etendue et ses variations, qui sont des choses purement Geometriques, il faut reconnoistre quelque chose de superieur, qui est la force; celle-cy n'est pas des moindres. Monsieur Newton reconnoist l'equivalence des

[49] *Oeuvres complètes de Christiaan Huygens* (The Hague, 1905), vol. 10 (correspondence, 1691–1695), p. 609.

Hypothese en cas des mouuements rectilineaires; mais a l'egard des Circulaires, il croit que l'effort que font les corps circulans de s'eloigner du centre ou de l'axe de la circulation fait connoistre leur mouuement absolu. Mais j'ay des raisons qui me font croire que rien ne rompt la loy generale de l'Equivalence. Il me semble cependant que vous meme, Monsieur, estiés autres fois du sentiment de M. Neuton à l'egard du mouuement circulaire.[50]

As this letter shows, Leibniz finds himself in a precarious situation, embracing the logical principle of kinematical relativity on the one hand, and the phenomenon of circular motion which demands the existence of absolute space, on the other. His "true motion," which differs from pure geometrical motion conceptually, is obviously an attempt at a compromise.

But Huygens is opposed to any compromise. Thus he writes in a letter dated August 24, 1694:

Pour ce qui est du mouvement absolu et relatif, j'ay admire vostre memoire, de ce que vous vous estes souvenu, qu'autrefois j'estois du sentiment de Mr. Newton, en ci qui regard le mouvement circulaire. Ce qui est vray, et il n'y a que 2 ou 3 que j'ay trouve celuy qui est plus veritable, duquel il semble que vous n'estes pas eloigne non plus maintenant, si non ence que vous voulez, que lorsque plusieurs corps ont entre eux du mouvement relatif, ils aient chacun un certain degre de mouvement veritable, ou de force, enquoy je ne suis point de vostre avis.[51]

Leibniz's reply of September 14, 1694, which brings this highly interesting exchange of ideas to an end, Huygens dying in 1695, shows his great interest in Huygens' solution of the problem of circular motion. He agrees that no special privilege attaches to circular motion as compared with uniform translational motion and that all reference systems should be treated as equivalent. In Leibniz's opinion it is merely the principle of simplicity that leads to the exclusive ascription of certain motions to certain bodies. No doubt this was borrowed by Leibniz from the realm of astronomy, where for many years it played an important role in the controversy between the Copernicans and their opponents. Leibniz not only realized the inherent similarity, or near identity,

[50] *Ibid.*, p. 639.
[51] Huygens, *Oeuvres complètes*, vol. 10, p. 609.

of the problem under discussion with the problem whether the Ptolemaic or the Copernican system is preferable, but he even composed a treatise, *Tentamen de motuum coelestium causis,*[52] whose intention is to show how the arguments with regard to the mechanical relativity of motion suggest the equivalence of the two rival cosmological systems. It seems that he originally intended to publish this work in Rome during his visit to the Holy City. But caution prevailed and he submitted only a *Promemoria,*[53] whose theoretical part begins with the statement: "Ut vero res intelligatur exactius, sciendum est Motum ita sumi, ut involvat aliquid respectivum et non posse dari phaenomena ex quibus absolute determinetur motus aut quies; constitit enim motus in mutatione situs seu loci."

We have mentioned Leibniz's last letter to Huygens which deals with the problem of absolute space. The letter is as follows:

> . . . Comme je vous disois un jour à Paris qu'on avoit de la peine à connoistre le veritable sujet du Mouuement vous me répondîtes que cela se pouuoit par le moyen du mouuement circulaire, cela m'arresta; et je m'en souuins en lisant à peu près la même chose dans le liure de Mons. Newton; mais ce fut lorsque je croyois déja voir que le mouuement circulaire n'a point de privilege en cela. Et je voy que vous estes dans le meme sentiment. Je tiens donc que toutes les hypotheses sont equivalentes et lorsque j'assigne certains mouuements à certains corps, je n'en ay ny puis avoir d'autre raison, que la simplicité de l'Hypotheses croyant qu'on peut tenir la plus simple (tout consideré) pour la veritable. Ainsi n'en ayant point d'autre marque, je crois que la difference entre nous, n'est que dans la maniere de parler, que je tache d'accomoder a l'usage commun, autant que je puis, salva veritate. Je ne suis pas même fort elogne de la vostre, et dans un petit papier que je communiquay à Mr. Viviani, et qui me paroissoit propre à persuader Messieurs de Rome a permettre l'opinion de Copernic, je m'en accommodois. Cependant si vous estes dans ces sentimens sur la realité du mouuement, je m'imagine que vous deuriés en avoir sur la nature du corps de differens de ceux qu'on a coustume d'avoir. J'en ay d'assez singuliers et qui me paroissent demonstrés.[54]

What is this singular conception of the nature of bodies on the basis of which Leibniz here claims to have found the solu-

[52] Gerhardt, *Leibnizens mathematischen Schriften,* vol. 6, p. 144.
[53] *Ibid.,* p. 146.
[54] Huygens, *Oeuvres complètes,* vol. 10, p. 681.

tion of the problem of circular motion? We do not know. Leibniz does not explain his solution, either here or elsewhere as far as is known.

We are in a more fortunate position with regard to Huygens' solution of the same problem. How could Huygens maintain, in the light of certain dynamical effects such as the rise of centrifugal forces in circular motion, the kinematical principle of relative motion, and at the same time dispense with the existence of absolute space and motion?

In 1886 L. Lange drew attention to the possibility of finding Huygens' solution among his posthumous papers in the archives of Leyden. It was, however, only in 1920 that D. J. Korteweg and J. A. Schouten, having found in the Leyden archives four loose sheets written by Huygens, and all dealing with circular motion, published the solution. The fourth paper, in which Huygens summarized his solution, is quoted in part:

> Diu putavi in circulari motu haberi veri motus "criterion" ex vi centrifuga. Etenim ad ceteras quidem apparentias idem fit sive orbis aut rota quaepiam me juxta adstante circumrotetur, sive stante orbe illo ego per ambitum ejus circumferar, sed si lapis ad circumferentiam ponatur projicietur circumeunte orbe, ex quo vere tunc et nulla ad aliud relatione eum moveri et circumgyrari judicari existimabam. Sed is effectus hoc tantummodo declarat impressione in circumferentiam facta partes rotae motu relativo ad se invicem in partes diversas impulsas fuisse, ut motus circularis sit relativus partium in partes contrarias concitatarum sed cohibitus propter vinculum aut connexum, an autem corpora duo inter se relative moveri possunt quorum eadem manet distantia?
>
> Ita sane dum distantiae incrementum inhibetur, contrarius vero motus relativus per circumferentiam viget.
>
> Plerique verum corporis motum statuunt cum ex loco certo ac fixo in spatio mundano transfertur, male nam cum infinite spatium undique extensum sit quae potest esse definitio aut immobilitas loci?
>
> Stellas affixas, in Copernicano systemate, forsan revera quiescentes dicent. Sint sane inter se immotae sed omnes simul sumtae alterius corporis respectu quiescere dicentur, vel qua in re different a celerrime motis in partem aliquam ? nec quiescere igitur corpus nec moveri in infinito spatio dici potest, ideoque quies et motus tantum relativa sunt.[55]

[55] D. J. Korteweg and J. A. Schouten, *Jahresbericht der Deutschen Mathematiker-Vereinigung* 29, 136 (1920).

This may be translated:

> For a long time I had thought that rotational motion by means of centrifugal forces contains a criterion for true motion. Indeed, with regard to other phenomena it is the same whether a circular disk or a wheel rotates near me, or whether I circle round the stationary disk. However, if a stone is put on the circumference this will be projected only if the disk rotates, and therefore I formerly thought that circular motion is not relative to any other body. Still, this phenomenon showed only that the parts of the wheel, owing to the pressure acting on the circumference, are driven in relative motion among themselves in different directions. Rotational motion is therefore only a relative motion of the parts, which are driven to different sides, but held together by a rope or other connection.
>
> Now, is it possible to move two bodies relatively without changing their distance? This is indeed possible if an increase in their distance is prevented. An opposite relative motion exists on the circumference. Most people suppose that the true motion of a body consists in its being transferred from a certain fixed place in the universe. This is wrong; for if space is unlimited in all directions, what then is the definition of the immobility of a place? It will perhaps be said that the fixed stars in the system of Copernicus are really at rest; well, they may indeed be mutually immobile with respect to each other; but taken together, relative to what other body are they said to be at rest or in what respect are they to be distinguished from bodies moving very fast in a certain direction? It is therefore impossible to state that a body is at rest in infinite space, or that it moves therein; rest and motion are therefore only relative.

Thus Huygens thought he had discovered that the dynamical effect of the appearance of centrifugal forces is merely an indication of the relative motion of the different parts of the disk. Yet the relative motion of these parts can be transformed away by taking as a reference system just that system which has the same angular velocity (and the same origin) as the rotating disk. In this rotating coördinate system the parts of the disk are at rest. The dynamic effect, however, referred to this system, does not vanish: the "pressure" exerted by centrifugal forces has not been transformed away, as it should be were the centrifugal force but a dynamical effect of the relative motion of the particles. Huygens' explanation, therefore, certainly does not pass the test of modern scientific criticism. Nevertheless, it is a historical fact that Huygens, inspired by his sound scientific insight, was the first physicist who believed in the exclusive validity of a principle of kinematic as well as dynamic relativity, two hundred years before the rise of modern relativity.