TITLE
ASP Conference Series, Vol. **VOLUME**, **PUBLICATION YEAR**
EDITORS

Implications of constraining the rate of period change with time in a pulsating White Dwarf

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Abstract. Pulsations are seen along the white dwarf (WD) cooling track in three temperature regions and are believed to be an evolutionary effect in otherwise normal white dwarfs. Measuring the rate of cooling for a pulsating WD constrains the evolution for all WDs at that temperature. We expect even the most stable pulsation periods to change on the evolutionary, or cooling, time-scale. Constraining the rate of period change with time, $\dot{P}$, thus constrains the theoretical evolutionary models. This helps in calibrating the cooling curve, which in turn, reduces one of the theoretical uncertainties encountered in WD cosmochronology. The rate of cooling is sensitive to the core composition and hence measuring a $\dot{P}$ effectively measures the mean core composition. WDs like G117-B15A and R548 have a measured $\frac{\dot{P}}{P}$ that makes them more stable than atomic clocks. If these WDs had an orbital planetary or sub-stellar companion, then that would cause a $\dot{P}_{orb}$, thus providing a means of detection of any such companion.

1. Introduction

The observed properties of the known classes of pulsating WDs place them in three different temperature regimes. The high temperature instability strip consists of the PNNV and the DOV stars at an effective temperature around 100 000 K and hotter. The DBV instability strip occurs around 25000 K, while the DAV instability strip is found between 11000 K to 12000 K (Winget 1998). The periods are typically 100 s to 1000 s, consistent with nonradial g-mode pulsations. WDs have high gravity ($\log g \approx 8$), so nonradial g-mode pulsations are energetically favored as they involve motion along equi-potential surfaces, rather than either radial or nonradial p-mode pulsations, which would involve relatively large radial motion. Pulsating WDs are not defective or special in any way; all DAs should pulsate in the ZZ Ceti instability strip (McGraw & Robinson 1976; Lacombe & Fontaine 1980, Giovannini et al. 1998), i.e., it is an evolutionary effect. So, when we measure the cooling rate for a pulsating WD, it applies to all WDs at that temperature. Cooling of the WD increases the pulsation period. For high overtone g-modes, we have (Winget, Hansen & Van Horn 1983),

$$\frac{\dot{P}}{P} \approx -\frac{1}{2}\frac{\dot{T}}{T} \tag{1}$$

where P is the pulsation period and T is the temperature of the driving region. Measuring the change in pulsation period with time, i.e., $\dot{P}$, is thus equivalent to constraining the rate of cooling.

2. Constraining the evolution of a White Dwarf

Constraining the rate of cooling for a WD has important implications in stellar evolution theory. These limits improve our calibration of the cooling curve and provide a way to study the core composition. They may also help in deciphering other phenomena like crystallization, phase separation and neutrino production, provided the pulsating WD has a suitable temperature and mass.

2.1. Calibration of the cooling curve

The best current observational determinations of the WD luminosity function for the disk of our galaxy indicate a turn-down in the space density of low luminosity stars (Liebert, Dahn & Monet 1988; Leggett, Ruiz & Bergeron 1998; Oswalt et al. 1996). We presume this to be a signature of the finite age of the disk (Winget et al. 1987). The luminosity where this turn-down occurs, in conjunction with theoretical cooling calculations, allows us to estimate the age of the galactic disk. The determination of the halo luminosity function would enable us to make the same sort of estimate for the halo. This process is referred to as WD Cosmochronology. It involves observational and theoretical uncertainties. The observational uncertainty comes from our inability to locate the turn-down in the luminosity function accurately, due to the fact that cool WDs are faint and consequently hard to detect. Most of the theoretical uncertainty in the age estimation comes from uncertainties in the constitutive physics and the basic parameters used in the cooling. These include calibration of the cooling curve, core composition, crystallization and phase separation. We can calibrate the cooling curve by measuring the rate of cooling for WDs at different temperatures. Nature has provided us with a way to measure the cooling rate of a WD by giving us pulsating WDs.

The DOV star PG 1159-035 revealed a rate of period change for the 516 s mode (Costa, Kepler, & Winget 1999) to be $(13.0 \pm 2.6) \times 10^{-11}$ s/s. At the cool end, the main periodicity of the DAV G117-B15A shows a $\dot{P} \leq (2.3 \pm 1.4) \times 10^{-15} s/s$ (Kepler et al. 2000). For another cool DAV, R548, the rate of period change we measured is $\dot{P} \leq (2.9 \pm 2.8) \times 10^{-15} s/s$ for the 213.132 s mode. We also found $\dot{P} \leq (1.0 \pm 4.7) \times 10^{-15} s/s$ for the 212.768 s mode. Both frequencies, separated by 8μhz, must sample the same rate of cooling. Hence, we can say that $\dot{P} \leq (2.9 \pm 2.8) \times 10^{-15} s/s$ for R548, as that is the number with a smaller error. The uncertainties in the $\dot{P}$ values for both R548 and G117-B15A are more important because these numbers represent limits and not yet true measurements. These values for $\dot{P}$, together, help in calibrating the cooling curve, as they are constraints on the rate of cooling for WDs at different temperatures. Measuring a $\dot{P}$ for a DBV would further improve this calibrated curve.

2.2. Core Composition

The rate of cooling of a WD depends on the core composition and on the stellar mass. For different core compositions there are families of cooling curves. The heavier the core, the faster the star cools. By estimating the rate of cooling for the variable WDs, and by comparing it to theoretical evolutionary models, we are effectively measuring the mean atomic weight of the core. Using a Mestel-like cooling law, Kepler et al. (1991) wrote a relation between the rate of period change and the mean core atomic weight A as:

$$\dot{P}(A) = 4.3 \times 10^{-15} \frac{A}{12} \tag{2}$$

Results for the DAVs R548 and G117-B15A, two white dwarfs with masses around 0.6 $M_{\odot}$, are consistent with a C/O core, as predicted by evolutionary models, and eliminate models with substantially heavier cores, as they would produce a faster rate of period change than observed.

2.3. Crystallization and Phase Separation

For a 0.6 $M_{\odot}$ Wood (1992) model, the onset of crystallization is at $T_e = 6000$ K for a C core ($t_{cool} \simeq 2$ Gyr, $L \simeq 10^{-3.8}\ L_{\odot}$), and at $T_e = 7200$ K for an O core. These temperatures are much cooler than the DAV instability strip from 11 000-12 000 K. So, ordinarily, one would not be able to study effects such as crystallization and phase separation using $\dot{P}$ values for the DAVs. However, massive stars like BPM 37093 ($M \simeq 1\ M_{\odot}$) should be crystallized pulsators (Winget et al. 1997; Montgomery & Winget 1999; Nitta et al. 1999) and provide us with a unique opportunity to study these effects. Crystallization affects $\dot{P}$ in the following ways; it releases latent heat and delays the cooling and, secondly, the outward moving crystallization front causes the periods to increase as described below.

Crystallization is also expected to make the existence of toroidal modes possible, but these should not be observable. Their calculations indicate that p-modes are affected to a few percent by crystallization, but the most dramatic effects are seen in g-modes. The nonradial g-modes involve fluid motion with a large shear, consequently the amplitude of fluid motions in a crystallized solid was found to be decreased by three orders of magnitude in their models. The kinetic energy density depends on the square of the displacement, so it is attenuated by six orders of magnitude in the solid core. The kinetic energy is an indicator of how a given mode samples different regions in a model and so g-modes can be considered to be excluded from the crystallized region. The structure of mode trapping is thus modified by crystallization; periods and period spacings are expected to increase as the crystallized mass fraction increases. The increase in period spacing can be as much as 25% in a 99% crystallized star, while the period itself can increase as much as 30% and more, depending on the mode. Though the fractional change in period is significant, it takes place on a large time-scale, preventing the resultant $\dot{P}_{cr}$ from being too large. Montgomery (1998) calculated $\dot{P}_{cr}$ to be $\sim 7 \times 10^{-15}$ s/s for periods less than 1000 s and $\sim 5 \times 10^{-15}$ s/s for periods between 500 - 700 s. The observed $\dot{P}$ is a sum of $\dot{P}_{cr}$ and $\dot{P}_{cooling}$ and should be measurable with a 10 year baseline. Fig. 9 of Montgomery et al. (1999) plots the calculated change in periods as a function of the crystallized mass fraction. There are a few kinks in the plot showing a large change in period on a faster time-scale. This is indicative of an avoided crossing, explained in the later sections. Such avoided crossings can have larger $\dot{P}$ values and it may be possible to detect them.

Recent calculations of phase separation indicate that as the star crystallizes, there will be an enhancement of oxygen content in the crystallized region, while the overlying fluid layer will be carbon enhanced, provided the WD interior was initially a mixture of carbon and oxygen (Ichimaru et al. 1988; Segretain & Chabrier 1993; Montgomery et al. 1999). This phase separation lowers the binding energy of the star and alters its chemical profile, both of which affect the observed $\dot{P}$. Phase separation also affects the effective temperature at which crystallization occurs and results in a release of latent heat. While the delayed cooling causes a decrease in $\dot{P}$ due to release of latent heat of crystallization and phase separation, crystallization causes an increase in the periods and therefore of $\dot{P}$.

Once the star is crystallized, the ionic heat capacity increases from $\frac{3}{2}\frac{k}{AH}$ to $3\frac{k}{AH}$ (Kepler & Bradley 1995). This decreases the cooling rate by a factor of 2 and we can expect a reduction in $\dot{P}$ by a factor of 2 also. Once the core reaches Debye temperature $\Theta_D \approx 2 \times 10^6$ K (Kepler & Bradley 1995), the specific heat decreases like T^3 and rapid cooling commences. This should imply a large $\dot{P}$. Unfortunately, none of the 0.6 $M_\odot$ WDs have cooled enough due to the finite age of the galactic disk (Kepler & Bradley 1995).

2.4. Neutrino Flux

The neutrino luminosity of a DOV can be a more efficient means of cooling than the photon luminosity. The ratio of neutrino to photon luminosity can vary from 0.1 to 3, depending on the effective temperature and mass of the star (O'Brien et al. 1998). For the DOV PG 0122+200, O'Brien et al. (1998) quote a limiting value for $\dot{P}$ to be 6×10^{-10} s/s. The $\dot{P}$ measurement for PG 1159-035 by Costa et al. (1999), on the other hand, revealed a value of $(13.0 \pm 2.6) \times 10^{-11}$ s/s. O'Brien et al. (1998) estimate that the ratio of neutrino to photon luminosity for PG 0122+200 is about 2.5 to 2.6, while for PG 1159-035, it is about 0.1. This implies that PG 0122+200 is a good candidate for obtaining the neutrino flux by a measurement of its $\dot{P}$ value. Such a measurement could prove to be a very important test for the standard model of particle physics.

3. Reliable Clock

Pulsation periods in WDs are found to be very stable. This can be theoretically justified as WD evolution is expected to be simple cooling at almost a constant radius. The hot DAV stars, like R548 and G117-B15A, exhibit extreme amplitude and frequency stability, making them the most stable optical clocks known. This stability may possibly be associated with two different effects, low k modes and/or mode trapping. G117-B15A and R548 are more stable than atomic clocks and most pulsars, although there is one millisecond pulsar that does match these amazing WDs in stability. G117-B15A can be used to measure the relative drift of atomic clocks, pulsars and even R548 and vice-versa. They can also be used to calibrate GPS software.

4. Motion of a Clock

G117-B15A and R548 are the most stable optical clocks known. Motion of a clock with uniform velocity towards or away from us would result in a Doppler effect and we would measure a different period than the true one. However, if the motion were accelerated, then there would be a resultant $\dot{P}$ associated with it. There are two possible ways to have such an accelerated motion: an orbital companion or a significant proper motion.

4.1. Orbital Companion

If G117-B15A or R548 had an unseen orbital companion, another star or a planet, then its motion about the center of mass of the system would manifest itself as a sinusoidal variation of the arrival time of maximum pulsation. This sinusoidal variation could, in principle, be distinguishable from the parabolic signature due to cooling of the WD. The period of the sinusoid would be the orbital period and the amplitude of the sinusoid would help in deducing the mass and/or distance of the orbital companion. The deviation from a sine curve would tell us something about the ellipticity of the orbit and the angle of inclination of the orbit in the sky. A Doppler effect that would result from the orbital motion of the clock would cause a $\dot{P}_{orb}$ (Kepler et al. 1991), given by

$$\dot{P}_{orb} = \frac{P}{c}\frac{Gm}{a^2} \tag{3}$$

where P is the pulsation period, m is the mass of the orbital companion and a is the separation between the components. The $\dot{P}_{orb}$ is not caused by the motion towards or away from us, but by the acceleration in the motion. Any uniform motion along the line of sight would just be interpreted as a correction in pulsation period, ΔP.

The observed minus calculated time of maxima (O-C) diagrams for R548, G117-B15A and L19-2 do not show any discernible sinusoidal variations. G117-B15A is in a binary system, but the orbital companion with a mass of 0.39 $M_\odot$ and a separation of 925 A.U. is not detectable using the O-C technique (Kepler et al. 1991). We can set the following limits on the physical parameters for an orbital companion. Suppose that the O-C diagram consists of points, that span 3 decades and have an average spacing of a year. We observe the DAV at about the same time of the year, so we are de-sensitized to observing an orbital period of a year, as we would find it to be in the same phase every orbit. Suppose the orbital period is shorter than a year, then every O-C point would sample it in a different phase and we would be able to uncover such a pattern eventually. So, the orbital period could only be longer than 6 decades, if no sinusoidal variations or the like are seen. Using this limit on the period and Kepler's third law, we then set a limit on the orbital radius r for the companion. The amplitude A of the sinusoid (orbital light travel time) is less than the average errors of the points in the O-C diagram and hence we do not detect it. This sets a limit on the orbital radius $r_\star$ for the DAV, which also has to go around the common center of mass of the system with the same orbital period P.

$$(2A)c = 2r_\star \sin(i) \tag{4}$$

where i is the angle of inclination. Note the role played by the intangible angle of inclination. If the plane of the orbit is perpendicular to the line of sight, then we will not see a sinusoid in the O-C diagram. So, in that context, the limits that we are setting on the mass and radius of the orbital companion have a factor from the angle of inclination, entangled with them. Remembering that the orbital radii should be in inverse ratio of their masses, we have

$$\frac{m}{M_\star} = \frac{r_\star}{r} \tag{5}$$

We can now set a limit on the mass m of the orbital companion. Note that the orbital separation a is the sum of r and $r_\star$. Let us find out the sensitivity of detecting an orbital companion using this technique. We could ask if we can detect planets like Earth at 1 A.U. For a period of 213.132 s, equation (3) gives

$$\dot{P}_{orb}^{\mathrm{Earth}} = \frac{213.132 \text{ s}}{3 \times 10^8 \text{ m/s}} \frac{6.6 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \cdot 6 \times 10^{24}\text{kg}}{(1.5 \times 10^{11} \text{ m})^2} \tag{6}$$

$$\dot{P}_{\mathrm{orb}}^{\mathrm{Earth}} = 12.5 \times 10^{-15} \text{ s/s} \tag{7}$$

This technique is ultimately sensitive enough to find planets like Earth at 1 A.U. as the $\dot{P}_{orb}$ is about 4 times larger than $\dot{P}_{cooling}$. For a planet like Jupiter ($M = 318\ M_\oplus$) at 5.2 A.U., we have

$$\dot{P}_{\mathrm{orb}}^{\mathrm{Jupiter}} = 12.5 \times 10^{-15} \frac{318}{(5.2)^2} \tag{8}$$

$$\dot{P}_{\mathrm{orb}}^{\mathrm{Jupiter}} = 1.5 \times 10^{-13} \text{ s/s} \tag{9}$$

So, planets the size of Jupiter are easier to detect than Earth like planets. If R548 were to be in a binary system with a brown dwarf ($M_\star = 0.08 M_\odot$), then what could be their maximum separation for the brown dwarf to be detectable, assuming that the plane of the orbit is not perpendicular to the line of sight. The angle of inclination plays a role in the size of $\dot{P}_{\mathrm{orb}}$, apart from the mass and size of the orbital companion.

$$6 \times 10^{-15} = 12.5 \times 10^{-15} \frac{26\,635}{a^2 \sin i^2} \tag{10}$$

where we have assumed the total upper limit of the observed $\dot{P} \simeq 6 \times 10^{-15}$ to be due to binary nature, and a mass for the companion of $26\,635\ M_\oplus$ ($0.08 M_\odot$).

$$a_{\max} \sin i = 235 A.U. \tag{11}$$

4.2. Proper Motion

Pulsating WDs also have a non-evolutionary secular period change due to proper motion. The size of this effect on $\dot{P}$ was evaluated to be of the order of 10^{-15} s/s (Pajdosz 1995). This effect is insignificant for the DOVs and the PNNVs. However, it is of the same order as the $\dot{P}$ measured for the hot DAVs.

Any motion (with a uniform velocity) of a DAV along the line of sight will manifest itself as a correction in the period estimate for the pulsations and will not affect the $\dot{P}$. However, motion perpendicular to the line of sight is equivalent to a centripetal acceleration, with Earth as the reference. This causes a $\dot{P}_{pm}$. Pajdosz (95) has evaluated this to be of the order of

$$\dot{P}_{pm} = 2.43 \times 10^{-18} P[s](\mu[''/yr])^2(\pi[''])^{-1} \tag{12}$$

where μ is the proper motion and π is the parallax. $\dot{P}_{pm}$ is always positive and should be subtracted out from $\dot{P}_{obs}$.

For the 213.132 s period in R548, the term $\dot{P}_{pm}$ is of the order of $(2.2 \pm 0.4) \times 10^{-15}$ s/s (Pajdosz 1995). So, the true $\dot{P}_{cooling}$ is $(0.7 \pm 2.8) 10^{-15}$ s/s. At 1σ level, we are placing a harder limit as $\dot{P} \leq 3.5 \times 10^{-15}$ s/s. While for G117-B15A, the magnitude of the correction is estimated to be $(0.92 \pm 0.5) \times 10^{-15}$ s/s (Kepler et al. 2000), smaller than in the case of R548, because G117-B15A is farther away. For G117-B15A, the corrected $\dot{P}_{cooling}$ then stands at $(1.4 \pm 1.5) \times 10^{-15}$ s/s, which places a 1σ limit of $\dot{P} \leq 2.9 \times 10^{-15}$ s/s. These are much tighter constraints on the evolutionary models. Bradley (1996) gives a 3σ upper limit of $\dot{P} \leq 6.5 \times 10^{-15}$ s/s, corresponding to a time-scale of 1.2 Gyr.

5. Implications related to Asteroseismology

5.1. Trapped Modes

In WDs, compositional stratification occurs due to prior shell burning stages and gravitational settling. Hydrogen, if present, floats on the surface. In such WDs, there is a mechanical resonance effect between the local g-mode oscillation wavelength and the thickness of one of the compositional layers. This mechanical resonance serves as a stabilizing mechanism. So, the trapped modes are more stable than un-trapped modes. The modes having nodes in the vicinity of the H/He interface tend to be reflected and are consequently trapped in the outer H layer. Such modes are energetically favorable, as the amplitudes of their eigenfunctions near and below the H/He interface are smaller than otherwise. Mechanical damping is more prominent in the core than in the envelope and therefore modes trapped in the envelope can have kinetic oscillation energies lower by six orders of magnitude, as compared to the adjacent non-trapped modes (Winget, Van Horn & Hansen 1981). This filter mechanism may help explain why all the modes expected from theoretical models are not actually observed in the ZZ Ceti stars (Winget, Van Horn & Hansen 1981; Brassard et al. 1992). Note that the H/He interface can also result in confinement or trapping of modes in the core, but these do not have dramatic effects like the modes trapped in the envelope (Bradley 1993).

If the 213 s doublet in R548 and the 215 s mode in G117-B15A consist of trapped modes, then indeed we could be measuring the stability of the trapping mechanism and not the cooling. Theoretical calculations indicate that trapped modes would have a $\dot{P}$ smaller than indicated due to cooling by a factor ≤ 2 (Bradley 1993). We do not have true measurements for the $\dot{P}$s, and only upper limits, so we cannot tell whether or not these modes are trapped. If we assume the mode identification made by Bradley (1998) that these are low k modes, then we are measuring $\dot{P}$s due to cooling, as the low k modes sample the deep interior and thus have a rate of period change that reflects cooling alone. High k modes have regions of period formation further out in the star. They can therefore be more easily affected by magnetic fields, rotation, convection and other non-linear interactions.

The uncertainties in measuring $\dot{P}$ go down as the square of the time-base, if we continue to get the same quality of data as before. This implies that to decrease the errors by a factor of 2, we would need about 52 years of data, for both R548 and G117-B15A. But, if we acquire very good quality data spanning 15 nights on a 4 m telescope, every decade, then such data sets would place tighter constraints on the $\dot{P}$, reducing the errors much faster than the square of time.

5.2. Avoided Crossings

Different pulsation modes sample different regions of the star. Hence, in general, the rates of period change need not be the same. Consider one trapped and one un-trapped mode, so one is changing faster than the other. If the two modes have frequencies very close to each other, then it is possible to interchange their natures. Such an interaction is termed as an avoided crossing (Aizenman, Smeyers and Weigert 1977; Christensen-Dalsgaard 1981). Trapped modes are more stable than un-trapped modes in general, but they do show an instability in the region of an avoided crossing. In other words, if you were monitoring the $\dot{P}$ for any of these modes, you would observe a rapid change during the crossing, i.e., the $\ddot{P}$ term would be important. The 274 s doublet in R548 could be undergoing such an avoided crossing, and we plan to investigate this possibility more thoroughly.

5.3. Ensemble Asteroseismology ?

We find that the 213 s doublet in R548 and the 215 s mode in G117-B15A show similar values for $\dot{P}$. The 274 s doublet in R548 shows a long-term detectable change in period on time-scales between 10^{-11} and 10^{-14} s/s, which is faster than cooling. The physical phenomenon causing the 274 s doublet to show variations at such time-scales is not yet understood. We need to investigate the 270 s mode in G117-B15A. If it behaves similarly, then that would prove to be a vital clue towards ensemble asteroseismology.

6. A Summary of the Results

Measuring a rate of change of period with time is crucial to understanding white dwarf evolution. We can measure the core composition of a pulsating

WD by comparing the $\dot{P}$ obtained to theoretical evolutionary models. We can learn about crystallization and phase separation by measuring a $\dot{P}$ for massive crystallized pulsators, with a baseline of 10 years. We can hope to detect the presence of orbital companions like planet Earth at 1 A.U. or brown dwarfs closer than 235 A.U., that DAVs like R548, G117-B15A or L19-2 might have. Given the reports of planets around neutron stars this may be a particularly important constraint. We also hope to learn about asteroseismological effects like mode trapping and avoided crossings from $\dot{P}$ values, as this is the only current technique to study them observationally.

We would like to acknowledge the NSF grant AST-9876730, NASA grant NAG5-9321 and Pronex (CNPq/Brazil) for supporting this work.

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