Classnotes 15

Evolution of stars beyond the main sequence depends, of course on the initial mass of a star.

$M > 8M_\odot$ – Star evolves to an iron core and explodes as a Type II supernova. Remnant is either a neutron star or a black hole.

$M > 0.8M_\odot$ but $< 8M_\odot$ – Star evolves to a red giant and sheds its outer envelope to form a planetary nebula with the core remaining to cool as a white dwarf.

$M < 0.8M_\odot$ – Evolution is too slow for any star to have consumed its supply of hydrogen.

The following piece on 'Evolution of low-mass stars' is adapted from "The Physical Universe" by F. H. Shu.

Ascending the Giant Branch

To fix ideas, let us first discuss a low-mass star like our Sun. After hydrogen has been exhausted in the core, heat continues to leak out. Since there is no more nuclear energy generation in the core to make up the deficit, the core must contract gravitationally, much as Kelvin and Helmholtz originally envisaged in the nineteenth century. As the core contracts, it heats itself up as well as the layers just above it. At the new higher temperatures, hydrogen can begin to burn in a shell just outside the hydrogen-exhausted core (Figure 8.3). The helium core itself still has no nuclear-energy generation, and as it continues to lose heat to the cooler overlying layers, it must continue to contract. This contraction is abetted as the surrounding hydrogen-burning shell drops more and more helium "ash" onto the core. The shrinkage of the core accompanied by the addition of more mass makes the gravity at the border of the core stronger and stronger, so the gravitational field felt by the hydrogen-burning shell becomes stronger and stronger. But the pressure in the shell equals the weight of a column of material of unit area above it. This pressure must therefore try to increase to counterbalance the increasing gravity of the core. The pressure of the ordinary gas in the shell can be increased, in accordance to the perfect-gas law, either by raising the density or by raising the temperature. In fact, both occur, and both increase the rate of hydrogen burning in the shell.

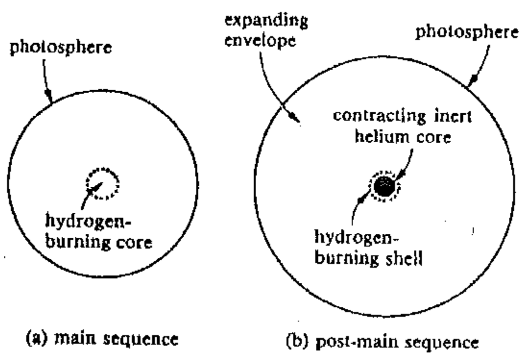

Figure 8.3 The structure of a star (a) on the main sequence and (b) as it begins to leave the main sequence because of core-hydrogen exhaustion.

However, not all the high luminosity generated in the shell finds its way to the surface. As long as the envelope remains **radiative**, the luminosity that it can carry is limited by the photon diffusion rate. The latter is nearly fixed for a star of given mass. The difference between the luminosity generated in the shell source and that leaving the surface goes into heating up the intermediate layers, causing them to expand. This expansion increases the total radius R; given a nearly constant value for the surface luminosity L, there must be a decrease of the effective temperature T_e, in accordance with the relation $L = 4\pi R^2 \sigma T_e^4$. The immediate post-main-sequence evolution of a radiative star therefore moves the star's position more-or-less horizontally to the right in H-R diagram, turning the dwarf star into a **subgiant**. The cooling and expanding surface layers cause the star to turn *red* in outward appearance.

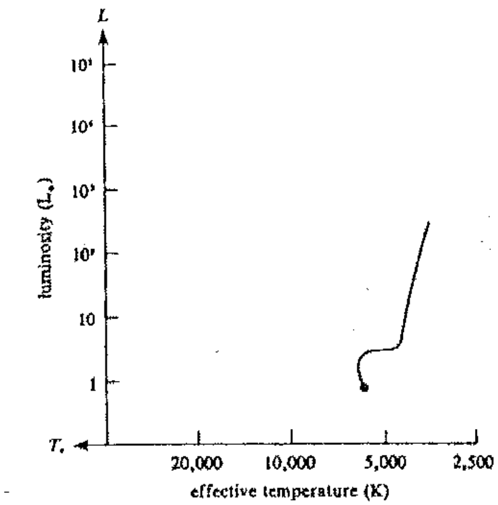

Figure 8.4 Ascent of a low-mass star to the red-giant branch. (adapted from Icko Iben, *Ann. Rev. Astr. Ap.*, 5, 1967, 571).

As the star expands, however, the effective temperature cannot continue to fall to arbitrarily low values. Sooner or late, the tracks of low mass stars travel almost vertically upward, turning the red subgiant into a **red giant** (Figure 8.4). The accompanying increase in the amount of shell luminosity which makes its way to the surface is too much for radiative diffusion to carry outward stably, and the entire envelope of the red giant becomes **convective** (Figure 8.5).

Meanwhile, the core continues to contract, and in a low-mass star, the free electrons become so tightly packed that they become degenerate. If we could artificially peel off the overlying layers of a red giant at this point, the core would be essentially a low-mass (about $0.4M_{\odot}$) helium white dwarf. The very large border gravities associated with this "white dwarf" cause the hydrogen in the shell source to burn furiously, sending the star quickly up the red-giant branch. At the tip of the red-giant branch (in Figure 8.4), the temperatures in the core rise to about 10^8 K, which is high enough to ignite helium and "burn" it into carbon via the triple-alpha process.

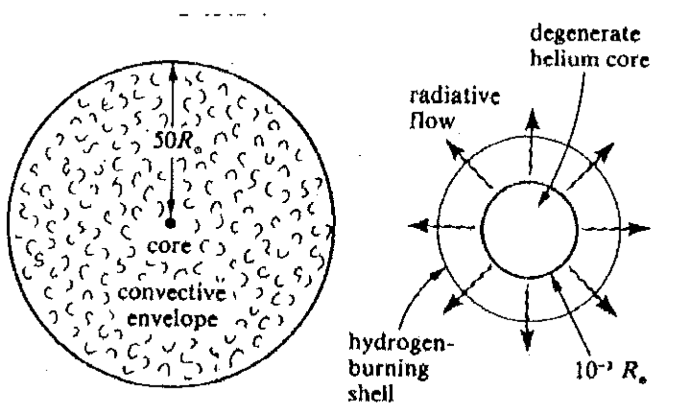

Figure 8.5 The structure of a red giant. The left figure shows the entire star from core to photosphere. The right figure shows an enlarged picture of the region near the core. Notice that the core, which may contain about half the total mass of a low-mass star at this point, occupies only one ten-billionth of the total value.

The Helium Flash and Descent to the Horizontal Branch

The ignition of helium in the core of a low-mass star, however, occurs under degenerate conditions; so it lacks the safety-valve feature which characterizes core-hydrogen burning on the main sequence (Thursday's Handout – also Seeds Box 10-1 – will explain what is meant by 'degenerate' conditions.) There, a slight increase in the core temperature will lead to a pressure change that decreases the temperature. Here, the pressure increases primarily because of degeneracy effects, not because of thermal motions; so an increase of the core temperature leads to an overproduction of nuclear energy without a compensating pressure increase and a compensating expansion. Hence, an increase of the temperature, once helium has been ignited, tends to lead to a runaway production of nuclear energy:

Higher temperatures → more nuclear energy

→ even higher temperatures, etc.

Therefore, helium burning turns on in low-mass stars with a "flash". So much energy is released in the "flash" that the core's temperature rises enough to remove the degeneracy. Normal thermal pressure then dominates over electron-degeneracy pressure, and the core expands. This expansion lowers the border gravity of the core, which weakens the hydrogen-shell source. Thus, although the star now has two nuclear energy sources — helium burning in the core, and hydrogen burning in a shell — the prodigious shell source is now so weakened that the star actually produces *less* luminosity than before. The lowered total luminosity is too little to keep the star in its distended red-giant state, and the star both shrinks in size and becomes intrinsically dimmer (Figure 8.6).

After the helium flash is completed, the core contains an ordinary (i.e., nondegenerate) helium plasma which is stably fusing helium into carbon. Surrounding this core is a hydrogen-burning shell, whose strength depends on the mass of the overlying envelope. This state of core-helium burning and shell-hydrogen burning is called the **horizontal branch** (Figure 8.7). The exact location of a horizontal-branch star in the theoretical H-R diagram depends not only on its initial mass and chemical composition of the main sequence, but also on the amount of envelope mass lost by the star as it ascended the red-giant branch. This mass loss can be expected because, with the low gravity at its distended surface, a red giant finds it even more difficult to retain coronal gas than the Sun. Theoretical reasoning alone cannot yet predict quantitatively the amount of mass loss to be expected, but

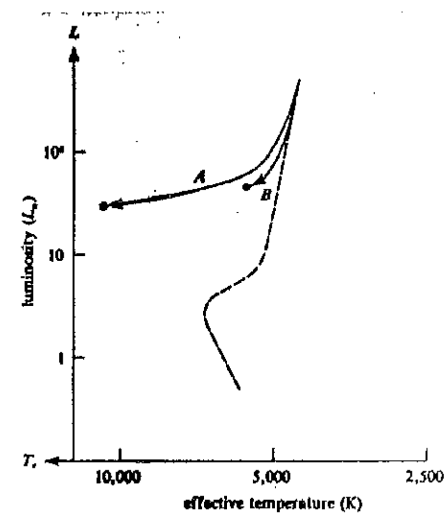

Figure 8.6 Descent of a low-mass star with poor heavy-element abundances (Population II star) from the tip of the red-giant branch to the horizontal branch. Track A corresponds to a star which suffered a relatively large loss of mass during the red-giant phase of stellar evolution. Track B corresponds to a star which suffered relatively little loss of mass. (Adapted from Icko Iben, *Ann. Rev. Astr. Ap.*, 5, 1967, 571.)

observations of red-giant stars show substantial mass loss. For a group of stars which start on the lower main sequence with similar initial masses and chemical compositions, the stars which lose more mass on the red-giant branch end up on the horizontal branch with smaller envelope masses and, therefore, weaker shell sources in addition to the core source. In particular, a horizontal branch star which had lost all its envelope mass would be a chemically homogeneous helium star burning helium into carbon at its center. Such a state is often called the "helium main sequence" by analogy with the usual (hydrogen) main sequence. Even a relatively low mass (say, $0.5M_{\odot}$) helium star appears quite blue because of its moderately large luminosity and moderately small radius. Most horizontal-branch stars, however, have a finite envelope mass on top of such a helium-burning core, and the hydrogen-burning shell associated with the weight of this envelope keeps the envelope relatively distended. Thus, true horizontal-branch stars tend to have slightly higher luminosities than a "helium-main-sequence" star of the same core mass, as well as appreciably lower effective temperatures. If we started with a group of starts, therefore, with similar initial masses and chemical compositions, and if this group suffered varying amounts of envelope-mass loss ascending the red-giant branch, we should expect them to end on the horizontal branch with nearly the same luminosities but with different effective temperatures. In other words, such a group would occupy a horizontal locus in the H-R diagram, and this feature gives these stars their name.

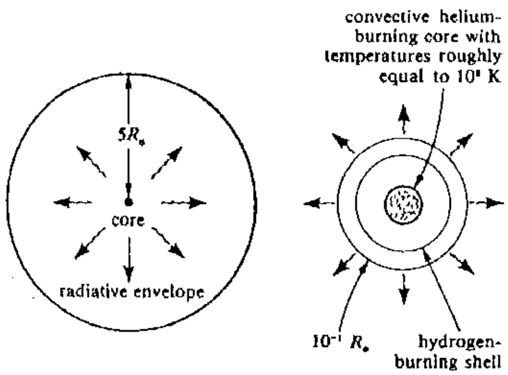

Figure 8.7 The structure of a horizontal-branch star. The left figure shows the entire star from core to photosphere. The right figure shows an enlarged picture of the region near the core.

Ascending the Asymptotic Giant Branch

What happens when helium in the core of a horizontal-branch star is exhausted (by burning into carbon and oxygen)? Well, obviously, the core must contract, which increases the pressure and temperature of the overlying layers. Thus, helium ignites in a shell just outside the core, and hydrogen burns in a shell outside of that. The star is now in a double-shell-burning stage. The mass of the inert carbon-oxygen core continues to increase, and it continues to contract just as the helium core did when the star ascended the red-giant branch again, and the double-shell-source phase is also known as the **asymptotic giant branch** (Figure 8.8). Eventually, the shrinkage of the core again causes the free electrons to become degenerate. If the overlying envelope were now to be stripped away (say, by mass loss), the core would be a hot carbon-oxygen white dwarf. Now, however, the mass of the degenerate core is larger than before because of the additional "ash" in the core, and the radius of the incipient white dwarf is smaller than its helium counterpart at the tip of the red-giant branch. Thus, the gravity of any overlying shell sources would be correspondingly larger, forcing them to generate higher luminosities yet. Stars at the end of the double-shell-burning phase may become red **super-giants**. At such tremendous rates of expenditures of energy, the star cannot live much longer.

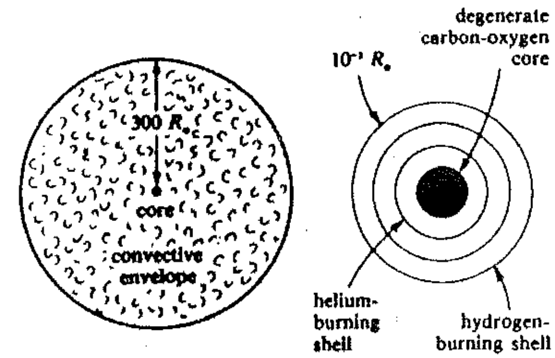

Figure 8.8 The structure of an asymptotic giant. The figure on the left shows the entire star from core to photosphere. The figure on the right shows an enlarged picture of the region near the core.

What happens to stars at these very late stages of stellar evolution is theoretically quite uncertain; several complications make detailed computation difficult. One of these is the onset of "thermal relaxation oscillations" when the helium-shell source becomes spatially very thin. The origin of this instability is very different from the "helium flash" discussed earlier. There, an initial overproduction of nuclear energy leads to a runaway because of the degeneracy of the nuclear-burning region. Here, an initial overproduction of nuclear energy also leads to a thermal runaway, but for an entirely different reason. Here, the nuclear burning region is nondegenerate, but it is a spatially thin shell. Thus, with the input of excess nuclear energy, the layer can and will expand. But the expansion of a thin shell does little to relieve the weight of the overlying material;

this material is lifted only a little. Thus, the weight hardly changes, and therefore the pressure that the thin shell has to maintain to offset this weight also hardly changes. Meanwhile, the temperature has increased, and if the rate of nuclear-energy generation sufficiently sensitive to temperature changes—as the triple-alpha reaction is—then it will also increase further before the excess heat has a chance to diffuse away. Thus, a thermal runaway ensues. The runaway is checked only after considerable expansion of the layer and the appearance of convection to carry away the excess heat. But the basic problem remains. After the runaway is checked, and when the star tries to adopt the "natural" double-shell-burning configuration appropriate for this stage of its evolution (i.e., when it tries to "relax" back to the "natural" stage of equilibrium), it finds itself in the same difficulty. Thus, the star undergoes a series of "thermal relaxation oscillations" which consist of one or more sharp pulses of extra energy generation followed by relatively long periods of quiet evolution (Figure 8.9). Each thermal runaway is followed by the development of a convection zone which extends from the helium-burning shell almost to the hydrogen-burning shell.

The inner convection zone may actually connect to the outer convective envelope through the hydrogen burning shell (see Figure 8.8). If this happens, the products of helium burning, carbon and oxygen, as well as *s*-process material, may be brought to the surface of the asymptotic red giant.

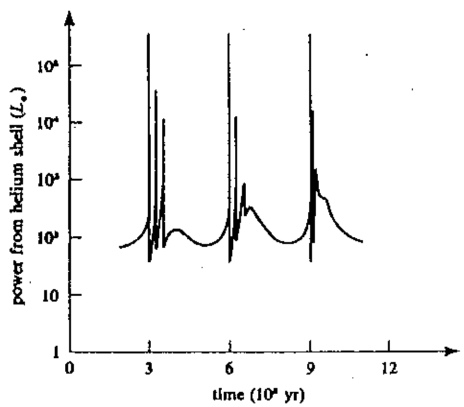

Figure 8.9 Thermal relaxation oscillations associated with helium-shell flashes. (Adapted from M. Schwarzschild and R. Härm, *Ap.J.*, 150, 1967, 961.)

Planetary Nebulae and White Dwarfs

Another complication is that considerable mass loss may occur on the asymptotic giant branch. Observations suggest that asymptotic branch stars beyond the red-giant tip lose mass very rapidly. Among the many promising mechanisms which have been considered is the suggestion that small specks of dust may form in the cool atmospheres and be driven out subsequently by the radiation pressure of the star. Quantitative calculations, unfortunately, are difficult. Observations indicate that stars that originally had less than about six solar masses seem to lose so much mass during such high-luminosity stages that they become (perhaps periodically) planetary nebulae illuminated by a hot central core. This hot central core is presumably an incipient white dwarf, with mass necessarily below the Chandrasekhar limit $1.4M_\odot$. From the central-star stage of planetary nebulae, the exposed core burns out its hydrogen and helium shells, loses its extended envelope, and descends the H-R diagram to enter the region occupied by white dwarfs proper. Figure 8.10 summarizes the complete evolution of a low-mass star from the main sequence to carbon-oxygen white dwarf. The final approach to a white dwarf from an asymptotic giant star is shown in dashed lines, to emphasize that the theory in incomplete for these late stages of stellar evolution.

Take a good look at Figure 8.10. You may be staring at the future of the Sun. When the Sun swells up to become a red giant for the first time, it will occupy about 30° in the sky. What a sunset that would make! When the Sun swells up for the second time (assuming it does not lose so much mass that it becomes a helium white dwarf), it may well engulf the Earth. Any inhabitants, of course, would long since have been roasted. Be consoled at least that the Sun itself will ultimately be able to rest in peace as a senescent white dwarf.

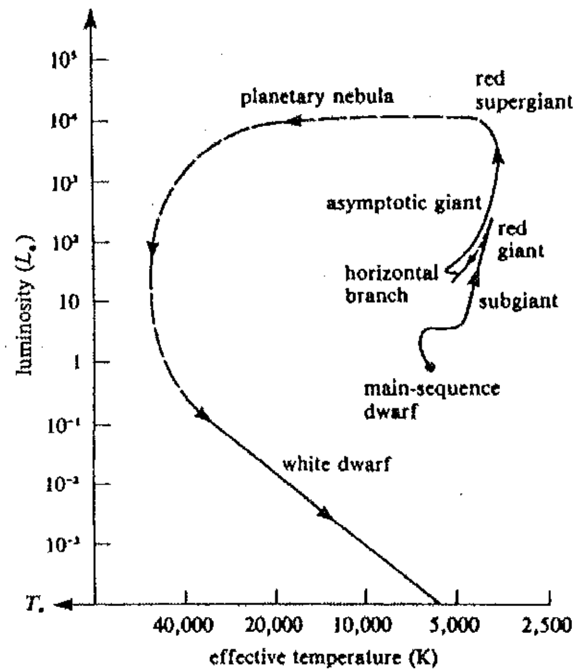

Figure 8.10 The complete evolution of a low-mass star from the main sequence to a white dwarf. The track from the asymptotic giant branch to a white dwarf (via a planetary nebula) is uncertain and is shown as a dashed curve.