CLASSNOTES 13

The next several classes will cover the topic 'Lives of the Stars' or 'Stellar Evolution'. Evolution of stars is almost entirely directed by their gravity (really, self-gravity) and, hence, by their mass. Although distinguishing characteristics such as chemical composition, rate of spinning (angular rotation rate), magnetic field play a rôle, the evolution of a single star is effectively directed by its mass at birth. Stars lose mass by a variety of mechanisms so that at their death the remaining mass is generally much less than the mass at birth.

1. **I shall begin by discussing the internal structure of the Sun and how we *know* it must be evolving, albeit slowly evolving. Here is a summary of the argument:**
   - The Sun is a massive object (terrestrially speaking), and being gaseous, should collapse under its own gravity in a *few minutes*.
   - The Sun is not collapsing this rapidly. Therefore, gravity must be opposed by another force. The only reasonable candidate is gas pressure (steam lifts the lid of the saucepan). If the Sun at each layer is hotter and denser on the inside of the layer than on the outside, there is a net (inside – outside) gas pressure directed outwards that may balance the inward force due to gravity. Then, it's a simple calculation – really, it is!—to show that the central temperature must be around 15 million degrees.

     This cannot be a *permanent* state of affairs. But as heat cannot escape quickly, an immediate collapse is avoided. Nevertheless, loss of heat reduces the gas pressures and induces a collapse – a slow collapse. Geological records, however, suggest the Sun has remained the same size for aeons. Hence, we must argue that the Sun must be generating energy to replace the escaping heat. This generation must take place at 15 million degrees.

   This argument does *not* identify the source of energy.

2. **Next, it is helpful to discuss how energy is transported from hot places to cool:**

   •radiation •convection •conduction

   The life of a star is controlled in very large part by the efficiency of the energy transport from the central regions to the surface. If energy is transported very efficiently, the core – in the absence of an energy source – would cool quickly and a collapse would quickly ensue. The star responds by burning fuel to replace heat as it is lost. Thus, more efficient transport of heat leads to fiercer burning and to a quicker exhaustion of fuel.

3. **What is the Sun's Energy Source?**

   A key constraint on a potential source is that it be adequate to power the Sun for what we think is its lifetime.

   A minimum requirement is that the estimated lifetime of the Sun exceed the age of the Earth where the latter is now accurately known to be 4 1/2 billion years.

   For a proposed energy source, we may predict a lifetime:

$$\text{Lifetime} = \frac{\text{(Fuel Supply in tons)} \times \text{(Efficiency as energy per ton)}}{\text{Energy output per second}}$$

   The required energy output per second is set by the observed luminosity of the Sun.

The Sun's Energy Source? It cannot be the astronomical equivalent of a terrestrial fossil fuel such as West Texas crude. The reason is that such fuels which involve rearrangement of molecules, atoms, and their electrons release very small amounts of energy for a given tonnage of fuel. In other words, their efficiency per ton is low. The Sun would run out of fuel very quickly, say in 1000 years.

Gravitation is a more serious possibility: i.e., a slow contraction of the Sun converting gravitational potential energy to heat and light. We call this a Kelvin-Helmholtz contraction. It is a fairly efficiently process. Contraction by 0.1 miles per year suffices to generate the observed energy output of the Sun. (Is this measurable?)

Contraction over the long haul adds up. About 20 million years ago, the Sun's radius would have been the size of the Earth's orbit had the Sun's luminosity been constant. This is a problem as we know the Earth is 4.5 billion or 4500 million years old. We shall discuss Lord Kelvin's estimates for the ages of the Earth and the Sun.

Although we can dismiss gravitation as the present source of the Sun's energy, keep gravitation at the forefront of your mind whenever we discuss stellar evolution.

Nuclear fusion energy is the present source of the Sun's energy. Einstein showed that energy and mass were equivalent. In principle, mass is convertible to energy and vice versa. Conversion proceeds according to the formula $E = mc^2$.

If the Sun converted mass with 100% efficiency to energy, it could live for about 10,000 billion years shining at its present rate.

A more probably process is for hydrogen to be converted to helium. This process is most simply written as

$$4H \rightarrow He + Energy.$$

A He nucleus is slightly lighter than four protons (nuclei of H atoms). The difference is 0.7% in mass. Then, the H to He conversion gives a lifetime of 0.7% of 10,000 billion years or 70 billion years. But only H in the hot core of the Sun can 'burn,' say the innermost 10% giving a lifetime of about 7 billion years.

Nuclear burning of hydrogen to helium does not occur through a simultaneous collision of 4 protons. As Seeds explains, the process is achieved through a series of collisions each involving two particles – see his notes on the pp chain and CNO-cycle. The net effect of the series is:

$$4H \rightarrow He + Energy.$$

4. **Lifetimes of Main-Sequence Stars**

The formula is, as before,

$$\text{Lifetime} \propto \frac{\text{Fuel Supply}}{\text{Rate of Fuel Consumption}}$$

The fuel supply is proportional to the stellar mass. The rate of consumption is indicated by the stellar luminosity. Then,

$$\text{Lifetime} \propto \frac{\text{Mass}}{\text{Luminosity}}$$

but the Mass-Luminosity relation tells us that $L \propto M^4$.
Then,

$$t \propto \frac{M}{L} = \frac{M}{M^4} = \frac{1}{M^3}$$

The table shows the estimated lifetimes of main sequence stars.

MASS ($M_\odot$)	$\frac{L}{L_\odot}$	$\frac{t}{t_\odot}$	t(yrs)
0.1	1/10000	1/1000	10,000 Billion
0.5	1/16	1/8	80 Billion
1	1	1	10 Billion
5	625	1/125	80 Million
10	10,000	1/1000	10 Million
50	6,250,000	1/125,000	80,000
100	100 Million	1/1,000,000	10,000

This table differs slightly from Table 13-2 on page 247 because I adopt $L \propto M^4$.

Note the enormous range in the lifetimes from 10,000 yr for M=100 $M_\odot$ to 10,000 billion years for M = 0.1 $M_\odot$. We shall see later that the age of the Universe is most probably about 15 billion years old.

5. **The 'fundamental forces'**

The following piece is again from Clark's book 'Stars and Atoms.'

The four fundamental forces of nature – gravity, electromagnetism, the strong nuclear force and the weak nuclear force – apply to every particle of matter in the Universe. In fact, it is through the fundamental forces that separate pieces of matter "communicate" with each other. The four forces are of different strengths and apply on different scales: planets orbit the Sun because of gravity, but electrons orbit atomic nuclei because of electromagnetism. Each of these forces is carried out by a different kind of virtual particle.

Of the four fundamental forces, we are most familiar with gravity and electromagnetism, because their efforts are most obvious in the world around us. The other two forces act only within atomic nuclei, so they are less noticeable.

Gravity is the natural force of attraction that acts between objects with mass. The greater the mass of two objects, the greater the force of their mutual attraction. The farther the objects are from each other, however, the weaker is the force of gravity between them. This is because gravity follow an inverse-square law: if the distance between the two objects is doubled, the force between them is quartered. Gravity is the weakest of the fundamental forces, yet it has an unlimited range. It shapes the Universe on its largest and most dramatic scales, because it acts over such vast distances throughout space.

The electromagnetic force acts between all particles with an electric charge, such as electrons, protons and ions. It is the driving force in all chemical reactions, which rely on interactions between electrons in order to form molecules. The force consists of two interconnected forces, electricity and magnetism. A moving particle with an electric charge creates a magnetic field, whereas a magnetic field around a conductive substance induces charged particles to move.

The electromagnetic force is different from gravity in that, as well as an attractive, it also has repulsive element. These are characterized by assigning positive and negative signs to charges to show their polarity. Unlike charges attract, whereas like charges repel. This is why negatively-charged electrons remain in orbit round positively-charged atomic nuclei. The electromagnetic force is similar to gravity, however, in that it, too, follows an inverse-square law. Although it is stronger than gravity, it does not dominate the structure of the Universe because, over large volumes, any overall positively- or negatively-charged regions cancel out each other.

The nuclear forces are extremely strong, but they are confined to the nuclei of atoms. The strong nuclear force – the strongest of the fundamental forces – acts only over a distance comparable to the diameter of a proton or a neutron about 10^{-15}m. It holds protons and neutrons together to form atomic nuclei.

This is the force that must be overcome in nuclear fission in order to "split" the atom.

The weak nuclear force has an even smaller range than the strong nuclear force – only about 10^{-18}m, the diameter of an electron. Within its range, it is stronger than gravity, but not quite as strong as electromagnetism. It governs the creation and interaction of the elementary particles known as neutrinos. These are created when neutrons become protons or protons become neutrons. Neutrinos interact weakly with atomic matter because it is necessary for them almost to touch the nucleus before the weak force can cause them to interact.

Here are familiar examples in which one of the few fundamental forces dominates:

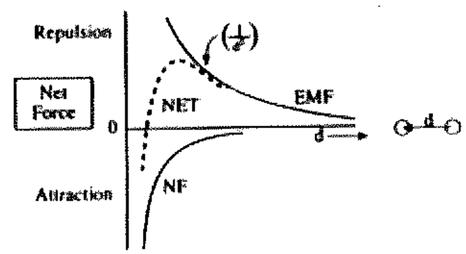
- Electromagnetism binds atoms as opposite charges attract. Lightning occurs when the ground and the clouds are at different electrical potentials. To equalize this, charges (electrons) move from the negative region to the positive region.
- All particles of matter are mutually attracted due to their mass, resulting in the fundamental force called gravity. The weakest of the forces, it controls the movement of planets, stars and galaxies, as well as holding objects on the Earth's surface.
- The weak nuclear force is stronger than gravity but weaker than either the strong nuclear force or electromagnetism. It governs the radioactive decay of some atoms, becoming very active every time a hadron (a particle composed of quarks, such as a proton) turns into a new hadron (such as a neutron) and a lepton (such as an electron, which is not composed of quarks). Some methods of radioactive dating rely on this decay process. Everytime such a reaction takes place, a neutrino is released or absorbed.
- The strong nuclear force, which holds together the nucleus of an atom, must be overcome to produce a nuclear explosion. When this happens, vast amounts of energy are liberated. Nuclear reactors do the same but in a controlled way, and use the heat to drive turbines and generate electricity.

6. Nuclear Fusion

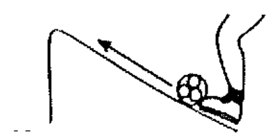
Let's return to the question of fusion. Note first the different ranges of the electromagnetic (the repulsion between the nuclei) and the nuclear forces: the former is 'long-range' and the latter very much short-range (nuclei in contact before force is effective).

Two nuclei, of course, positively charged must repel each other due to what we call the electromagnetic force. Why then may the nuclei come into contact and fuse? The answer is that the nuclear force is attractive (i.e., pulls the nuclei together). The nuclei move in response to the net force: nuclear – electromagnetic.

We can depict the changing situation as two nuclei approach by the following graph:



An analogy may help:



In other words, it takes an initial high energy (high temperature) for the two nuclei to get to the 'crest of the hill' and to feel a net attractive force.

Is fusion guaranteed to release energy? Main sequence stars "live" by fusing 4 H nuclei (protons) into a He nucleus (also called α-particles). The gas has to be very hot before the fusion reactions are successful. As we discussed, the nuclei must collide at high velocities in order to overcome the em repulsion and so get them within the short range of the sn force. Recall the analogy of kicking a ball up a hill: a gentle kick is insufficient to get the ball to the crest and it rolls back down.

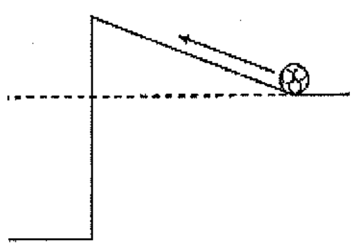
The strength of the repulsion depends on the *product* of the charges on the two nuclei:

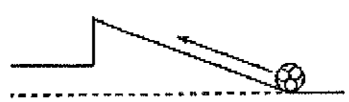
$^{1}H + {}^{1}H$	Force $\propto$	$1 \times 1 = 1$
$^{2}He + {}^{4}H$		$2 \times 2 = 4$
$^{12}C + {}^{12}C$		$6 \times 6 = 36$
$^{12}C + {}^{1}H$		$6 \times 1 = 6$

If the nuclei get very close, the sn force will exceed the em repulsion and the nuclei are pulled together. Crudely speaking this pull releases energy. For a large nucleus, say uranium, there are lots of protons and neutrons, so you might think the sn force provides a lot of energy in assembling the nucleus. But recall the short range of the sn force. In effect, the uranium nucleus is so large that protons and neutrons on the far side of the nucleus from an approaching nucleus do not provide much attraction, but the em force is a long-range force and so these protons do provide a repulsion on the incoming nucleus.

Nuclear fusion reactions operating in stars release energy as long as nuclei lighter than iron are synthesized. We shall shortly summarize the series of reactions that culminate in iron production. Thanks to the above effect, the fusion of iron nuclei to create a heavier nuclei requires a net input of energy. (See Figure 14-9, Seeds). I hope you see that this must cause a crisis for a star. We are going to discuss the crisis.

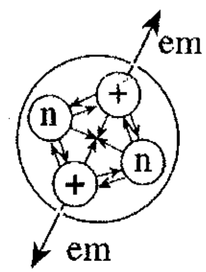
The 'iron problem' may be set in the context of the earlier analogy. H-burning and other burning stages prior to synthesis or ironmay be represented as:





Here the drop of the left releases more energy than was spent in kicking the ball to the crest (kindly ignore friction).

Why is a nucleus stable? In general, a nucleus consists of a mix of protons and neutrons. Stable nuclei have roughly the same number of protons and neutrons. The number of protons is called the atomic number (Z) and determines the chemical element: Z = 1 is hydrogen, Z = 2 is helium, Z = 6 is carbon, Z = 8 is oxygen and Z = 26 is iron, for example. Nuclei with identical Z but different numbers of neutrons are said to be isotopes of the same element. Carbon has two stable isotopes: Z = 6 (of course) for both but the neutron number N = 6 for the commonest form of C and N = 7 for the rare form. These are distinguished by writing them as ^{12}C and ^{13}C where C denotes Z = 6 and the preceding superscript is Z + N.



So the nucleus is a mix of protons and neutrons. If it is stable the forces of repulsion and attraction must balance. What are these forces. Gravity exists between the particles, but is too weak to be a factor. The weak force is not a factor either. We are left with the electromagnetic (em) force that is repulsive between protons. We are also left with the strong nuclear (sn) force. This is attractive and operates equally effectively between p, p and n, and n and n. Hence, the presence of some neutrons increases the attractive force within the nucleus, as the sketch of ^{4}He at the left shows.

I hope you can now explain why a nucleus like ^{2}He or ^{6}C (i.e., no neutrons) is *very* unstable. Can you also explain why a very neutron-rich nucleus is also very unstable, say ^{10}He or ^{20}C (even ^{5}He and ^{14}C are unstable)? In thinking about these questions keep in mind that the em force is a long range force, but the sn force is a very short range force.

7. **In what forms is energy released in nuclear reactions?**
Nuclear reactions release energy as:

- photons, especially gamma-rays.
- kinetic energy, products of a reaction fly apart. Of course, those fast moving products collide with neighboring particles, lose energy to them and, as a result, all are speeded up (heated up).
- neutrinos. These are special products in the sense that, in all but the dense cores of supernovae, the neutrinos escape their stellar interior and trundle off on a long journey through space. Neutrinos do not normally heat a stellar interior.
- Positrons. These are positively charged counterparts of the normal electrons. Very quickly, positrons encounter an electron and this is a mutual annihilation: $e^- + e^+ \rightarrow 2\gamma$ where the γ-rays (photons) are added to the pool of photons in the core.

'Heat' without further description is **not** an adequate description of form of energy.

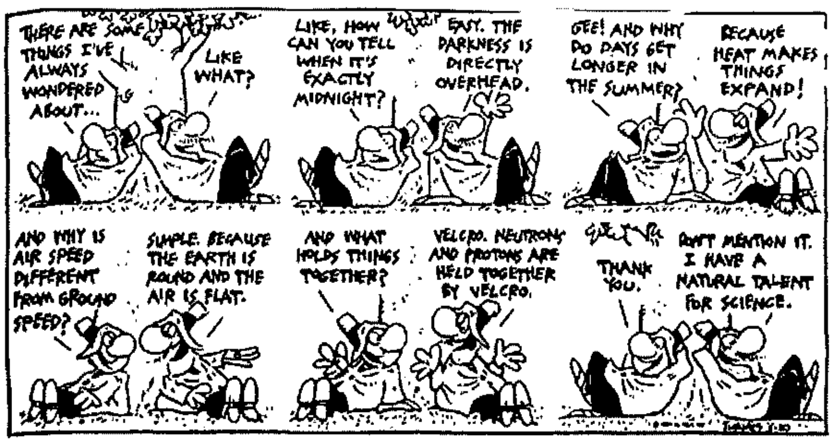


Fig. 9.9 Frank and Ernest discuss some physics problems including how velcro binds neutrons and protons into nuclei.