## CLASSNOTES 9

**Here are some notes on the structure of atoms to supplement the text.**
Around the turn of the century, it was appreciated that atoms consist of electrically charged particles, both positive and negative. The physicist J.J. Thomson proposed a model for the structure of the atom based upon a relatively large, positively charged, amorphous mass in which much smaller, negatively charged electrons are embedded. This was known as the 'plum-pudding' model. The sizes of atoms were known to be about an Ångström or so (1 Å = $10^{-8}$cm). Some of the earliest sensible conclusions concerning atomic structure were based on experiments carried out or inspired by Geiger and Marsden at the direction of E. Rutherford. These experiments involved

directing a stream of α-particles (helium nuclei) towards a piece of gold foil (Fig. 1). Radium, encased in a lead block as shown, provided a parallel beam of α-particles which carry positive charge and move at very high speeds.
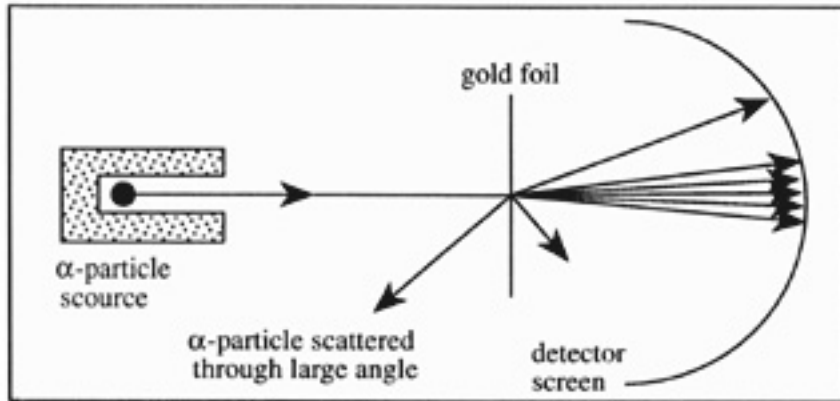


*Figure 1. Rutherford's apparatus for an analysis of α-particle scattering by gold foil.*
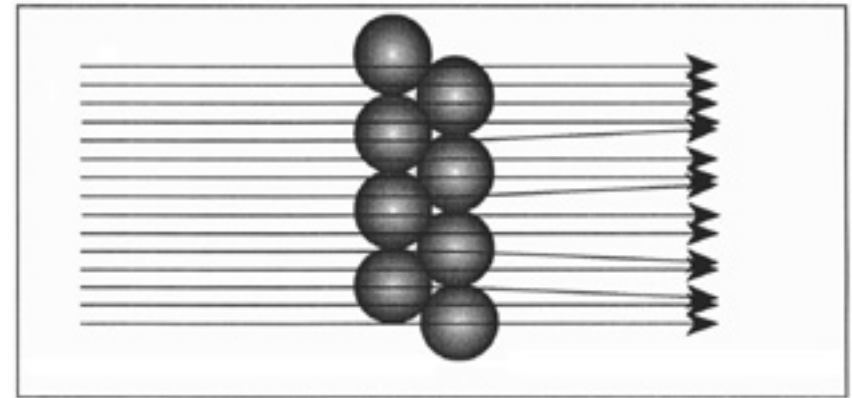


*Figure 2. The expected result of the Rutherford alpha-particle scattering experiment, assuming the Thomson model.*

It was anticipated, based upon the Thomson atomic model, that the stream of α-particles would pass straight through the foil (Fig. 2 ), with perhaps just a few particles being slightly deflected. This is because the Thomson model postulates a rather uniform distribution of matter and positive/negative charge throughout the atom. The deflections arise because the
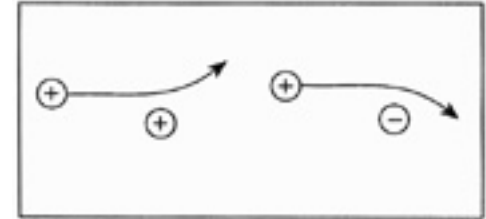


positively charged α-particles pass close to individual electrons or regions of positive charge. Recall that like charges repel and unlike charges attract as in the sketch here.

The results obtained were a great surprise to Rutherford. Certainly, most of the alpha-particles did pass straight through the foil. However, a few were deflected through quite large angles, sometimes greater than 90°. Such large deflections would not occur if the Thomson model was reasonably correct. The experimental results led Rutherford to suggest that an atom consists of a very small, but relatively massive, positively charged nucleus, surrounded by electrons (Fig. 3). The components of the atom are held together by electrostatic forces. Virtually all the mass of the atom is associated with the positively charged nucleus, even though it is so small relative to the size of an atom. The close approach of an α-particle to the massive nucleus of a Rutherford atom occasionally results in a large scattering angle.
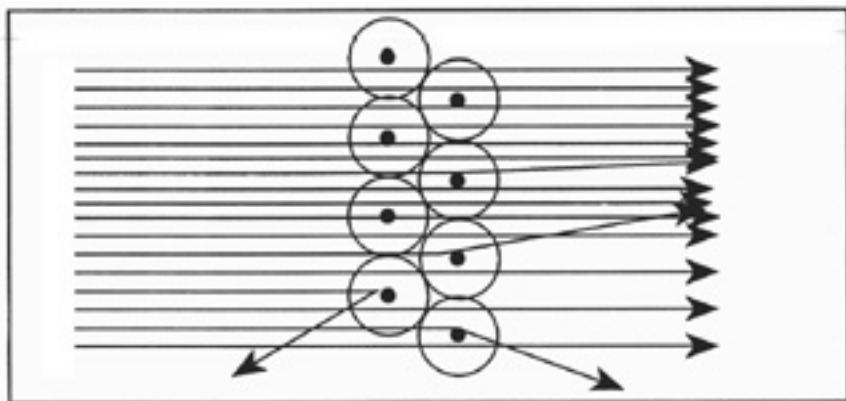
*Figure 3. The results of the Rutherford alpha-particle scattering experiment.*

Rutherford suggested that the reason most α-particles pass through gold foil with no or little deviation is simply that the atom is largely empty space and just doesn't interact much with the intruding high-speed alpha-particle. However, just occasionally, one of the alpha-particles collides with, or comes very close to the 'massive' positively charged nucleus. The gold nucleus has a positive charge of 79 units and a mass of about 197 units. The α-particle has a positive charge of 2 units and a mass of about 4 units. The repulsive force between the α-particle and the gold nucleus scales as the product of their charges and inversely as the distance between them squared. In a direct hit, the massive gold nucleus is barely moved by the colliding α-particle, which will be deflected through a large angle, even to the extent of deflecting it back along the direction from which it came. The diameter of the nucleus was shown to be about $1/10^5$ the size of the atom itself or $10^{-13}$ cm or so. Clearly these ideas portray an atom very different from the Thomson model.

Rutherford in characteristic style expressed his surprise at the results of the scattering experiment:

> *"It was quite the most incredible event that has ever happened in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you."*

We'll return to discuss the nucleus when we discuss stellar evolution. The immediate problem with Rutherford's model atom was to account for the stability of the electron cloud. According to well-tested ideas, moving electrical charges such as electrons in orbit must lose energy by emitting electromagnetic radiation, and then spiral quickly into the positively charged nucleus. But atoms are stable! The first step to the solution was made by Niels Bohr. Basically, the rules of physics are different at the atomic level. Seeds gives an adequate description of current ideas on atoms but omits to mention the names of the great pioneers.

As we shall discuss in class, the nucleus of an atom is composed of protons and neutrons. The neutron is just a little bit more massive than the proton. Both are roughly 2000 times more massive than the electrons that make up the electron cloud around the very compact nucleus. An electron carries one unit of negative charge. The proton has one unit of positive charge. The neutron, as its name implies, has no charge. A free (isolated) proton is stable -- experiments have shown that, if it decays, it lives at least $10^{33}$ years or so before decaying. A free neutron decays spontaneously in about 11 minutes. A neutron inside a nucleus can be stable. Neutrons and protons basically retain their identities when placed together to form a nucleus.

Scattering experiments done in the spirit of Rutherford's original experiment – i.e., firing high velocity charged particles at a target and observing their scattering – have shown that neither the proton nor the neutron are simple point masses with charge. Both exhibit structure. Each has been shown to be made up of three more elementary particles called Quarks.

Quarks have fractional electrical charges of either positive two-thirds or negative one-third. They can combine in twos or threes: protons, with positive charge of one, consist of two 'up' quarks and a 'down' quark; neutrons, with no electrical charge, consist of one 'up' quark and two 'down' quarks.

Our concerns end with the protons, neutrons, and electrons. There are several popular accounts of elementary particles. If I tell you the names of the six quarks – up, down, strange, charm, bottom, and top – you may be intrigued to read more. Each has an anti-particle: antiup and antitop, for example.

Quarks were once thought to be 'fundamental,' that is not themselves composed of even more fundamental particles. Now, theories suggest that quarks are made of 'strings' which are the fundamental units if matter. For a readable account of strings, and much else see 'The Elegant Universe' by Brian Greene (paperback).

**Proportions**

In discussions of many topics, it is convenient to introduce 'proportions.' As many students seem to be unduly puzzled by the concept, I offer these notes.

1. Mathematically, there is a simple relation between an equation and the concept of proportions. Consider the case of Newton's law of gravitation. Take two stars A and B. The gravitational force between them, as given by Newton's law, is

$$F = G\frac{M_A M_B}{R^2_{AB}}$$

Here the masses of A and B are $M_A$ and $M_B$ respectively and the distance between them is $R_{AB}$.

Suppose we have many pairs of stars but the separation of stars for each pair is the same. If we want to describe how the gravitational force varies from one pair to the next, we might say:

> *the gravitational force between a pair of stars is proportional to the product of the individual masses.*

It is not necessary to say anything about the separation of the two stars because it is same for all pairs. This expression does not tell us the strength of the force in any one case but simply that, from one pair to the next, the force scales as the product of the masses $M_A$ and $M_B$. If we wish to calculate the strength of the force, we do need to know the separation $R_{AB}$ and the constant G.

If, in a second family of binary stars, the masses were the same in each pair but the separations between the stars differed, we might wish to say that *the gravitational force between a pair of stars is inversely proportional to the square of the distance between them.*

 (Equivalently, '. . . is proportional to one over the square of the distance between the two stars.')

As in the first example, if we wish to calculate the strength of the force, we need to know the masses $M_A$ and $M_B$, and the constant G.

The words 'are proportional to' are represented by the symbol $\propto$, and so an equivalent statement for the second of the above examples is

$$F \propto \frac{1}{R_{AB}^2}$$

2    In this section, I attempt to show how we often use a proportionality in simple problems. I choose the case of the relation between luminosity and brightness of a star. Recall that:

**Luminosity (L)**: the total amount of energy a star given off by a star in a fixed amount of time.

**Brightness (B)**: the amount of energy from a star collected here on Earth in a fixed amount of time and, usually, we also specify the area over which the light is collected.

You will appreciate that L and B are related by the distance of the star from Earth. Two stars of the same L but at different distances will have different B; the more distant star appearing fainter.

In class we derived (Classnotes 6) the relation between L, B, and distance d, which I write as

$$B \propto \frac{L}{d^2}$$

By using the symbol $\propto$, I avoid having to introduce discussion of the units used for the B, L, and d.

I will now work a simple problem using the above expression and then do the same problem using 'the formula.'

**Problem**: Two stars are equally luminous but at different distances. One star is at 10 light years from Earth and the other at 20 light years.

*Which star is fainter?* To answer this question, we should not need to manipulate either the proportionality or the equation! The more distant star must be fainter. Its light is spread out more!

*How many times fainter is the more distant than the nearer star?* In this case, we need the proportionality (or the equation). Looking back up the page we see that, for stars of the SAME L, B scales as $1/d^2$. At 20 light years the more distant star is twice as far as the nearer one. If d increases by a factor of 2, $d^2$ increases by 2 x 2 = 4 times. Then, the star at 20 light years is 4 times fainter than the one at 10 light years.

Equation lovers and formulae fanatics might look at the problem differently. The formula required is

$$B = a\,\frac{L_1}{d^2}$$

where a is a numerical constant that ensures that $L/d^2$ is in the same physical units as B.

Number the stars 1 and 2. For star 1 we may write

$$B_1 \;=\; a\,\frac{L_1}{d_1^2}\text{, and for star 2}\;\; B_2 \;=\; a\,\frac{L_2}{d_2^2}$$

Now divide one equation by the other. This cancels out the constant a and gives the general result

$$\frac{B_1}{B_2} = \frac{L_1}{L_2}\,\frac{d_2^2}{d_1^2}$$

In the particular problem here, $L_1 = L_2$, and we are asked to find the ratio $B_1/B_2$. Then,

$$\frac{B_1}{B_2} = \frac{d_2^2}{d_1^2} = \left(\frac{d_2}{d_1}\right)^2$$

Now we may substitute 20 light years for $d_1$ and 10 light years for $d_2$ to obtain

$$\frac{B_1}{B_2} = \left(\frac{10}{20}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

which tells us that the brightness of star 1 (20 light years) is 1/4 that of star 2 (10 light years) or equivalently the more distant star is four times fainter than the nearer one. Just the answer we got using the proportionality relation.

3.  Other points about proportionality:

The proportionality

$$F \propto \frac{M_A M_B}{R^2}$$

describes how the force due to gravitation depends on the masses and their separation.

This relation cannot be expressed directly as the equation

$$F = \frac{M_A M_B}{R^2}$$

The correct equation is, as stated earlier,

$$F = G \frac{M_A M_B}{R^2} \quad \text{where G is a constant.}$$

Proportionalities can be manipulated like equations, as we saw in the example of the brightnesses of the two stars. Here is a second example.

We shall show that the luminosity (L) of a star depends on its radius (R) and surface temperature (T) such that $L \propto R^2 T^4$.

This is equivalent to

$$R^2 \propto \frac{L}{T^4}; \quad T^4 \propto \frac{L}{R^2}; \quad T \propto \left(\frac{L}{R^2}\right)^{\frac{1}{4}} \quad \text{etc.}$$

I happen to think that for us a proportionality is easier to understand and manipulate than the corresponding equation. But you may well think otherwise. No problem -- as long as we arrive at the same answer!!