CLASSNOTES 8

In class, I mentioned the relationship between frequency (f), wavelength λ, and the velocity of a wave (c): $f\lambda = c$. The derivation of this relationship is straight-forward.

You observe a wave to pass you. In 1 second you find that f waves pass you: this f is then the frequency (number of waves per second, now referred to as Hz where Hz = Hertz.) The total length of the wave that passed you is the number of waves times the length of each wave, or $f \times \lambda$ where λ is the wavelength. The speed of the wave (c) is the distance travelled in a certain period of time. In 1 second the wave travelled $f\lambda$, so speed is

$$c = \frac{\text{distance}}{\text{time}} = \frac{f\lambda}{1} = f\lambda$$

You do not need to be able to prove this result for a quiz. etc. Its significance for us lies in the fact that we may refer to a light wave by either its wavelength or its frequency. When one is specified, the other is too.
Since the speed of light is constant (in vacuum), the product $f\lambda$ is constant across the electromagnetic spectrum. The spectrum may be ordered by increasing wavelength or by increasing frequency:

$$0 \rightarrow \lambda \rightarrow \infty$$
$$\infty \rightarrow f \rightarrow 0$$

Question: There are FM and AM radio stations. Which are of longer wavelength -- FM or AM?
Example: KUT-FM has a frequency of 90 MHz. What is the broadcasting wavelength? Rearranging $f\lambda = c$, we have $\lambda = c/f$, which with $c = 3 \times 10^{10}$ cm/s and f = 90 MHz = 9×10^{7} Hz gives λ (in cm) = $(3 \times 10^{10}) / (9 \times 10^{7})$ = 333 cm or approximately 3 meters. Now do the same for your favorite AM station. Do you make the wavelength longer or shorter?

The Doppler Effect
The relative line of sight (radial) motion of source and observer leads to a shift of the recorded wavelength (or frequency, as $f\lambda = c$) of a wave.

You should be able to explain why the shift arises, why 'relative' and 'line of sight' are key terms, and why motion of approach (recession) leads to a shift to higher frequencies (shorter wavelengths) and motion of recession leads to a shift to lower frequencies (longer wavelengths).
Mathematically, the key result is that a radial velocity (v) leads to a wavelength (or frequency) shift such that

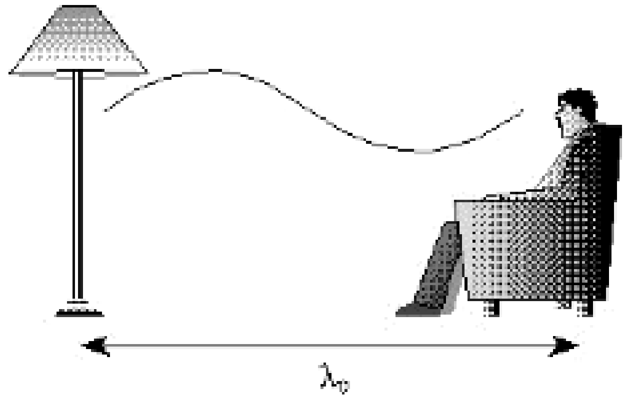
$$\frac{v}{c} = \frac{\lambda_0 - \lambda}{\lambda} = \frac{f_0 - f}{f_0}$$

where c is the speed of light, f_0 is the frequency (wavelength) in the absence of motion, and $f\lambda$ is the observed frequency (wavelength).

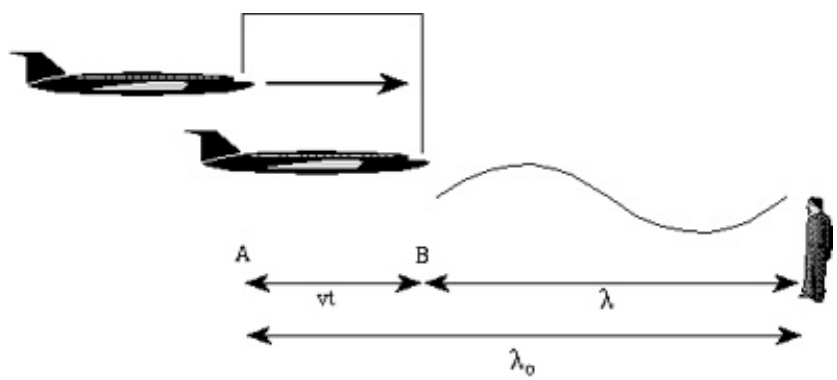
This equation is correct as long as v is very much less than the speed of light.

While it suffices for the class to understand why the Doppler shift arises and to remember the above formula, it may be of interest to know how this formula comes about.

The next figure shows a wavelength emitted by a lamp as seen by an observer who is stationary. (The length of the wave is *greatly* exaggerated.)



Now consider the situation when the lamp is mounted on a vehicle. Position A shows where the vehicle is when the beginning of the wave is emitted. Position B when the end of the wave was emitted. Recall that the speed of the light wave is not changed when it is emitted from a moving source.



The wavelength as recorded by the observer is shorter because the vehicle moved from A to B. The time for one wave to be emitted is the time taken for the wave to go the distance (lambda)0 at a speed c: $t = \lambda_0/c$. In this time, the vehicle went a distance $vt = v\lambda_0/c$.
Looking at the figure, we see that

$$\lambda_0 = \lambda + vt = \lambda + \frac{v\lambda_0}{c}$$

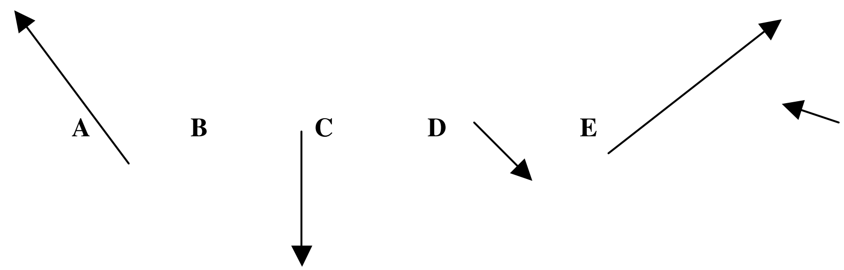
$$\text{or } \lambda_0 - \lambda = \frac{v\lambda_0}{c}$$

$$\text{or } \frac{\lambda_0 - \lambda}{\lambda_0} = \frac{v}{c}$$

which proves the formula quoted at the outset.

This derivation does not work when v approaches c. Then, one must worry about Einstein's Theory of Special Relativity.

Exercise:
The following sketch shows five distant stars. The length and direction of the arrows represent the total space velocity of the stars.



Which star, as observed from Earth, has

- the largest radial velocity of recession?
- the largest blueshift?
- the smallest total space velocity?

1. Suppose Earth to be off to the right at a very large distance.
2. If the Earth is off to the bottom of the page, should your answers be different? If so, what are they?

Our experiences of the Doppler effect involve sound not light; i.e., we hear a change of pitch as a siren first approaches us and then recedes but we do not see colored lights change color as we drive at high speed toward it. Why this difference between sound and light? (The Highway Patrol uses radar (radio waves) and the Doppler effect to track your speed!)

We haven't the money, so we've got to think.

Never say, 'I tried it once and it did not work.'

Lord Rutherford (1871-1937)

When it comes to atoms, language can be used only as in poetry. The poet too, is not nearly so concerned with describing facts as with creating images.

An expert is a man who has made all the mistakes, which can be made, in a very narrow field.

Niels Bohr (1885-1962)