AST 393F
Assignment 4 **Due Monday 10/19**

1. Consider a molecular cloud, which for simplicity we will take to be a slab with an H_2 density of n_{H2} and an HCN column density of N_{HCN}. Initially, we'll also simplify the radiative transfer by only considering radiation propagating perpendicular to the cloud surface. Assume the cloud temperature, T = 30K, is large enough that kT >> hν for the HCN J=1-0 line. This means that we are on the Rayleigh-Jeans side of the blackbody curve, so $I_\nu = 2\nu^2/c^2\, kT$ for a blackbody of temperature T. Assume the cloud has a Doppler (Gaussian 1/e) linewidth parameter of b = 1 km/s.
a) Assume the line-center optical depth is 0.1, $n_{H2} >> n_{cr}$, there is no background radiation source, and HCN is the only opacity source in the cloud. Sketch the emergent spectrum (i.e. the shape of the line) at wavelengths close to that of the line. Label your spectrum with km/s on the horizontal axis and Kelvins (with $kT = c^2/2\nu^2\, I_\nu$) on the vertical axis.
b) Sketch the spectrum of this line for the case of a line-center optical depth of 2.
c) Sketch the spectrum for $\tau = 0.1$ and $n_{H2} = n_{cr} / 10$.
d) Sketch the spectrum for $\tau = 2$ and $n_{H2} = n_{cr} / 10$. Read Tielens' discussion of escape probability before answering this.
e) Sketch the spectrum for $\tau = 2$, $n_{H2} >> n_{cr}$, with a background blackbody source with T = 40 K.
f) Go back to the first case, with $\tau = 0.1$, $n_{H2} >> n_{cr}$, but now consider radiation propagating at angle θ from the perpendicular to the cloud surface. How does the emergent line intensity depend on θ? Integrate over the outgoing hemisphere. What is the ratio of the mean intensity, J, of the outgoing radiation to the intensity at θ = 0? What is the ratio of the flux, F, to I(θ=0)? (You could simplify the integrals by taking the optically thin limit for I at all angles θ.)
g) What if the line is very optically thick, $\tau >> 1$ at its center? What are the ratios of the outgoing mean intensity and flux to the outgoing intensity at θ = 0 (all at line center)?

2. Let's put some real numbers into the calculation for the HCN J=1-0 line. Most of the numbers you'll need are in Tielens table 2.4.
a) Derive a formula for the relationship between the line-center optical depth and the column density in J=0. (Tielens has a formula for the average optical depth over a line [which I think is off by a factor ~2]. Formulas in chapter 1 of Rybicki & Lightman, notably those connecting the Einstein coefficients and the definition of $\phi(\nu)$, may be useful for deriving the formula for the line-center optical depth.)
b) Assuming $n_{H2} >> n_{cr}$, give the formula connecting $N_{J=0}$ and N_{HCN}. For $N_{HCN} = 10^{17}$ cm^{-2} and T = 30 K, what is $N_{J=0}$ and what is τ at line center?
c) If $n_{H2} = 10^4$ cm^{-3}, how do $N_{J=0}$, τ at line center, and I in the line at θ = 0 change? (I'm not sure I know how to do this exactly. See how far you can get, probably making some approximations.)

3. I need help calculating the temperature of the gas in a protoplanetary disk. My modeling program currently calculates the dust temperature from the heating by starlight and cooling by emission of infrared radiation. And I know how energy is transferred between the dust and the gas by collision between molecules and dust grains. But my

observations indicate that the gas is hotter than the dust, so there must be additional gas heating (and cooling) mechanisms.

a) I think the dominant gas heating mechanisms are due to photoelectric ejection of electrons from dust grains and PAHs, and absorption of x-rays by the gas. I know how to calculate the flux of UV and x-rays from the star at each point in the disk. Can you give me the formulas I need to calculate the heating rates? Have I missed any heating mechanisms that could be relevant in a disk around a low-mass pre-main-sequence (T Tauri) star? If there are constants I need in the formulas, give them. I want a formula I can drop into my modeling program.

b) I think cooling of the gas is more difficult to calculate. I think the dominant cooling mechanism is emission of far-infrared lines by H_2O molecules. Tielens has graphs of the cooling rate by H_2O at temperatures of 10K and 40K, but my gas is at a temperature of about 1000K. I'm willing able to assume the lines are optically thin, and I can use published chemical models of the abundance of H_2O in protoplanetary disks. I really don't have the formulas I need, and it may not be easy to find or derive them, so you are welcome to work together on this or ask whoever you want. If you (as a group) can find the formulas we'll give you an acknowledgement in our paper.