AST383D (Fall 2007)

STELLAR STRUCTURE AND EVOLUTION

Problem Set 3

Due Wednesday, October 24, 2007

(worth 10/100)

1. Adiabatic Temperature Gradient for a Radiation-Ideal gas Mixture

Assume that the pressure in a given region of a star is due to both radiation and gas: $P = P_{\rm rad} + P_{\rm gas}$. The respective proportions are given by $\beta = P_{\rm gas}/P$ and $1 - \beta = P_{\rm rad}/P$. As usual, assume $P_{\rm rad} = 1/3aT^4$ and $P_{\rm gas} = \rho\frac{k_{\rm B}T}{\mu m_{\rm H}}$.

a. Evaluate the 1st Law of Thermodynamics for an adiabatic process ($ds = 0$). Assuming that the ideal gas is monatomic (3 degrees of freedom), show that:

$$\frac{3}{2}\beta d\ln T + 12(1-\beta)d\ln T - 3(1-\beta)d\ln\rho = d\ln\rho$$

b. Evaluate the expression $dP = dP_{\rm rad} + dP_{\rm gas}$ to get a second relation between the differentials $d\ln T$, $d\ln\rho$, and $d\ln P$. Combine this equation with the one obtained in a. to eliminate all terms with $d\ln\rho$. Now, show that the adiabatic gradient can be expressed as:

$$\nabla_{\rm ad} = \left(\frac{\partial\ln T}{\partial\ln P}\right)_s = \frac{1 + \frac{(1-\beta)(4+\beta)}{\beta^2}}{\frac{5}{2} + \frac{4(1-\beta)(4+\beta)}{\beta^2}}$$

c. Verify that you recover the correct limiting cases for pure radiation ($\beta = 0$) and for a pure ideal gas ($\beta = 1$).

2. Grey atmosphere

a. Do Exercise 4.10 in HKT (2nd edition) !

b. Assume for simplicity that we are dealing with an atmosphere located on top of a Sun-like star, where the opacity is entirely due to electron scattering (which of course is not the case for the real Sun). How deep (in km) does one have to go from the "surface" (where the density has dropped to roughly the ISM value) until the photosphere is reached (at $\tau = 2/3$)? Make simple assumptions where necessary. You are *not* asked to carry out an involved calculation here!

3. Reynolds number

Consider a flow with Reynolds number $Re = VL\rho/\mu$, where V and L are the characteristic velocity and spatial scale of the flow, respectively.

a. Assume Kolmogorov turbulence, where a turbulent eddy on the length scale l has a typical velocity:

$$v \simeq V \left(\frac{l}{L} \right)^{1/3} \quad .$$

Show that the dissipation length, where $v_{\rm d} l_{\rm d} \simeq \mu/\rho$, is approximately:

$$l_{\rm d} \simeq Re^{-3/4} L \quad .$$

b. Consider riding in your car along Mopac on a beautiful Sunday afternoon with no traffic jams present. Estimate the Reynolds number for the air streaming around your car! What do you conclude for the nature of the flow? Could such a flow be simulated with a state-of-the-art supercomputer?