AST383D (Fall 2007)

STELLAR STRUCTURE AND EVOLUTION

Problem Set 2

Due Wednesday, October 3, 2007

(worth 10/100)

1. Completely Degenerate Electron Gas

In class, we have derived expressions for the e.o.s. for various limiting cases. Here, you are asked to consider a slightly more complicated case: a *completely degenerate (cd)* electron gas with *arbitrary* particle speeds.

a. Show that the (electron) pressure for this more general case is:

$$P = \frac{1}{3}\int_0^{p_\mathrm{F}} v p f_\mathrm{cd}(p) dp = \frac{8\pi c}{3h^3}\int_0^{p_\mathrm{F}} \frac{p/(m_e c)}{\sqrt{1+p^2/(m_e^2 c^2)}} p^3 dp \quad ,$$

with m_e being the electron rest mass, and p_F the Fermi momentum.

b. Show that this expression can be simplified to the limiting cases discussed in class: the non-relativistic (NR) case (for $p_\mathrm{F} \ll m_e c$), and and the ultra-relativistic (UR) case (for $p_\mathrm{F} \gg m_e c$)!

c. Derive the critical mass density (ρ_crit) for the NR–UR transition from the condition: $p_\mathrm{F} \sim m_e c$, assuming for simplicity $n_e \simeq \rho/m_\mathrm{H}$ in evaluating the electron number density.

d. Evaluate the integral in part a (either analytically by using integral tables or numerically) for a range of densities. Plot the resulting (electron) pressure (in dyne cm^{-2}) vs. mass density (in g cm^{-3}), over the range $\rho = 10^{-2} - 10^{10}$ g cm^{-3}. Make sure to choose the scaling of the axes appropriately, and you should use a logarithmic scaling (given that the range of values is huge). Finally, overplot on the same figure (but using different line styles) the $P - \rho$ relations for the two limiting cases (NR and UR) from part b. In evaluating the electron number density, again assume for simplicity: $n_e \simeq \rho/m_\mathrm{H}$.

[**NOTE**: You will need to use one of the standard plotting programs for this exercise, such as IDL or PGPLOT. If need be, ask your peers for help.]

2. Saha Equation

a. Solve the Saha equation, assuming for simplicity the case of a *pure* hydrogen gas, for different layers in a typical low-mass star. For each of the cases below, calculate the degree of ionization (x)! In evaluating the partition functions, approximate them by using only the first term: $u_0 \simeq g_{0,0}$ (neutral H), and $u_1 \simeq g_{1,0}$ (ionized H).

(i) Stellar photosphere: $P_{\rm gas} \simeq 10^5$ dyne cm^{-2}, and $T \simeq 6,000$ K.

(ii) Deeper layer: $P_{\rm gas} \simeq 10^{12}$ dyne cm^{-2}, and $T \simeq 10^6$ K.

(iii) Central core: $P_{\rm gas} \simeq 10^{17}$ dyne cm^{-2}, and $T \simeq 10^7$ K.

b. Do your results make sense physically? In particular, what is going on in the stellar core (case *(iii)*)?