AST383D (Fall 2005)

STELLAR STRUCTURE AND EVOLUTION

Problem Set 2

Due Tuesday, November 1, 2005

(worth 30/100)

1. Adiabatic Temperature Gradient for a Radiation-Ideal gas Mixture

Assume that the pressure in a given region of a star is due to both radiation and gas: $P = P_{\rm rad} + P_{\rm gas}$. The respective proportions are given by $\beta = P_{\rm gas}/P$ and $1 - \beta = P_{\rm rad}/P$. As usual, assume $P_{\rm rad} = 1/3aT^4$ and $P_{\rm gas} = \rho \frac{k_{\rm B}T}{\mu m_{\rm H}}$.

a. Evaluate the 1st Law of Thermodynamics for an adiabatic process ($ds = 0$). Assuming that the ideal gas is monatomic (3 degrees of freedom), show that:

$$\frac{3}{2}\beta d\ln T + 12(1-\beta)d\ln T - 3(1-\beta)d\ln\rho = d\ln\rho$$

b. Evaluate the expression $dP = dP_{\rm rad} + dP_{\rm gas}$ to get a second relation between the differentials $d\ln T$, $d\ln\rho$, and $d\ln P$. Combine this equation with the one obtained in a. to eliminate all terms with $d\ln\rho$. Now, show that the adiabatic gradient can be expressed as:

$$\nabla_{\rm ad} = \left(\frac{\partial \ln T}{\partial \ln P}\right)_s = \frac{1 + \frac{(1-\beta)(4+\beta)}{\beta^2}}{\frac{5}{2} + \frac{4(1-\beta)(4+\beta)}{\beta^2}}$$

c. Verify that you recover the correct limiting cases for pure radiation ($\beta = 0$) and for a pure ideal gas ($\beta = 1$).

2. Hydrogen Burning

a. Assume that a star of solar composition with a central density of $\rho_c = 100$ g cm^{-3} undergoes core hydrogen burning. For what temperature (at the center) would the energy generation rates for the pp-chain and the CNO cycle become equal?

b. Do Exercise 6.9 in HKT (2nd edition) !

3. Adiabatic Radial Pulsations

Re-read Section 8.1 in HKT (2nd edition).

a. Do HKT (2nd ed) Exercise 8.2 (linear adiabatic wave equation)! You will notice that the HKT notation is slightly different from the lecture notes, but you should be able to easily see the correspondence. E.g., $\Gamma_1 = \Gamma$, $\zeta = x$, $\sigma = \omega$.

b. Do HKT (2nd ed) Exercise 8.3 (constant density model)!