AST383D (Fall 2005)
STELLAR STRUCTURE AND EVOLUTION
Problem Set 1
Due Tuesday, October 4, 2005

(worth 20/100)

1. Linear density model

Assume a star obeys a linear density model so that

$$\rho(r) = \rho_c(1 - r/R) \ ,$$

where ρ_c is the central density and R is the radius of the star.

a. Find an expression for the central density in terms of R and the mass M of the star.

b. Use the equation of hydrostatic equilibrium and zero boundary conditions to find the pressure as a function of radius. Your answer will be in the form $P(r) = P_c \times f(r/R)$, where $f(x)$ is a function you will determine. What is the dependence of the central pressure P_c in terms of M and R? Express P_c numerically with M and R in solar units.

c. Assuming an ideal gas equation of state, what is the central temperature T_c? You can use $\mu \simeq 0.5$ for the mean molecular weight.

d. Find the ratio of the radiation pressure to the gas pressure at the center of this star as a function of the total stellar mass (expressed in units of $M_\odot$). At what mass does the radiation pressure become comparable to the ideal-gas pressure?

e. Estimate the mass at which the electron degeneracy pressure becomes important.

f. Write down an explicit expression for the total gravitational potential energy of this toy star, and verify that the virial theorem is exactly satisfied.

2. Mean molecular weights

Re-read Section 1.4.1 in HKT (2nd edition).

a. Assuming a fully ionized mixture, with a composition described by the usual mass fractions X, Y, Z, derive general expressions for the mean molecular weight μ, and the mean molecular weight per free electron μ_e. These quantities are defined by: $\rho = \mu m_H n$, where

m_H is the mass of a hydrogen atom, and n is the total particle number density, and ρ is the total mass density. Similarly: $\rho = \mu_e m_H n_e$, where now n_e is the number density of free electrons. You should arrive at expressions similar to HKT equ. (1.53) and equ. (1.55). Clearly state all your assumptions and approximations in arriving at these expressions.

b. Calculate μ and μ_e for solar composition (Population I): $X \simeq 0.70$, $Y \simeq 0.28$, $Z \simeq 0.02$, and for primordial composition (Population III): $X \simeq 0.76$, $Y \simeq 0.24$, $Z \simeq 0$.

c. Very briefly explain why these abundances are different!

3. Saha equation

Re-read Section 3.4 in HKT (2nd edition). In particular, make sure that you understand how HKT equ. (3.39) is derived.

a. Do HKT (2nd ed) Exercise 3.1 (He ionization fractions)

4. Convective stability of grey atmosphere

a. Do HKT (2nd ed) Exercise 4.10 !