Homework #9

1. $Density = \frac{Mass}{Volume}$. For a sphere, $Volume = \frac{4}{3}\pi R^3$.
The typical density of a white dwarf:
$D_1 = \frac{M}{V} = \frac{M_\odot}{\frac{4}{3}\pi R_{Earth}^3} = \frac{2.0\times10^{30}kg}{\frac{4}{3}\pi(6{,}500{,}000m)^3} \approx 1.74\times10^9 kg\cdot m^{-3}$
The typical density of a neutron star:
$D_2 = \frac{M}{V} = \frac{2.0\times M_\odot}{\frac{4}{3}\pi R^3} = \frac{2.0\times2.0\times10^{30}kg}{\frac{4}{3}\pi(10{,}000m)^3} \approx 9.55\times10^{17} kg\cdot m^{-3}$
The density of a proton $D_3 = \frac{M}{V} = \frac{M_p}{\frac{4}{3}\pi R^3} = \frac{1.67\times10^{-27}kg}{\frac{4}{3}\pi(10^{-15}m)^3} = 3.99\times10^{17} kg\cdot m^{-3}$

2. The escape speed is $V_{Esc} = \sqrt{\frac{2GM}{R}}$, where G is the gravitational constant, M is the mass of central object and R is the distance between the escaping object and the center object. Since we are considering the escape speed from the surface of a white dwarf or a neutron star, the distance between objects will be the radius of the white dwarf or the neutron star.

The escape speed from the surface of a white dwarf:
$V_{Esc1} = \sqrt{\frac{2GM_\odot}{R_{Earth}}} = \sqrt{\frac{2\times6.67\times10^{-11}N\cdot m^2 kg^{-2}\times2\times10^{30}kg}{6{,}500{,}000m}} = 6.4\times10^6 m\cdot s^{-1}$
Compared with the speed of light, $\frac{6.4\times10^6 m\cdot s^{-1}}{3\times10^8 m\cdot s^{-3}} = 0.021 = 2.1\%$

The escape speed from the surface of a neutron star:
$V_{Esc2} = \sqrt{\frac{2G(2M_\odot)}{R}} = \sqrt{\frac{2\times6.67\times10^{-11}N\cdot m^2 kg^{-2}\times2\times2\times10^{30}kg}{10{,}000m}} \approx 2.3\times10^8 m\cdot s^{-1}$
Compared with the speed of light, $\frac{2.3\times10^8 m\cdot s^{-1}}{3\times10^8 m\cdot s^{-3}} = 0.77 = 77\%$

3. a). The gas shell is expanding and we are looking at its front side coming towards us and its back side going away fromus. If the expansion is spherically symmetric, the wavelength shifts of the emission line caused by the motions of the front side and the back side should be approximately same, except the light from the front side is blueshifted and the line coming from the back side is redshifted. Here we only concern about the seperation (the absolute value) between these two shifted lines ($\Delta\lambda = abs(\lambda_1 - \lambda_2) = 0.01\%\lambda_0$), it equals to the amount of a shift caused by a speed which is twice of the expanding speed (v_{ex}).
$\frac{\Delta\lambda}{\lambda_0} = \frac{2v_{ex}}{c}$, so
$v_{ex} = \frac{c}{2}\frac{\Delta\lambda}{\lambda_0} = \frac{c}{2}\times0.01\% = 0.005\%c = 0.00005c = 15km\cdot s^{-1}$

b). 100 years ago, the size (the diameter) of the shell is $58''$. It's $1' = 60''$ now, which means the shell expands $1''$ in 100 years. If the expanding speed is constant, it will take 30 times longer for the shell to expand from $0''$ (the beginning) to $30''$ (today's radius), so $30 \times 100 = 3,000 yrs$.

c). $r = v_{ex}t = 15 km \cdot s^{-1} \times 3 \times 10^3 yrs = 15 km \cdot s^{-1} \times 3 \times 10^3 \times 365 \times 24 \times 3600 s = 1.42 \times 10^{12} km = 9486 AU$
$d = 2r = 18972 AU$

d). 1AU subtends $1''$ at a distance of $1 pc$. $9486 AU$ subtends $30''$ at how many pc away? $\frac{9486(AU)}{30('')} = 316.2 pc$