Homework #2

1. Venus' orbit is inside Earth's orbit and it moves faster than Earth does. From Table 3A at the end of the textbook, you can find out that Venus uses only 0.62 tropical Year to make one revolution around Sun, so Venus's orbit velocity measured by angle is about 1.6 times of that of Earth. Based on this and the relative size of their orbits, you can mark out their positons on the orbits at a series of equally spaced instants. You will see when Earth moves about $\pm 30°$ from the postion where the Earth passes the Venus, Venus's motion will change from prograde to retrograde or from retrograde to prograde. It corresponds one month before or after the pass happens (See attached figure).

2. a) From the definitions, Perihelion distance $d_p = a_H - c_H$, Aphelion distance $d_a = a_H + c_H$, so the semimajor axis $a_H = (d_p + d_a)/2 = 18.18\ AU$.

b) From the second Kepler's law, an imaginary line connecting the Sun to any planet sweeps out equal areas in the same amount of time. It works for Halley's comet too. At the Perihelion and Aphelion points, the velocities are perpendicular to the lines, so the area is proportional to the product of the speed and the length of the line. We have $area = v_a d_a = v_p d_p$. $v_a = v_p d_p / d_a = 70 \times 0.36/36 = 0.7\ km/sec$.

c) According to the third Kepler's law, the square of a planet's orbital period is proportional to the cube of its semi-major axis. We can apply this to Halley's comet and use the information about Earth, which we should be familar with. $T_H^2/a_H^3 = T_\oplus^2/a_\oplus^3 = some\ constant$. We know $a_\oplus = 1AU$, $T_\oplus = 1Yr$, $a_H = 18.18\ AU$, so $T_H = (18.18^3 \times T_\oplus^2)^{0.5} \approx 77.5\ Yrs$ (it's actually 76 Yrs).

d) The circular orbit speed $v^2 = GM/R$. If we apply this to the Earth, we will have $v_E^2 = GM/R_E$, thus $v_{0.36AU}^2/v_E^2 = R_E/R$. Because $v_E \approx 30\ km \cdot s^{-1}$ and $R_E = 1\ AU$, we get $v_{0.36AU} = 50 km \cdot s^{-1}$.

e) Using the same method as in d), we can get the circular orbit speed at 36 AU is $5 km \cdot s^{-1}$.

f) The circular orbit speed at the perihelion distance is smaller than the comet's speed because it should move faster at the point in order to move against the gravitational pull of Sun and get farther away from the circular orbit. The circular orbit speed at the aphelion point is bigger than the comet's orbit speed at this distance in order to keep the circular motion.

3.a) Your acceleration is $a = F/m = 10/50 = 0.2\ m{\cdot}s^{-2}$.

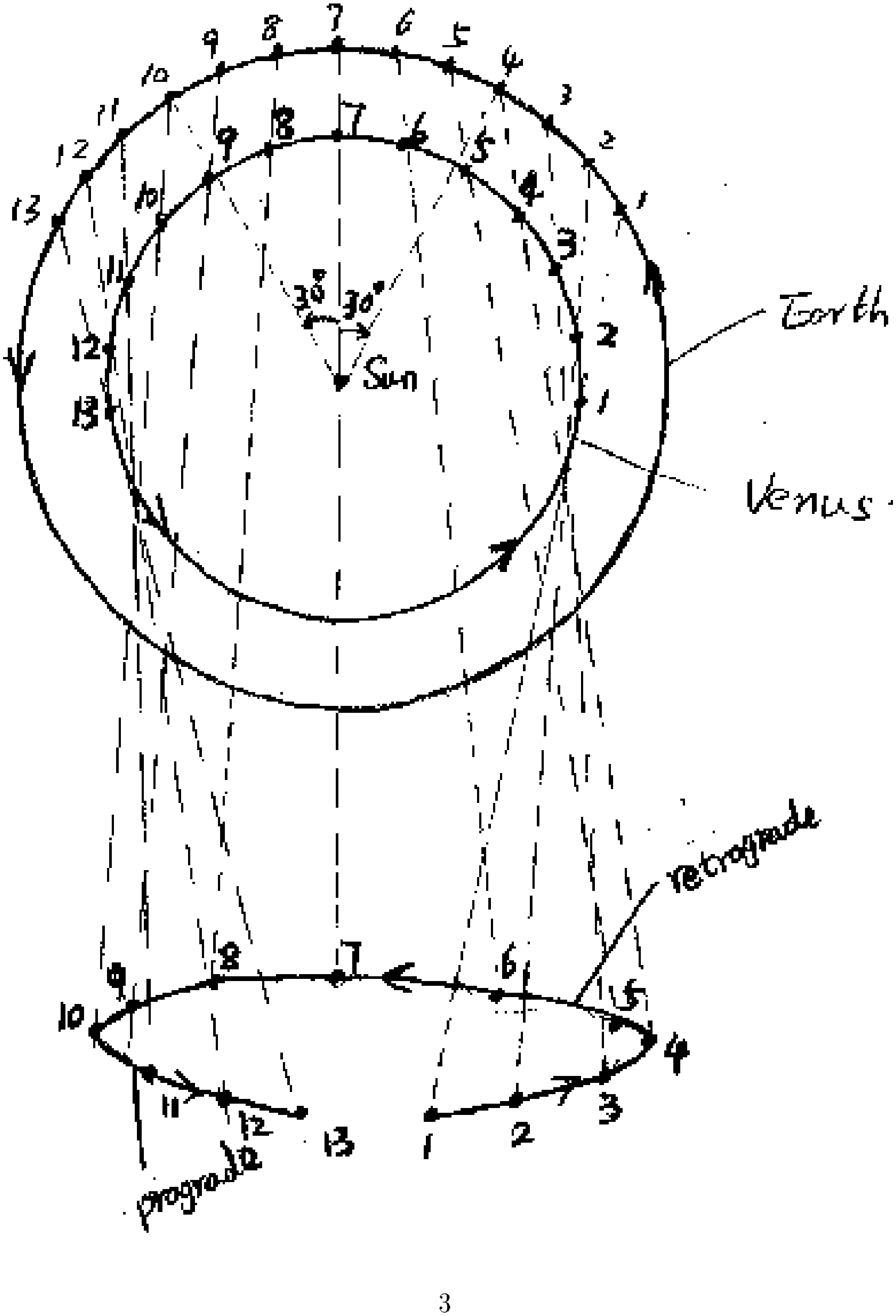
b) The momentum you gain from pushing is $m{\cdot}v = F{\cdot}t$, so $v = 10{\times}1/m = 0.2m{\cdot}s^{-1}$. Or, because your acceleration is $0.2\ m{\cdot}s^{-2}$, which lasts for one second and you started from $0\ km{\cdot}s^{-1}$, your final speed will be $v_f = v_0 + a{\times}t = 0 + 0.2{\times}1 = 0.2\ km{\cdot}s^{-1}$.

c) The spaceship gains the same amount of momentum, but at the opposite direction, so $v_s = 10{\times}1/M = 0.02m{\cdot}s^{-1}$. Be aware of that the total momentum (yours and spaceship's) is zero.

d) Since you and the spaceship are moving at the different directions, the relative speed between you and the spaceship is the sum of two speeds, so $t = 10/(0.2 + 0.02){\approx}45\ secs$.

e) Because all forces we have been talking about are internal force inside the system (you and the spaceship), there is no extra momentum generated. During the whole process, the total momentum is conserved and equals to zero. At the end, you and the spaceship have the same velocity. The total momentum is $(M + m)v$, which should equal to the momentum of the initial state, thus the velocity of the system is zero.

f) Ignoring the distance the spaceship moved during the acceleration (when you are pushing), during the rest of time, the spaceship is either stationary or moving with the speed of $0.02\ m{\cdot}s^{-1}$, so the spaceship has moved $0.02{\times}45 = 0.9m$.



Earth
Venus
Sun
30°
30°
retrograde
prograde