

ASTRONOMY 301
PROBLEM SET NUMBER 3
DUE IN CLASS
Friday, November 7, 2003

Read these instructions first. Print your name on each page of your answer sheets and staple them together. It is important to show the reasoning you used to solve the problem and any formulae you used and algebraic manipulation that you did. You are encouraged to work together to figure out the method of solution, but you must write out the solutions independently in your own words. Do all five problems; each problem counts 1 point. Turn in your answers in class on the due date.

1. The brightest star in the night sky is the star Sirius, Alpha Canes Majoris. It has a parallax angle of 0.379 arcsecond. What is its distance in parsecs?
2. The star Sirius is classified as an A1 V star, whereas the Sun is a G2 V star. This tells us that Sirius is hotter than the Sun. If the Sun has a surface temperature of 5780 K and its spectrum peaks at 500 nm, at what wavelength does the spectrum of Sirius peak if it has a surface temperature of 9200 K? (Hint: use Wien's Law.)
3. Sirius also has a measured radial velocity of -18 km/sec. Given that the rest wavelength of the reddest Balmer line of hydrogen, called H β , is 656.30 nm, at what wavelength would it appear in Sirius' spectrum? (The 'rest wavelength' is the wavelength measured in a lab on Earth.)
4. A prominent absorption line in the spectrum of Sirius is H γ , the second reddest line of the Balmer series of hydrogen. What are the upper and lower orbits involved in the absorption of light by a hydrogen atom to make this line? H γ has a rest wavelength 486.1 nm. What is the difference in energy in ergs between the two orbits? Work this problem with units of cm and seconds and you'll get the answer in ergs. Recall that frequency can be calculated from the wavelength, that the speed of light in cm/sec is 3×10^{10} , and that Planck's constant is 6.626×10^{-27} erg-sec.
5. From the luminosity and surface temperature, we can calculate the radius of Sirius. It is 26 times as luminous as the Sun and has a surface temperature of 9200 K. The Sun has a surface temperature of 5780 K. What is Sirius' radius in solar radii? You will need the equation $L = 4\pi R^2 \sigma T^4$. (Hint: take a ratio to the sun before entering any numbers.)

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SOLUTIONS**

#1. The parallax angle for Sirius is 0.379 arcsecond. By definition, the distance to Sirius, measured in parsecs, is one over the parallax angle in arcseconds. So the distance is 2.64 parsecs.

#2. We know from Wien's Law that the peak in the spectrum of a star will be proportional to one over the temperature in kelvins. Because we are given the solar temperature and peak wavelength for the Sun, we have two simple ways to solve the problem. One way is to use the Sun's values to determine the constant of proportionality:

wavelength of peak = constant ÷ T, so for the Sun

500 nm = constant ÷ 5780 K, so the constant is

constant = 2890000 nm · K

then we solve the equation for Sirius with this constant

wavelength of peak = 2890000 nm·K ÷ 9200 K
= 314 nm

An alternate way to solve this is to take ratios between Sirius and the Sun:

$$\frac{\text{Wavelength for Sirius}}{\text{Wavelength for Sun}} = \frac{\text{constant} \div 9200 \text{ K}}{\text{constant} \div 5780 \text{ K}}$$

$$\text{So, } \frac{\text{wavelength for Sirius}}{500 \text{ nm}} = \frac{5780}{9200}$$

$$\text{Wavelength for Sirius} = 500 \times 5780 \div 9200 \text{ nm} = 314 \text{ nm}$$

#3. This is a Doppler Effect problem. We start with Doppler's equation:

$$\frac{\text{change in wavelength}}{\text{emitted wavelength}} = \frac{\text{speed}}{c}$$

We're given that the emitted wavelength is 656.30 nm and that the speed is -18 km/sec. We know the speed of light, c = 300,000 km/sec. Substituting,

$$\frac{\text{change in wavelength}}{656.30 \text{ nm}} = \frac{-18 \text{ km/sec}}{300,000 \text{ km/sec}}$$

$$\begin{aligned} \text{Then, the change in wavelength} &= 656.30 \times (-18) \div 300,000 \text{ nm} \\ &= -0.0393 \text{ nm} \sim -0.04 \text{ nm} \end{aligned}$$

The wavelength we will observe is the change added to the emitted wavelength, *i.e.*, 656.26 nm. As a check we note that this is blue-shifted from the emitted wavelength, as it should be for a negative speed.

#4. From lecture, we know that the Balmer lines of hydrogen occur in absorption when the electron is lifted from the second orbit to a higher orbit (Oct 27, 29 lecture notes, slide 20). From the quantum theory of light, energy = $h f$ where f is the photon's frequency (Oct 27, 29 lecture notes, slide 13). The lowest energy corresponds to the lowest frequency which in turn corresponds to the longest wavelength. Thus, the reddest line of the Balmer series (longest wavelength) will occur for the lowest energy photon and the second reddest line, for the second lowest energy photon. The lowest energy photon will move the electron from orbit 2 \rightarrow 3, and the second lowest, from 2 \rightarrow 4. Thus, H α corresponds to orbit 2 \rightarrow 4.

To determine the energy difference between orbits 4 and 2, we use the relation

$$E = h f, \text{ where } h \text{ is Planck's constant and } f \text{ is the frequency}$$

Back when we first discussed light, we learned that the frequency and wavelength are related by

$$f \lambda = c, \text{ where } \lambda \text{ is the wavelength and } c \text{ is the speed of light.}$$

Substituting the second equation into the first one,

$$E = h c / \lambda$$

We're told to use units of cm and seconds to get an answer for the energy in ergs. We are given that $h = 6.626 \times 10^{-27}$ erg-sec, $\lambda = 486.1$ nm, and $c = 3 \times 10^{10}$ cm/sec. First convert the 486.1 nm to cm. We know that a nm is 10^{-9} m and a cm is 10^{-2} m, thus a nm is 10^{-7} times smaller than a cm and $\lambda = 486.1 \times 10^{-7}$ cm.

Now substitute all the values,

$$E = 6.626 \times 10^{-27} \text{ erg-sec} \times 3 \times 10^{10} \text{ cm/sec} \div 486.1 \times 10^{-7} \text{ cm}$$

$$E = (6.626 \times 3 / 486.1) \times 10^{-10} \text{ ergs}$$

$$E = 0.0409 \times 10^{-10} \text{ ergs}$$

$$E = 4.09 \times 10^{-12} \text{ ergs}$$

#5. We know that the luminosity of a star is related to its radius and temperature by

$$L = 4\pi R^2 \sigma T^4$$

First we write this for Sirius and then for the Sun,

$$L_s = 4\pi R_s^2 \sigma T_s^4$$

$$L_o = 4\pi R_o^2 \sigma T_o^4$$

As the hint suggests, we'll take the ratio of the equations:

$$\frac{L_S}{L_o} = \frac{R_S^2 T_S^4}{R_o^2 T_o^4}$$

And all the constants have cancelled out.

Now we substitute the values given, $L_S = 26 L_o$, $T_S = 9200 \text{ K}$ and $T_o = 5780 \text{ K}$,

$$26 = (R_S \div R_o)^2 (9200 \div 5780)^4$$

Solving for R_S / R_o we get 2.0. The radius of Sirius is $2.0 R_{\text{sun}}$