ASTRONOMY 301
PROBLEM SET NUMBER 2
DUE IN CLASS
Friday, October 10, 2003

Read these instructions first. Print your name on each page of your answer sheets and staple them together. Turn in your answers in class on the due date. It is not necessary to show your arithmetic on the answer sheet, but it is important to present clearly the reasoning by which you achieved the solution. Do all five problems; each problem counts 1 point.

#1. The resolving power of a telescope is given by Dawes' criterion:

Smallest detail in arcseconds = 0.021 x wavelength (in nanometers) / diameter (cm).

Suppose you are viewing in green light (500 nm) with a 25 cm telescope. Can you resolve a basketball at 50 miles distance? (A basketball at 50 miles is about 0.6 arcsecond across.)

#2. Use Kepler's Third Law to determine the semimajor axis of a planet orbiting the Sun with a period of 4.6 years.

#3. Your weight is the force with which gravity attracts you to the surface of a planet:

$$F = \frac{Gm_1 m_2}{r_{1,2}^2}$$

The planet Venus has a mass 0.815 times that of the Earth and a radius 0.949 times that of the Earth. How would your weight there compare to your weight on Earth? [Hint: Take a ratio of the force attracting you to the Earth to that attracting you to Venus **before** you do any calculations.

#4. Use Newton's version of Kepler's Third Law: $G (m_1 + m_2) P^2 = 4 \pi^2 a^3$ to determine the mass of Mars from its satellite Phobos. Phobos has a period of 7 hours 39 minutes and a semimajor axis of 9.378×10^6 meters. Facts you need to know: Phobos is tiny compared to Mars; if you use units kilograms, meters, seconds, then $G = 6.67 \times 10^{-11}$; and $\pi = 3.14$. The mass of Mars will come out in kilograms.

#5. The radioactive element Uranium 235 decays to Lead 207 with a half-life of $704. \times 10^6$ years. You are told that a certain rock contains 97 atoms of Lead 207 for every 3 atoms of Uranium 235. Assuming that the rock had no Lead 207 in it to begin with and that it has been undisturbed over its lifetime, how old is the rock?

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SOLUTIONS

#1. The smallest detail in arcseconds is given by Dawes' criterion

0.021 x wavelength (in nanometers) / diameter (cm).

Substitute the wavelength (500 nm) and the telescope aperture (25 cm) in to Dawes' criterion, and we have

Smallest detail = 0.021 x 500 / 25 arcsecond.
= 0.42 arcsecond

A basketball at 50 miles is about 0.6 arcsecond, which is slightly larger than the smallest detail visible in green light with this telescope. Thus, you will barely resolve the basketball.

#2. Kepler's Third Law says that the orbital period squared is proportional to the semimajor axis cubed. If we use the units years for the period and AU for the semimajor axis, then

$$P^2 = a^3$$

With a period of 4.6 years, $P^2 = (4.6)^2 = 21.16$. Then $= a^3 = 21.16$. The cube root is 2.76 AU. A planet with a period of 4.6 years will have a semimajor axis of 2.76 AU. This is in fact the orbit of the first asteroid discovered; the asteroid Ceres.

#3. Your weight on Earth is the force of Earth's gravity upon your mass. Your weight on Venus is the force of Venus' gravity on your mass. These forces are given by Newton's Law of Gravity:

$$F = \frac{Gm_1m_2}{r_{1,2}^2}$$

Since you know the ratio of Venus' mass to Earth's and the ratio of Venus' radius to Earth's, the easiest way to do this problem is to take the ratio of the forces so you end up with the ratio of the masses and the ratio of the radii. First, put m_1 = Earth's mass, m_2 = your mass, and $r_{1,2}$ = Earth's radius. Then write the equation again with m_1 = Venus'mass, m_2 = your mass, and $r_{1,2}$ = Venus' radius. Take the ratio of the equations

$$F(Venus)/F(Earth) = G\, m_{Venus}\, m_{you} / r_{Venus}^2 \div G\, m_{earth}\, m_{you} / r_{earth}^2$$

Do the cancellations,

$$F(Venus)/F(Earth) = (m_{Venus}/ m_{earth}) \div (r_{Venus}/ r_{earth})^2$$

Substitute the values that you know, 0.815 and 0.949, into the equation,

$$F(Venus)/F(Earth) = (0.815) \div (0.949)^2 = 0.905$$

Your weight on Venus would be 90% of your weight on Earth.

#4. Newton's version of Kepler's Third Law is $G\,(m_1 + m_2)\,P^2 = 4\,\pi^2\,a^3$. We know the value of G, the Period of Phobos, the semimajor axis of Phobos' orbit, the value of π, and that the mass of

Phobos (m_2) is tiny compared to the mass of Mars (m_1). If m_2 is tiny compared to m_1, then we make only a tiny error by putting $m_1 + m_2 = m_1$ in the equation. Now we have,

$$G\, m_1 P^2 = 4\, \pi^2\, a^3$$

Now we have to be sure that we have the units right: kilograms, meters, seconds. Convert Phobos' period from 7 hours 39 minutes to its value in seconds = 27540 seconds. Substitute all the known quantities and we have

$$(6.67 \times 10^{-11})\, m_1\, (27540)^2 = 4\, (3.14)^2\, (9.378 \times 10^6)^3$$

Doing the calculations, we get 6.43×10^{23} kg for the mass of Mars.

#5. We are told that there are 97 atoms of Lead 207 for every 3 atoms of Uranium 235 in the rock. Because the rock started with no Lead 207, all the Lead 207 must have started as Uranium 235. Thus, the 97 Lead atoms and the 3 Uranium atoms must have been 100 atoms of Uranium at the beginning of the rock's existence. After 1 half-life, there would be 50 of these 100 atoms left as Uranium. After 2 half-lives, 25 atoms. After 3 half-lives, 12.5 atoms. After 4 half-lives, 6.25 atoms. And after 5 half-lives, only 3.125 atoms of Uranium 235 would be left and there would be 100 – 3.125 = 96.875 atoms of Lead 207. This is really close to what we have left – 3 atoms and 97 atoms. Thus, the rock must be very close to 5 half-lives old. The age must then be close to $5 \times 704 \times 10^6$ years. This is 3.5×10^9 years. (Doing the calculation any more exactly than this requires the use of a logarithmic equation that is beyond the math for this course.)