CONTRIBUTION OF $\Lambda$ TO THE BENDING OF LIGHT

(HAS RELEVANCE TO GRAVITATIONAL LENSING IN COSMOLOGY)

<table>
<thead>
<tr>
<th>Reference</th>
<th>Authors</th>
<th>Journal/Source</th>
</tr>
</thead>
</table>
WIDELY ACCEPTED "RESULT":

etc., etc

"Λ DOES NOT AFFECT BENDING"

STRANGE NEGLECT:

EDDINGTON 1923 ("Math. Th. of Rel")
ALREADY HAS CONTRIBUTION
OF Λ TO ADVANCE OF PERHELION

WHY NOT TO BENDING OF LIGHT?

[ 1" per century ↔ Λ = 5 \times 10^{-42} \text{ cm}^{-2} ]
FIELD OF MASS-POINT:

WITHOUT $\Lambda$ - FIELD Eqs $\Rightarrow$

SCHWARZSCHILD:

$$ds^2 = (1 - \frac{2m}{r})dt^2 - (\frac{r'}{r})^2dr^2 - r^2\left\{d\theta^2 + \sin^2\theta d\phi^2\right\}$$

WITH $\Lambda$

KOTTLER: [OR SCHWARZSCHILD-DE SITTER]

$$ds^2 = (1 - \frac{2m}{r} - \frac{\Lambda r^2}{3})dt^2 - (\frac{r'}{r})^2dr^2 - r^2\left\{} \right\}$$

STATIC METRICS:

$$\left(1 - \frac{2m}{r} - \frac{\Lambda r^2}{3}\right) = e^{2\phi} \approx 1 + 2\phi$$

$$-2\phi$$

$$-\nabla \Phi = -\frac{Gm}{r^2} + \frac{c^2\Lambda}{3} \frac{r}{r}$$

RADIAL FORCE FIELD

Expect less bending!
Simple Argument for No $\Lambda$-Bending:

Orbit Eq (Geodesic) in $\theta = \frac{\pi}{2}$ (equatorial) "plane"

$$\frac{d^2u}{d\phi^2} + u = \frac{m}{r^2} + 3mu^2 - \frac{\Lambda}{3h^2u^3}$$

Exact!

$$\rho = r^2 \frac{d\phi}{ds} = \text{const}$$

For $ds \neq 0$ $\Lambda$ contributes

For $ds = 0$ $\Lambda$ drops out!
BUT THE $\phi, \nu$ EQ. IS NOT EVERYTHING!

$\phi, \nu (r)$ ARE MERE COORDINATES

ONLY THE METRIC TELLS US WHAT IS MEASURED!
usual 1st approximation to
Solution of photon orbit D.E.:

\[ \frac{1}{r} = \frac{\sin \phi}{R} + \frac{3m}{2R^2} \left( 1 + \frac{1}{3} \cos 2\phi \right) \]

(Schw+Ko)

Schwarzschild bending angle:

\( r \to \infty \quad \phi \to \phi_\infty \) (SMALL!)

I: \( \frac{1}{r} = \frac{\sin \phi}{R} + \frac{3m}{2R^2} \left( 1 + \frac{1}{3} \cos 2\phi \right) \)

\[ \phi_\infty = \frac{3m}{2R} \cdot \frac{4}{3} = \sqrt{\frac{2m}{R}} \]

Total bending: \( 2\phi_\infty = \frac{4m}{R} \) (Einstein)
WITH $\Lambda$:

SCHWARZSCHILD $\rightarrow$ KOTTLER

cannot let $r \rightarrow \infty$!

**Ex:**

As a simple example, calculate $\gamma_0$

measured angle in 2 or 3-D Riem. space:

$$\cos \gamma_0 = \frac{\delta^i_j}{\sqrt{\delta_{ij}}} = \frac{g_{ij} \, d \xi^i \, d \xi^j}{\sqrt{(g_{ij} \, d \xi^i \, d \xi^j)^2}}$$

Put $\xi = 0$ in (I) to find $r = R^2/2m$

then (to lowest order) find from (II):

$$\gamma_0 = \frac{2m}{R} \left( 1 - \frac{\Lambda R^4}{24m^2} \right)$$

EINSTEIN
A specific application to lensing: Einstein Ring

\[ \psi_0 = \frac{2m}{R} (1 - \frac{\Delta R^4}{24 m^2}) \]

\( m, R \) same in Schw. and Kottler

\[ \Rightarrow \text{Hence formulae comparable!} \]

If lens is huge cluster of galaxies, cosmological expansion must be taken into account if observer recedes, say, at speed \( V \)

\[ 24_0 \rightarrow 24_0 \sqrt{\frac{1+V}{1-V}} \]

\[ \Rightarrow \text{Aberration factor} \]

\( \Rightarrow \text{Ratio of Einstein:}\Delta\text{-bending unchanged} \)
ANOTHER INTERPRETATION
OF $\gamma_o$:

AFFECTS DISTANCE FROM APPARENT SIZE

\[ 2\gamma_o D_s = 2R \]

\[ D_s = \frac{R}{\gamma_o} = \frac{R^2}{2m} \left( 1 + \frac{\Lambda R^4}{24 m^2} \right) \]
PROBABLE BEST HOPE OF DETECTION: LARGE!

LENSING BY CLUSTERS OF GALAXIES

$M \approx 10^3 - 10^4 \, M_\odot$

BENDING:

$\frac{\lambda \text{-term}}{m \text{-term}} \sim \frac{1}{200}$

$\frac{\lambda \text{-term}}{2 \text{nd order } m \text{-term}} \sim 10$
Q: IMPORTANT QUESTION: HOW FAR TO INTEGRATE TO GET TOTAL BENDING BY A GIVEN SOURCE

SCHWARZSCHILD:
   a) Space gets flat \( \{ \text{as } r \to \infty \} \)
   b) \( m - \text{attraction} \to 0 \)

KOTTLER:
   a) Space never gets flat \( \{ \text{as } r \to \infty \} \)
   b) \( \Lambda - \text{repulsion increases} \)

A: "SWISS-CHEESE" COSMOLOGY "
FRIEDMAN UNIVERSE
+ SCHWARZSCHILD (KOTTLER) BUBBLES
(JUNCTION CONDITIONS SATISFIED)

All bending takes place in bubble
No further bending outside

\[ \alpha = \frac{4mG}{Rc^2} - \frac{\Lambda R r_B}{3} \]  
(R = radius of lens)