Twin Paradox

- There are twins, A and B
- B moves relative to A
  - A’s point of view
    - B is moving at speed \( v \)
    - B’s clock ticks more slowly by \( \gamma \).
    - Therefore, B appears to be aging more slowly.
  - B’s point of view
    - A is moving at speed \( v \)
    - A’s clock ticks more slowly by \( \gamma \).
    - Therefore, A appears to be aging more slowly.
- So, which one is older, when they meet?
  - Twin Paradox

Case 1 (a different point of view)

- C’s point of view
  - C is moving to the left with respect to the original frame
  - A and B are moving to the right together until event 1.
  - Then A is at rest but B speeds up.
  - A turns around earlier than B.
    - Then B is at rest but A moves to the right faster than before
  - A and B finally meet at event 3.
  - In this case, A’s and B’s worldlines are still symmetric. (point symmetry)
    - A and B have traveled the same spacetime distance; thus, A and B have aged the same years

Case 2

- A remains at rest at all times.
- B leaves home at event 1, turns around at event 2, and finally meets A at event 3.
- In this case, A’s and B’s worldlines are not symmetric!
- What happens?
  - The answer is that A has aged more than B.
- Why?
  - B’s spacetime distance is shorter than A’s
    - Remember, \( ds^2=c^2dt^2-dx^2 \)
Case 2 (a different point of view)

- In C’s frame, A is moving to the left at all times. B is initially moving to the left, together with A.
- B becomes at rest at event 1, and then moves to the left faster than before.
- A and B meet at event 3.
- In this case, A’s and B’s worldlines are still not symmetric!
  - A has aged more than B.
  - B’s spacetime distance is still shorter.

So, what was it?

- Motion of A and B remains completely relative only when both are moving at constant velocity.
  - Motion has to be inertial for a “perfect relativity” to be valid
- However, for two people to know their initial ages and then meet later again, the motion cannot stay inertial ➞ motion is no longer perfectly relative.

They meet here, but they don’t know each other’s initial age