**Special Relativity (1905)**

- **Two Invariants**
  - Speed of light, $c$
  - Spacetime distance \( ds^2 = c^2 dt^2 - dx^2 \)
- **Unification of space and time**
  - No absolute space or time exists: *Relativity*
- **Special relativity does not include gravity**
  - Spacetime is not yet Riemannian, but *Minkowskian*

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**Relativity of Space and Time**

- A’s space coordinate, $x$, does not coincide with B’s, $x'$. Rather, $x$ is a combination of $x'$ and $ct'$.
- The same is true for time coordinate.
  - This means that simultaneous events in A’s coordinate would not appear simultaneous in B’s coordinate!
- But, spacetime distance remains unchanged.

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**Time Dilation and Length Contraction**

- When A sees B moving, B’s time interval appears to be longer (clock ticks more slowly; *time dilation*) and B’s length appears to be shorter (*length contraction*). And vice versa.

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**Intuitive way to understand it**

- From your point of view, the ball appears to move faster; however, light cannot travel faster!
- Therefore, it must take light *more time* to come back down to the laser
  - *Time Dilation*
Formulae
\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]

- \( \gamma \) is always greater than 1.
- As \( v \) approaches \( c \), \( \gamma \) becomes large.
- When \( v=c \), \( \gamma \) is infinite.

- B’s unit time in A’s frame equals A’s unit time in A’s frame multiplied by \( \gamma \). (Hence time dilation)
  – Be careful! The time actually elapsed in B’s frame gets shorter because the unit time gets longer.
- B’s unit length in A’s frame equals B’s unit length in B’s frame divided by \( \gamma \). (Hence length contraction)

Mass Increase

- A pushes B (whose mass at rest is \( m \)) by applying a force \( F \).
  – Acceleration is given by \( a=F/m \).
  – Velocity acquired would be \( v=a\ dt=F\ dt/m \)
- When B is moving, the clock ticks more slowly
  – B feels the force for a shorter time
  – \( v'=a\ dt'=F\ dt'/(m\gamma) \)
- Thus, the mass of B appears to be bigger by \( \gamma \!
- Nothing can be accelerated to the speed of light, because the mass becomes infinite.