Lifetime of the Sun

- Newton’s law of gravity plus his 3rd law of motion gives the mass of the Sun
  - $2 \times 10^{30}$ kg
- Apparent size of the Sun (32 arc-minutes=0.0093 radian) times the distance to the Sun (150 million km) gives the diameter of the Sun
  - 1.4 million km
- Apparent brightness of the Sun and the distance give the solar luminosity
  - $3.8 \times 10^{26}$ watts
- How long could the Sun shine?
  - Gravitational energy is not enough
    - The Sun lasts for only 20 million years (Helmholtz in 1854; Kelvin in 1887)
  - Something other than gravitational energy was necessary

Nuclear Energy

- If the Sun could convert all of its mass into energy via Einstein’s formula, $E=mc^2$...
  - The Sun would shine for about 15 trillion years!
  - In reality, nuclear reaction in the Sun can convert only a tiny fraction (~0.07%) of the total mass into energy.
    - Only 0.7% of hydrogen gets converted to energy.
    - Not all mass is hydrogen (H:75%, He:25%)
    - Nuclear reaction occurs only in the central region.
- Calculations show that the Sun shines for about 10 billion years by burning hydrogen.

p-p chain (0.7% efficiency)

- $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu_e$
- $^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma$
- $^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H}$
  - $^1\text{H}$: proton, $^2\text{H}$: deuteron
  - $^3\text{He}$: Helium-3, $^4\text{He}$: Helium-4
  - $e^+$: positron, $\nu_e$: neutrino, $\gamma$: gamma ray
- In total, four protons are fused into one helium-4 and produce energy:
  - $^1\text{H} + ^1\text{H} + ^1\text{H} + ^1\text{H} \rightarrow ^4\text{He} + \text{“binding energy”}$
    - One proton weighs $1.6726 \times 10^{-27}$ kg
    - One helium-4 weighs $6.643 \times 10^{-27}$ kg
    - Four protons minus one helium = $4.7 \times 10^{-29}$ kg -> energy
      - This is only 0.7% of the original mass of four protons.

Nuclear Reaction Close-up

- p-p chain is a slow process because...
  - Electric repulsive force prevents one proton from colliding another one. Quantum tunneling (or “barrier penetration”) takes a long time to occur.
  - The first reaction, $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu_e$, involves weak interaction, which is a very slow process.
- Nuclear reaction occurs only in the central region of the Sun because...
  - Density is high (protons easier to meet)
  - Temperature is high (protons easier to penetrate the barrier)
  - Fusion occurs within 2% of the total volume
Hydrostatic Equilibrium

- Gas is pushed outward by pressure.
  - Nuclear energy heats up gas -> high pressure
  - Pressure = $k_B \times \text{number density} \times \text{temperature}$
  - Pressure is highest at the center and decreases at larger distances.
- Gas is pulled inward by gravity.
- "Hydrostatic equilibrium" = "Pressure force balances gravitational force"
  - Gravity = $GM^2/R^2$
  - Pressure = $k_B nT$
  - Calculations show that temperature should be ~20 million K
  - High temperature is necessary to balance gravity.

Energy Transfer

- Core (0-0.25 R$_{solar}$): ~15 million K
  - Energy is produced by fusion
- Radiative zone (0.25-0.70 R$_{solar}$): 2 to 8 million K
  - Energy is carried by radiation (photons) up to near the surface of the Sun
  - Photons frequently scattered by electrons (random walk)
- Convective zone (0.70-1 R$_{solar}$): < 2 million K
  - Photons are absorbed by atoms near the surface; unable to carry energy
  - Energy is then carried by “convection” (clue: boiled water)
- Surface: 5,800 K
  - It takes about a million years from the core to the surface.
- Depth of convective zone depends on stellar mass.
  (Low mass stars have deeper convective zone.)