Homework #10

1. The orbital speed at the distance of \( r \) from an object of mass \( M \) can be found through the equation: \( v = \sqrt{\frac{GM}{r}} \), where \( G \) is the gravitational constant.

a). Your orbital speed around the black hole is:
\[
v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{5 \times 6.67 \times 10^{-11} (N m^2 kg^{-2}) \times 5 \times 2 \times 10^8 (kg)}{150,000 (m)}} \approx 2.98 \times 10^7 (m \cdot s^{-1})
\]

b). The gravitational force can be calculated from the formula \( F = \frac{GMm}{r^2} \).
Assuming the weight is \( m = 80 \) kg.
The force on feet is:
\[
F_{\text{feet}} = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} (N m^2 kg^{-2}) \times 5 \times 2 \times 10^8 (kg) \times 40 (kg)}{(749,000 (m))^2} = 4.743124 \times 10^{10} (N).
\]
The force on head is:
\[
F_{\text{head}} = \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} (N m^2 kg^{-2}) \times 5 \times 2 \times 10^8 (kg) \times 40 (kg)}{(150,000 (m))^2} = 4.743111 \times 10^{10} (N).
\]

c). The tidal force is:
\[
F_{\text{tide}} = (4.743124 - 4.743111) \times 10^{10} \ (N) = 130,000 \ (N) \approx 286,600 \ (pounds).
\]
Nobody wants to do that, the high tidal force will pull the body apart.

2. a). With a constant acceleration \( a \), starting from the zero speed, an object will reach a speed \( v = at \) after time \( t \), so it will take:
\[
t = \frac{v}{a} = \frac{3 \times 10^9 (m \cdot s^{-1})}{9.8 (m \cdot s^{-1})} = 30, 612, 245 \ (s) = 0.97(yrs)
\]
to accelerate up to the speed of light.

b). The distance from us to the center of the Milky Way is \( 8kpc \) (pp607).
We can convert that into value in the unit of lightyear (ly), remember that \( 1ly \) is the distance you can travel in one year with the speed of light.
\( 8kpc = 8, 000 \times 3.3 (ly) = 26, 400 (ly) \), which means it will take 26,400 years to travel this far at the speed of light. Now we are travelling at the speed 0.99c, 1% slower, so we need a little more time, that’s \( \frac{100}{99} \times 26, 400 = 26, 666.7(yrs) \).

c). The slowing factor is:
\[
\gamma = \sqrt{[1 - \frac{v^2}{c^2}]^{-1/2}} = \sqrt{[1 - \frac{0.99^2}{c^2}]^{-1}} = \sqrt{[1 - 0.99^2]^{-1}} = 7.09.
\]
So the trip will still last about \( 26, 666.7/7.09 = 3, 761.8(yrs) \), we definitely don’t want to try it.