The determination of the basic properties of the planets (sizes, masses, motions) is an important first step for the observational astronomer. It is upon such information that a more complete understanding of the solar system is built. Sometimes direct observation suffices, but in other cases it provides misleading information. The rotational period of Mars was successfully determined in this manner. However, similar attempts for Mercury made by Bessel, Schiaparelli, and others produced conflicting results. Because Mercury is a small planet whose surface features have low contrast and because it is so close to the Sun that it is rarely visible against a dark sky, it is difficult to determine how fast it is rotating merely by looking at it from the Earth. Bessel deduced a period of about 24 hours, but long-term observations by Schiaparelli (and confirmed by others) placed its period at 88 days. This is the same rotational period as its period of revolution, and the planet was described as being in \textit{synchronous rotation}, with one face of the planet always towards the Sun.

About 1900 the first attempts were made to use spectroscopy to determine rotational periods of planets. Keeler first applied it to the rings of Saturn, and attempts were made by Slipher and others to determine Mercury’s rotational period in this manner. The slit of a spectrograph was laid parallel to the equator of the planet and the lines from the receding edge would be redshifted while those from the approaching edge were blueshifted, obeying the classic Doppler formula. The results obtained by this method indicated that Mercury took several days to rotate, but a precise determination could not be made.

A much more powerful method became possible during the 1960’s with the successful use of radar signals bounced from planetary surfaces. The basic idea involves using a radio telescope to send a short pulse of electromagnetic radiation of known frequency towards the planet and then record the spectrum (frequency vs. intensity) of the returning echo.

In this exercise, you will use actual spectra of a radar echo from a pulse sent to Mercury to calculate for yourself the rotational period of the planet.

Objectives:
- Read a pulse spectrum to find the frequency shift of the pulse
- Measure the Doppler shift to interpret the change in frequency between the outgoing pulse and the reflection
- Determine the radial velocity of Mercury
- Calculate the rotational period of Mercury

Equipment:
- Lab notebook
- Pencil or pen
- Calculator

Set-Up:
It is not necessary to work with anyone else, but if it helps you, by all means do so. Remember to credit your collaborators in your write-up. Skim through the exercise before you begin and get familiar with what you’ll be doing. Write up any preliminary information and then begin. Clearly mark answers to questions along the way – don’t make me search for them.
I. Technique

Depending on the relative position of the Earth and Mercury, the radar pulse takes between 10 and 30 minutes to travel to Mercury, bounce off, and return. Pettengill, Dyce, and Shapiro produced an accurate rotational period for Mercury using this method. In 1965, they used the 1000 ft. radio telescope at Arecibo, Puerto Rico, to beam a series of 0.0005 second and 0.0001 second radar pulses toward Mercury at a frequency of 430 MHz. [One MHz (10^6 Hz) is equal to one million hertz or cycles per second. The FM radio band ranges in frequency from 88 to 108 MHz.] Since the round trip travel time of the pulses was much greater than the pulse length, it was possible to see how the pulses were broadened by reflection from the rotating planet. Frequency shifts also resulted from the relative motions of the planet and the Earth’s rotation, but these were corrected for using careful timing and computer compensation.

To understand how radar signals produced the desired information, examine Figure 1. When a radar signal is reflected from a rotating spherical planet, the echo is spread out in time as well as in frequency. By the time the pulse has reached Mercury, it has spread out to cover the entire planet. Because the planet’s surface is a sphere, the pulse hits different parts of the planet at different times. The pulse first hits the surface at a point directly on a line between the centers of the Earth and Mercury (the “sub-radar point”). After a small time delay (a few microseconds, µs) the echo is received from points farther back toward the edges of the planet. Thus we wait for the first echo, from the sub-radar point, and then by looking at the returning echoes at succeeding times, each a few microseconds later than the next, we get information about different parts of Mercury’s surface.

![Figure 1. Doppler shift as a result of the rotation of Mercury.](image)

The frequencies of the returning echoes are different from the frequency of the pulse sent out because they have bounced off the moving surface of Mercury. The part of the signal returned from the approaching edge will come back with an increase in frequency (blueshift) and the part returned from the receding edge will show a decrease in frequency (redshift). Any time a source of radiation is moving radially (towards or away from the observer) there will be a Doppler shift.
in the received frequency (or wavelength; recall $\lambda = c/f$) that is proportional to the velocity along the line of sight. This can be expressed like so:

$$\frac{v_o}{c} = \frac{\Delta f}{f}$$

where $c$ is the speed of light ($3 \times 10^8$ m/s); $\Delta f$ is the observed change or shift in frequency ($\Delta f = f - f_{obs}$); $f$ is the original frequency that the source would emit were it not moving; and $v_o$ is the speed of the source with respect to you along that line of sight.

There are two motions of the planet that can produce such a shift: its orbital velocity as a whole around the Sun, and its rotation about its axis. The first echo, from the sub-radar point, is shifted in frequency only by the orbital velocity of the planet as a whole. We can calculate how fast the planet is moving with respect to the Earth from the amount of the shifts, but we can’t tell how fast it’s rotating (spinning) because the component of the rotational velocity of the surface of Mercury is perpendicular to our line of sight at this point (see Figure 1), and so there is no additional frequency shift. The echoes that come in after the sub-radar echo, however, show additional shifts because they come from farther back where the rotational velocity is more directly along our line of sight. Because of the rotation of Mercury, one edge of the planet is moving towards us a little faster than the planet as a whole, and the other edge is moving away from us a little slower than the planet as a whole. So due to the Doppler effect, part of the returning echo (from the faster moving edge of the planet) is at a slightly higher frequency and part of the returning echo (from the slower moving edge) is at a slightly lower frequency as shown in Figure 2. We measure the amount of this frequency shift and apply our knowledge of

**Figure 2. Frequency of returning echo.**
the Doppler effect to calculate the velocity of the surface of Mercury and from this, its period of rotation.

**ANSWER THE FOLLOWING QUESTIONS IN YOUR LAB NOTEBOOK.**

1. Until the radar measurements, what was the accepted rotation period of Mercury? Do you know of any other celestial body that exhibits equal periods of rotation and revolution?
2. To what wavelength does 430 MHz correspond?
3. Why can’t you just examine the echo from the sub-radar point to determine the rotational period of the object?
4. If Mercury is 0.6 AU from the Earth, how long does it take an electromagnetic wave to reach Mercury and come back to Earth?

**II. THE RADAR ECHO AND SPECTRUM**

In principle, it ought to be easy to determine the rotational velocity of Mercury’s limb (by knowing the planet’s circumference) and calculate the rotational period. However, as the echo weakens towards the edge of the planet, the signal returned from the limb becomes undetectable. We will use the echo from a point intermediate between the sub-radar point and the limb to calculate a line-of-sight component of Mercury’ rotational velocity and from this, find its true rotational velocity.

Figure 3 shows the spectrum of the radar echo for five different time delays ($\Delta t$). Each one is a “snapshot” of the spectrum of the returning echo beginning at the instant of reception, followed by another 120 $\mu$s later, and three more at successive 90 $\mu$s intervals. Note that the longer the time delay the broader the return signal in frequency. These pulse spectra show a certain amount of noise, or static, which increases with the later-arriving pulses since they are weaker. Compare the appearance of the received pulse with the initial pulse. You should note that the initial pulse, which is of course stronger, appears slightly smoother and sharper.

To make your measurements, examine the radar signals in Figure 3. Pick one of the time-delayed signals and mark the leftmost and rightmost points where the relative power begins to drop down to the baseline. Subtract one from the other and take the absolute value, i.e. $|f_{\text{left}} - f_{\text{right}}|$. The difference between these points is the shift due to rotational velocity: one side of Mercury is rotating toward you as fast as the other side is rotating away. So the difference in the frequency shifts from the two extremes, $f_{\text{left}}$ and $f_{\text{right}}$, is twice the shift due to rotational velocity alone. We must also correct for the fact that this is an echo: the shift is twice that produced by a source that is simply emitting at a known frequency. This is because the pulse arriving at Mercury appears shifted as seen from the surface, and then it is shifted again because the surface of Mercury is moving as seen from Earth. So to correctly calculate the Doppler frequency shift $\Delta f$, we must take $|f_{\text{left}} - f_{\text{right}}|/2$ and then divide by 2 again, or $\Delta f = |f_{\text{left}} - f_{\text{right}}|/4$. You can now calculate the observed line-of-sight component of the rotational velocity by using Equation 1 above, because we know that the frequency of the transmitted signal is 430 MHz ($4.3 \times 10^8$ Hertz). Record your measurements and calculations in a table similar to that in Figure 4.
III. GEOMETRY OF MERCURY’S ROTATION

Figure 5 gives a schematic of the way to determine Mercury’s rotational period from reflected radar pulses. Since you know the anticipated time delays of the four echoes, you can calculate the following quantities:

a) the distance $d$ that the beam has traveled beyond the sub-radar point,
b) $x$, the distance parallel to our line of sight from the center of the planet to the point from which the echo comes back, and
c) $y$, the distance perpendicular to our line of sight to the extreme edge of the region of Mercury from which the echo comes back.

\[ d = \frac{1}{2} c \Delta t \quad \text{Equation 2} \]

\[ x = R - d \quad \text{Equation 3} \]

\[ y = \sqrt{R^2 - x^2} \quad \text{Equation 4} \]

Do this using Equations 2, 3, and 4 and record your numbers in your table. Show your work in your lab notebook. Here, $c$ is the speed of the radar wave, one $\mu$s = $10^{-6}$ s, and $R$ is the radius of Mercury ($2.42\times10^6$ m). Note that Equation 2 is just “distance equals rate multiplied by time” and that $y$ is just one side of a right triangle whose hypotenuse is the radius of Mercury and whose other side is $x$.

Now inspect Figure 5 again and note that the triangle containing $x$, $y$, and $R$ is similar to the triangle containing $v_o$ and $v$. Hence we can calculate the true rotational velocity $v$ of Mercury in meters per second from the line-of-sight component $v_o$ and the rule of similar triangles, Equation 5. Do this for your echo and record the numbers in your table.

\[ \frac{v}{v_o} = \frac{R}{y} \quad \text{Equation 5} \]

Now that we have the true rotational velocity of the planet, it is trivial to calculate the rotational period. It is simply a rate equation again: “rotation rate equals distance divided by time.” Here, the distance is the circumference of the planet given by $C = 2\pi R$ (Equation 6).
Now repeat these calculations for each of the time-delayed echoes and record the data in your table. When you have calculated $P_{rot}$ for each of the delayed echoes, examine your work to make sure your answers are reasonable. Then compute an average period of rotation in days for Mercury from your values. Also compute the standard deviation of your dataset and the percent error in your value (Equation 7) given that the accepted value for the rotation period of Mercury is 59 days.

\[
\frac{P_{rot} - P_{accepted}}{P_{accepted}} \times 100\% = \% \text{ error} \quad \text{Equation 7}
\]

ANSWER THE FOLLOWING QUESTIONS IN YOUR LAB NOTEBOOK.
5. What is the average rotation period in days for Mercury, its standard deviation, and the percent error in your calculated value?
6. If you did not get 59 days for your rotation period, explain why.
7. What may be some possible reasons that Schiaparelli observed an 88 day rotation period?

IV. THE ORBITAL VELOCITY OF MERCURY

You can use your value of the frequency shift for the echo from the sub-radar point to calculate the orbital velocity of Mercury. Apply the Doppler formula to the shift to calculate the speed of the planet. Negative speeds are speeds of recession (redshift) and positive speeds are speeds of approach (blueshift). Show the data you use and the calculations in your data book. Express your answer in kilometers per second.

\[
P_{rot} = \frac{C}{\nu} = \frac{2\pi R}{v} \quad \text{Equation 6}
\]

Figure 5. Geometry of Mercury’s rotation.
ANSWER THE FOLLOWING QUESTIONS IN YOUR LAB NOTEBOOK.
8. What is the orbital velocity of Mercury as calculated from your data?
9. Does this, in conjunction with your value for the rotational velocity, support the synchronous rotation scenario? Why or why not?

V. SUMMARY
Do not forget to fasten any loose pages into your notebook (you must include your data, i.e. spectra, and anything else you made plots or drawings, notes, etc. on).

ANSWER THE FOLLOWING QUESTIONS IN YOUR LAB NOTEBOOK.
10. Venus’ rotation is considerably slower than Mercury’s. If this method were used on Venus, would the frequency change be larger or smaller than that for Mercury? (Keep in mind that the planets are not the same size.)

VI. REFERENCES