

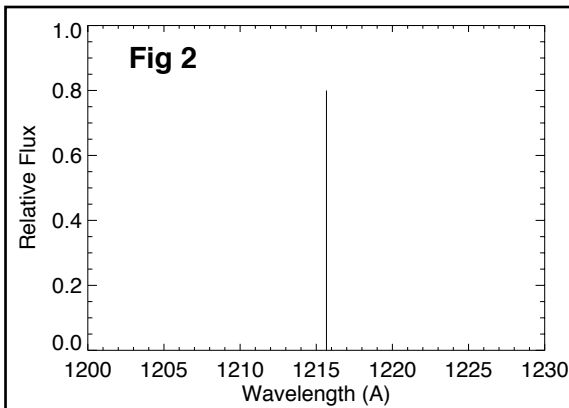
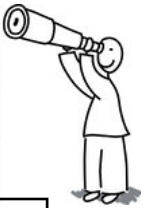
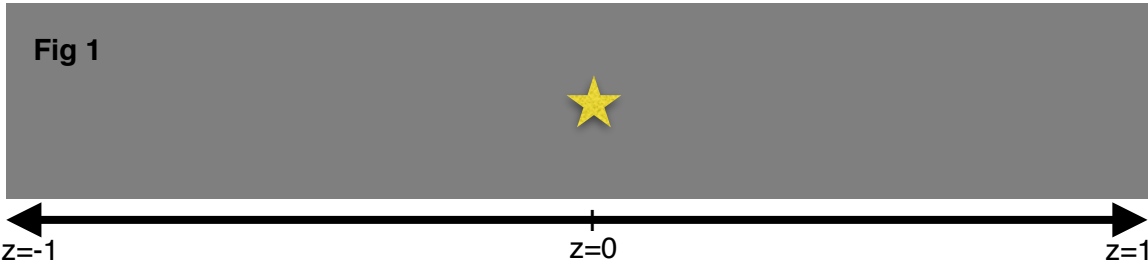
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AST358: Lyman Alpha Emission

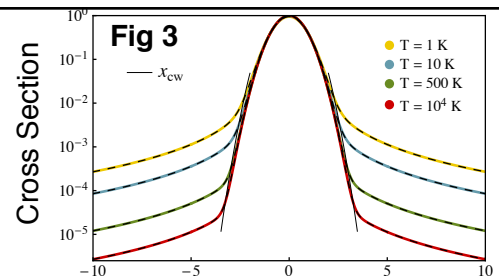
In this activity you will learn first-hand the complicated lives which Lyman alpha photons live as they try to escape from neutral hydrogen gas in galaxies. Our physical scenario can be summed up by Figure 1:



In this diagram, the star represents a source of Lyman alpha ($\text{Ly}\alpha$) photons. For simplicity, we will assume that all of these photons come out exactly at line center (1215.67 \AA or $2.46 \times 10^{15} \text{ Hz}$). We're also going to assume that the photons only move in one dimension (labeled as the z-direction - note that z has nothing to do with redshift here!). If we were to plot the spectrum of this line, it would look something like Figure 2.

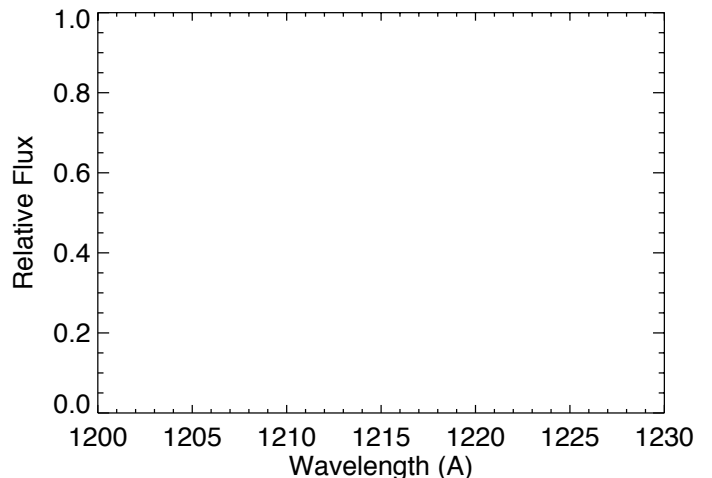
We often denote the amount of frequency shift from line center using the dimensionless photon frequency x , defined by the equation to the right (where thermal is the thermal velocity, which is $\sim 15 \text{ km/s}$ near star-forming regions). Figure 3 shows the cross section for absorption of $\text{Ly}\alpha$ by hydrogen in units of x (where $x=0$ is line center). The optical depth for absorption is proportional to this cross section:

$$\tau_{abs} \propto \sigma n_{gas}$$



$$x = \left(\frac{\nu_{obs} - \nu_{Ly\alpha}}{\nu_{Ly\alpha}} \right) \left(\frac{c}{v_{thermal}} \right)$$

Part 1: If you were to observe the photons from Figure 2 through the gas cloud from the position shown in Figure 1, what do you think the observed emission line profile would look like? Remember two things: first, gas particles have random motions with a velocity dispersion given by $v_{thermal}$. Second, when $\text{Ly}\alpha$ is absorbed, it is almost immediately re-emitted, only now in the rest-frame of the absorbing atom. Draw this in the empty figure to the right (focus on the shape - you may approximate the wavelengths):



Part 2: Now, you will work out for yourself what this line profile should look like. While this activity is somewhat simplified from state-of-the-art supercomputer simulations, it is not all that different from those calculations which were done ~50 years ago. You will empirically derive the observed Lyman alpha emission profile our friend with the telescope in Figure 1 would see, by building up the distribution of frequencies for photons which escape the gas slab. In the one dimension we're considering, photons can escape the cloud if they reach a distance in the z-direction of unity ($z=1$). At that point, they stream through space to our telescopes without changing frequency.

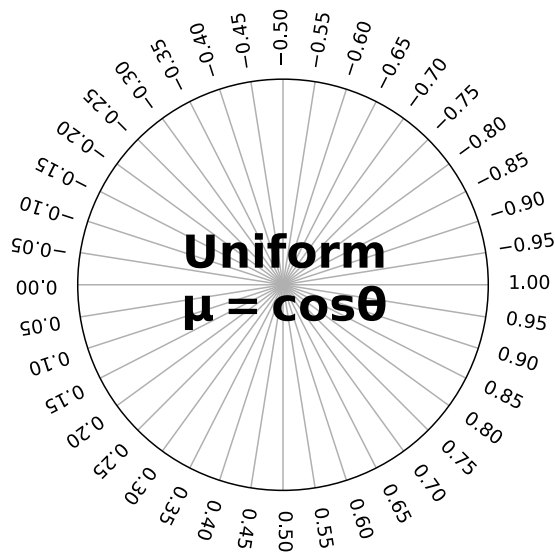
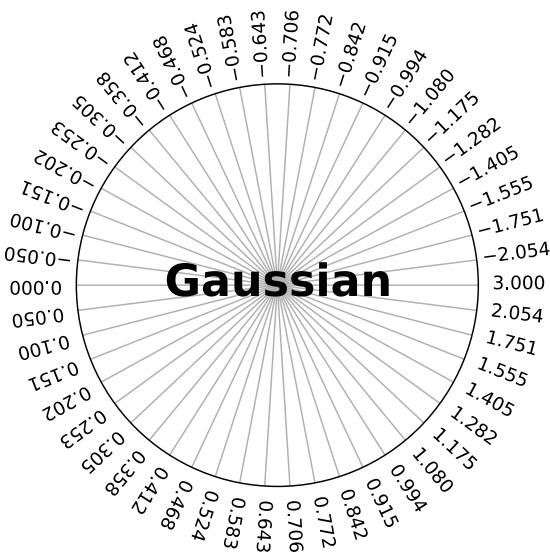
Each group will simulate **10 photons** until escape ($z=\pm 1$), and when done, will add their results (the dimensionless frequency, x , at escape) to the plot on the white board. Together as a class, we will discover if your predictions of the shape of the observable Lyman alpha emission line was correct.

Each photon will start at line center ($x=0$), and at the center of the slab ($z=0$). You will then use game-board spinners to simulate the shifting in frequency and in space (known as diffusion) caused by the resonant scattering, learning what the frequency of the photons are by the time they escape.

One simplification we will make is that anytime a photon is in the line "core" ($|x| < 3$), we will move it out beyond the core to $x = \pm 4$. To decide whether x is positive or negative, spin the spinner labeled "uniform"; set the sign of your new value of x to the sign of the value on the spinner. If the frequency is out of the core ($|x| > 3$), then the new value of x (which I'll call x_i , denoting the value of x at iteration i) is given by:

$$\text{Equation 1: } x_i = x_{i-1} - \frac{1}{x_{i-1}} + G$$

The variable **G** is a randomly drawn value from a Gaussian distribution - you will get this value by flicking the "Gaussian" spinner. This Gaussian random process represents the random motion of the particles of the gas (some are moving faster than others).



For each photon, after you calculate its new dimensionless frequency (\mathbf{x}_i) after scattering, you will use the new frequency to figure out how far the photon moves before it scatters again. The further a photon is from line center, the farther it can move before it scatters again. We will denote this distance gain as \mathbf{dz} , so the new position \mathbf{z}_i is:

$$\text{Equation 2: } z_i = z_{i-1} \pm dz(x_i)$$

The distance gain \mathbf{dz} depends on somewhat complicated radiative transfer physics (i.e., we'd need a lot more spinners), so I've gone ahead and tabulated $\mathbf{dz}(|x|)$ for you on the next page. For a given value of \mathbf{x} , grab the value of \mathbf{dz} off this table. As you might expect, \mathbf{dz} gets larger for larger values of $|x|$, as the photon is farther from the line core. The values of \mathbf{dz} in the table are always positive, but the photon has an equal chance of scattering forwards or backwards, so we need to spin the random spinner to get a sign again, which you will apply to the value of \mathbf{dz} .

You will then repeat this process again and again (and again and again), until $|z| > 1$. At this point, the photons escapes, and you'll record the value of \mathbf{x} on the board.

Here is a quick example of the journey of a Ly α photon, highlighted in the table below:

Scatter #1: We start at $(x,z)=(0,0)$, As I'm in the core, I immediately move out to ± 4 ; I spin the uniform spinner to decide if \mathbf{x} is positive or negative — it lands on a negative value, so I set $\mathbf{x} = -4$. To calculate my new position, I find $\mathbf{dz}(x=4)$ from the table, which is 0.284. I then spin the uniform spinner to see whether I move forwards or backwards, and find +0.65; I don't care about this value, just the sign, which is positive. I then calculate my new value of \mathbf{z} using Eqn 2, finding $\mathbf{z}=0.284$. I record this in the table.

Scatter #2: For the next scattering, I'm out of line center, so I use Eqn 1 to find my new value of \mathbf{x} . I spin the Gaussian spinner, which lands on 0.524, so $\mathbf{x}_i = -3.23$ (ack, moving back towards line center!). The value of \mathbf{dz} at this \mathbf{x} is 0.182, and I spin the uniform spinner getting +0.55, so Eqn 2 gives $\mathbf{z}_i=0.466$.

Scatter #3: I spin and get $\mathbf{G}=-0.994$, giving $\mathbf{x}_i=-3.91$. At this \mathbf{x} , $\mathbf{dz}=0.27$, and I spin the uniform spinner getting 0.20, so Eqn 2 gives $\mathbf{z}_i=0.736$ (working our way out!).

Scatter #4: I spin and get $\mathbf{G}=-0.772$, giving $\mathbf{x}_i=-4.42$. At this \mathbf{x} , $\mathbf{dz}=0.334$, and I spin the uniform spinner getting a negative value, so we move backwards! Eqn 2 gives $\mathbf{z}_i=0.402$.

Scatter #5: I spin and get $\mathbf{G}=-0.706$, giving $\mathbf{x}_i=-4.90$. At this \mathbf{x} , $\mathbf{dz}=0.427$, and I spin the uniform spinner getting a positive value (+0.60), so Eqn 2 gives $\mathbf{z}_i=0.829$.

Scatter #6: I spin and get $\mathbf{G}=-1.175$, giving $\mathbf{x}_i=-5.87$. At this \mathbf{x} , $\mathbf{dz}=0.619$, and I spin the uniform spinner getting a positive value (+0.30), so Eqn 2 gives $\mathbf{z}_i=1.448$. So, this photon escapes!! I did get a little lucky, as I only moved backwards one time, so I expect it to take more like 10 scattering events per photon on average for you.

Guided by this example, work through ten photons until escape, and mark their \mathbf{x} -value at escape (when $\mathbf{z} > 1$) on the board.

# Scatters	x	z
0	0	0
1	-4.000	0.284
2	-3.23	0.466
3	-3.91	0.736
4	-4.42	0.402
5	-4.90	0.829
6	-5.87	1.448

Photon 1

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Trial Photon

# Scatters	x	z
0	0	0
1	+/- 4	
2		
3		
4		

We'll do this one together as a class - do *not* record it on the board.

Photon 2

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Photon 3

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Photon 4

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Photon 5

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Photon 6

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Photon 7

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Photon 8

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Photon 9

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Photon 10

# Scatters	x	z
0	0	0
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

x	dz(x)
3.0	0.160
3.1	0.170
3.2	0.181
3.3	0.193
3.4	0.205
3.5	0.217
3.6	0.230
3.7	0.243
3.8	0.256
3.9	0.270
4.0	0.284
4.1	0.298
4.2	0.313
4.3	0.328
4.4	0.343
4.5	0.359
4.6	0.375
4.7	0.392
4.8	0.408
4.9	0.426
5.0	0.443
5.1	0.461
5.2	0.479
5.3	0.498
5.4	0.517
5.5	0.536
5.6	0.556
5.7	0.576
5.8	0.596
5.9	0.617
6.0	0.638
6.1	0.660
6.2	0.681
6.3	0.703
6.4	0.726
6.5	0.749
6.6	0.772
6.7	0.796

#x	dz(x)
6.8	0.820
6.9	0.844
7.0	0.869
7.1	0.893
7.2	0.919
7.3	0.945
7.4	0.971
7.5	0.997
7.6	1.024
7.7	1.051
7.8	1.078
7.9	1.106
8.0	1.134
8.1	1.163
8.2	1.192
8.3	1.221
8.4	1.251
8.5	1.281
8.6	1.311
8.7	1.342
8.8	1.373
8.9	1.404
9.0	1.436
9.1	1.468
9.2	1.500
9.3	1.533
9.4	1.566
9.5	1.600
9.6	1.633
9.7	1.668
9.8	1.702
9.9	1.737
10.0	1.772
10.1	1.808
10.2	1.844
10.3	1.880
10.4	1.917
10.5	1.954

Use these tables to get the value of dz (the shift in position) for a given value of x. Note that it doesn't matter whether x is positive or negative, so the value of dz is the same for +x or -x.

Remember, to calculate z, you need to spin the uniform spinner to see whether the photon moves forwards or backwards.

Equation 1:

$$x_i = x_{i-1} - \frac{1}{x_{i-1}} + G$$

unless $|x_{i-1}| < 3$, then $x_i = \pm 4$.

Equation 2:

$$z_i = z_{i-1} \pm dz(x_i)$$