1. From the tight correlation between BH mass $M_{BH}$ and bulge velocity dispersion $\sigma_B$, shown in class (see slide 16 of posted powerpoint lectures; Fig 1), we can infer:

$$M_{BH} \sim 5 \times 10^8 M_\odot \text{ for } \sigma_B \sim 300 \text{ km/s}$$

Eddington luminosity of BH:

$$L_{\text{edd}} = 1.3 \times 10^{38} \text{ erg s}^{-1} \left( \frac{M_{BH}}{M_\odot} \right)$$

$$= 6.5 \times 10^{46} \text{ erg s}^{-1}$$

2. Circular orbital speed $V_c$ at radius $R$ is related to the mass $M(R)$ enclosed inside radius $R$ by:

$$\frac{V_c^2}{R} = \frac{GM(R)}{R^2}$$

$$M(R) = \frac{V_c^2 R}{G} = 222 \left( \frac{V_c}{\text{km s}^{-1}} \right)^2 \left( \frac{R}{\text{pc}} \right) M_\odot$$

$$= 222 \times (10,000)^2 \times 2$$

$$= 4.4 \times 10^{10} M_\odot$$

- If this mass $M$ was a compact stellar cluster, then
  - the average no $N$ of stars of mass $1 M_\odot$ is
  $$N \sim 4.4 \times 10^{10}$$

  The no density of stars $n$ is
  $$n = \frac{N}{\frac{4}{3} \pi R^3} = 4.4 \times 10^{10} \left/ \frac{4}{3} \pi \times (2 \times 3 \times 10^6 \text{ m})^3 \right.$$
\[ n \approx 5 \times 10^{-4} \text{ m}^{-3} \]

Stellar collision timescale is given by

\[ t_{\text{coll}} = \frac{1}{n \sigma A} = \frac{\text{mean force path}}{A} \]

where \( \sigma \) = typical speed of star
\( A \) = cross section of star
\[ = \pi R_0^2 \]
\[ = \pi \left( 7 \times 10^8 \text{ m} \right)^2 = 1.5 \times 10^{18} \text{ m}^2 \]

Take \( \sigma \approx V_c \approx 10^4 \text{ kms} = 10^7 \text{ m/s} \)

\[ t_{\text{coll}} = \frac{1}{n \sigma A} = 1.3 \times 10^{15} \text{ s} \approx \left( \frac{1.3 \times 10^{15}}{3 \times 10^7} \right) \text{ yrs} \]
\[ \approx 4 \times 10^7 \text{ yr} \]

So \( t_{\text{coll}} \ll \text{Hubble time } \approx 10^{10} \text{ yrs} \)

This means that a stellar cluster of mass \( M \approx 4 \times 10^6 M_\odot \) and radius \( R \approx 2 \text{ pc} \) would be dynamically unstable. Stellar collisions would cause such a system to be disrupted: the central part would undergo core collapse and the outer stars would be flung out and escape. Thus, the object is not a massive stellar cluster and is likely to be a black hole.
Q3

e) \[ 1 + z = \frac{\lambda_{\text{obs}}}{\lambda_{\text{rest}}} = \frac{3281.4}{656.3} = 5 \]

\[ z \approx 4 \]

d) Lookback time \( T_b = \text{Age today} - \text{Age at redshift } z + 1 \)

\[ = 13.7 - 1.5 \text{ from Fig 2} \]

\[ = 12.2 \text{ yr} \]

\[ T_b \] as \% of present age \( = \frac{12.2}{13.7} \times 100 \approx 89 \% \)

c) \[ I = I_0 e^{-R/R_d} \]

At point where \( I = \frac{I_0}{10} \), \[ \frac{R}{R_d} = \ln(10) \]

\[ \Rightarrow R = 2.3 R_d \]

\[ = 2.3 \times 0.5'' \]

\[ = 1.15'' \]

From Fig 2, angular diameter distance \( D_A \) at \( z + 1 \) is 1400 kpc

From class: \( (R \text{ in kpc}) = \frac{(R \text{ in } '' \times D_A \text{ in } \text{kpc})}{206.3} \)

\[ = \frac{1.15 \times 1400}{206.3} = 7.8 \text{ kpc} \]
(d) Observed surface brightness = Intrinsic surface brightness \cdot (1+z)^{-4}

\[ I_{\text{obs}} = I_{\text{int}} (1+z)^{-4} \]

\[ m_{\text{obs}} - m_{\text{int}} = -2.5 \ \lg \left( \frac{I_{\text{obs}}}{I_{\text{int}}} \right) \]

\[ = -2.5 \ \lg \ (1+z)^{-4} \]

\[ = -2.5 \times (-4) \ \lg (5) \]

\[ = 7 \]

Observed surf. brightness = 25 \text{ mag arcsec}^{-2} + 7 \text{ mag arcsec}^{-2}

= 32 \text{ mag arcsec}^{-2}