

## I.1 The stars

## I.1.1 Star light, star bright ...

All the information we have about stars more distant than the Sun has been deduced by observing their electromagnetic radiation, mainly in the ultraviolet, visible, and infrared parts of the spectrum. The light that a star emits is determined largely by its surface area, and by the temperature and chemical composition – the relative numbers of each type of atom – of its outer layers. Less directly, we learn about the star's mass, its age, and the composition of its interior, because these factors largely determine the conditions at its surface. As we decode and interpret the messages brought to us by starlight, we rely on knowledge gained in laboratories on Earth about the properties of matter and radiation as the basis for our theory of stellar structure.

The *luminosity* of a star is the amount of energy it emits per second, measured in watts, or ergs per second. Its *apparent brightness* or *flux* is the total energy received per second on each square meter (or square centimeter) of the observer's telescope; the units are  $\text{W m}^{-2}$ , or  $\text{erg s}^{-1} \text{cm}^{-2}$ . If a star shines with equal brightness in all directions, we can use the *inverse-square law* to estimate its luminosity  $L$  from the distance  $d$  and measured flux  $F$ :

$$F = \frac{L}{4\pi d^2}. \quad (1.1)$$

Often, we do not know the distance  $d$  very well, and we must remember in subsequent calculations that our estimated luminosity  $L$  is proportional to  $d^2$ . The Sun's total or *bolometric* luminosity is  $L_{\odot} = 3.86 \times 10^{26} \text{ W}$ , or  $3.86 \times 10^{33} \text{ erg s}^{-1}$ . Stars differ enormously in their luminosity: the brightest are over a million times more luminous than the Sun, while we observe stars as faint as  $10^{-4} L_{\odot}$ .

Lengths in astronomy are usually measured using the *small-angle* formula. If, for example, two stars in a binary pair at distance  $d$  from us appear separated on the sky by an angle  $\alpha$ , the distance  $D$  between the stars is given by

$$\alpha \text{ (in radians)} = D/d. \quad (1.2)$$

The orbit of two stars around each other can allow us to determine their masses. If the two stars are clearly separated on the sky, we use Equation 1.2 to measure the distance between them. We find the speed of the stars as they orbit each other from the *Doppler shift* of lines in their spectra; see Section 1.2. Newton's equation for the gravitational force, in Section 3.1, then gives us the masses. The Sun's mass, as determined from the orbit of the Earth and other planets, is  $M_{\odot} = 2 \times 10^{30} \text{ kg}$ , or  $2 \times 10^{33} \text{ g}$ .

Stellar masses cover a much smaller range than their luminosities. The most massive stars are around  $100 M_{\odot}$ . A star is a nuclear fusion reactor, and a ball of

gas more massive than this would burn so violently as to blow itself apart in short order. The least massive stars are about  $0.075M_{\odot}$ . A smaller object would never become hot enough at its center to start the main fusion reaction of a star's life, turning hydrogen into helium.

**Problem 1.1:** Show that the Sun produces only about  $10^{-4}$  times as much energy per unit mass as an average human giving out about  $1 \text{ W kg}^{-1}$ .

The radii of stars are hard to measure directly. The Sun's radius  $R_{\odot} = 6.96 \times 10^5 \text{ km}$ , but no other star appears as a disk when seen from Earth with a normal telescope. Even the largest stars subtend an angle of only about 1/20 of an arcsecond: one arcsecond ( $1''$ ) is 1/60 of an arcminute ( $1'$ ), which is 1/60 of a degree. With difficulty, we can measure the radii of nearby stars with an interferometer; in eclipsing binaries we can estimate the radii of the two stars by measuring the size of the orbit and the duration of the eclipses. The largest stars, the red supergiants, have radii about 1000 times larger than the Sun, while the smallest stars that are still actively burning nuclear fuel have radii around  $0.1R_{\odot}$ .

A star is a dense ball of hot gas, and its spectrum is approximately that of a *blackbody* with a temperature ranging from just below 3000 K up to 100 000 K, modified by the absorption and emission of atoms and molecules in the star's outer layers or *atmosphere*. A blackbody is an ideal radiator or perfect absorber. At temperature  $T$ , the luminosity  $L$  of a blackbody of radius  $R$  is given by the *Stefan-Boltzmann* (SB) equation:

$$L = 4\pi R^2 \sigma_{\text{SB}} T^4, \quad (1.3)$$

where the constant  $\sigma_{\text{SB}} = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$ . For a star of luminosity  $L$  and radius  $R$ , we define an *effective temperature*  $T_{\text{eff}}$  as the temperature of a blackbody with the same radius, which emits the same total energy. This temperature is generally close to the average for gas at the star's 'surface', the *photosphere*. This is the layer from which light can escape into space. The Sun's effective temperature is  $T_{\text{eff}} \approx 5780 \text{ K}$ .

**Problem 1.2:** Use Equation 1.3 to estimate the solar radius  $R_{\odot}$  from its luminosity and effective temperature. Show that the acceleration of gravity  $g$  at the surface is about 30 times larger than that on Earth.

**Problem 1.3:** The red supergiant star Betelgeuse in the constellation Orion has  $T_{\text{eff}} \approx 3500 \text{ K}$  and a diameter of  $0.045''$ . Assuming its distance  $d \approx 140 \text{ pc}$ , show that its radius  $R \approx 700R_{\odot}$ , and that its luminosity  $L \approx 10^5 L_{\odot}$ .

Generally we do not measure all the light emitted from a star, but only what is emitted in a given interval of wavelength or frequency. We define the *flux per unit wavelength*  $F_\lambda$  by setting  $F_\lambda(\lambda)\Delta\lambda$  to be the energy of the light received between wavelengths  $\lambda$  and  $\lambda + \Delta\lambda$ . Because its size is well matched to the typical accuracy of their measurements, optical astronomers generally measure wavelength in units named for the nineteenth-century spectroscopist Anders Ångström:  $1 \text{ \AA} = 10^{-8} \text{ cm}$  or  $10^{-10} \text{ m}$ . The flux  $F_\lambda$  has units of  $\text{W m}^{-2} \text{ \AA}^{-1}$  or  $\text{erg s}^{-1} \text{ cm}^{-2} \text{ \AA}^{-1}$ . The *flux per unit frequency*  $F_\nu$  is defined similarly: the energy received between frequencies  $\nu$  and  $\nu + \Delta\nu$  is  $F_\nu(\nu)\Delta\nu$ , so that  $F_\lambda = (\nu^2/c)F_\nu$ . Radio astronomers normally measure  $F_\nu$  in *janskys*:  $1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$ . The apparent brightness  $F$  is the integral over all frequencies or wavelengths:

$$F \equiv \int_0^\infty F_\nu(\nu) d\nu = \int_0^\infty F_\lambda(\lambda) d\lambda. \quad (1.4)$$

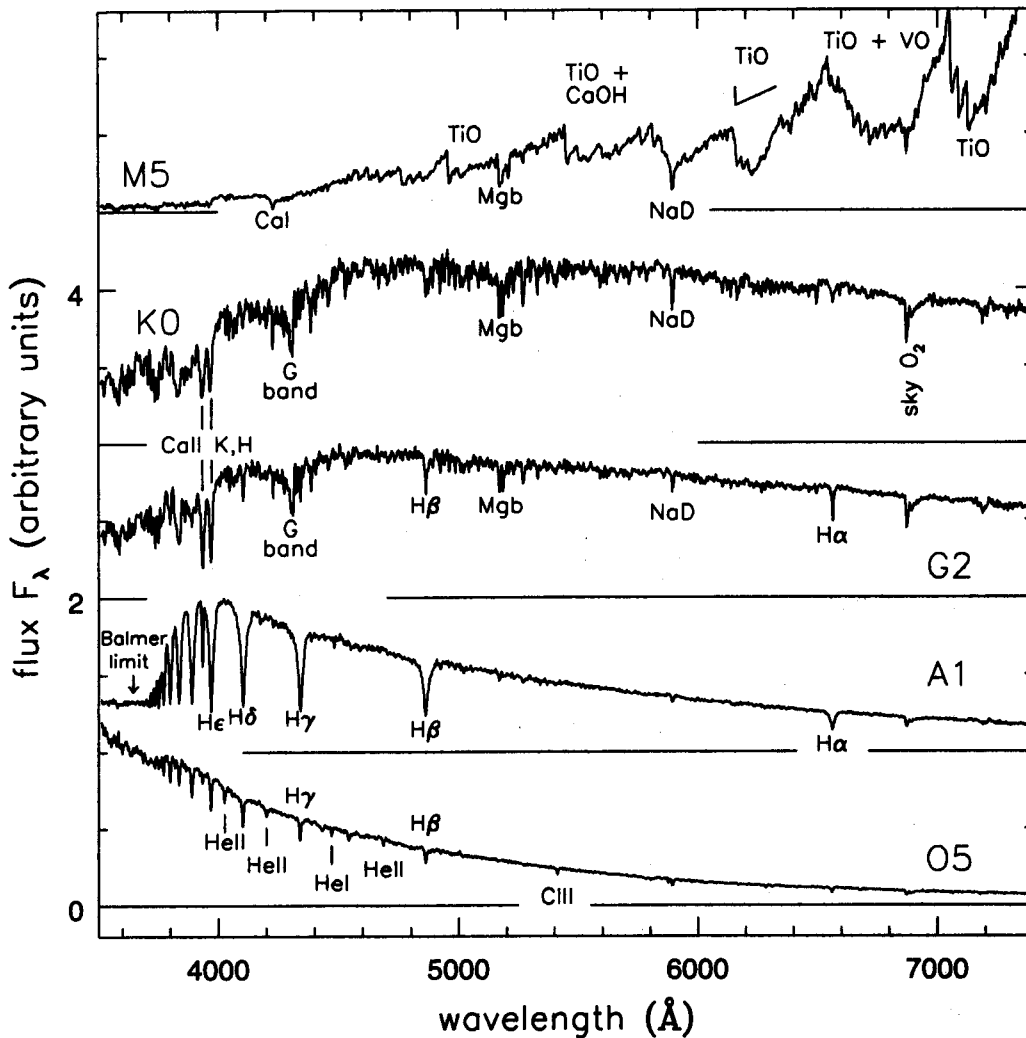
The hotter a blackbody is, the bluer its light: at temperature  $T$ , the peak of  $F_\lambda$  occurs at wavelength

$$\lambda_{\text{max}} = [2.9/T(\text{K})] \text{ mm}. \quad (1.5)$$

For the Sun, this corresponds to yellow light, at about  $5000 \text{ \AA}$ ; human bodies, the Earth's atmosphere, and the uncooled parts of a telescope radiate mainly in the infrared, at about  $10 \mu\text{m}$ .

### 1.1.2 Stellar spectra

Figure 1.1 shows  $F_\lambda$  for a number of commonly observed types of star, arranged in order from coolest to hottest. The hottest stars are the bluest, and their spectra show absorption lines of highly ionized atoms; cool stars emit most of their light at red or infrared wavelengths and have absorption lines of neutral atoms or molecules. Astronomers in the nineteenth century classified the stars according to the strength of the *Balmer* lines of neutral hydrogen HI, with A stars having the strongest lines, B stars the next strongest, and so on; many of the classes subsequently fell into disuse. In the 1880s, Antonia Maury at Harvard realized that when the classes were arranged in the order O B A F G K M, the strength of all the spectral lines, and not just those of hydrogen, changed continuously along the sequence. The first large-scale classification was made at Harvard College Observatory between 1911 and 1949: almost 400 000 stars were included in the *Henry Draper Catalogue* and its supplements. We now know that Maury's spectral sequence lists the stars in order of decreasing surface temperature. Each of the classes has been subdivided into subclasses, from 0, the hottest, to 9, the coolest: our Sun is a G2 star. Recently class L has been added to the system, for the very cool stars discovered by infrared observers. Astronomers often call stars at the beginning of this sequence 'early types', while those toward the end are 'late types'.



**Figure 1.1** Optical spectra of main-sequence stars with roughly the solar chemical composition. From the top in order of increasing surface temperature, the stars have spectral classes M5, K0, G2, A1, and O5 – G. Jacoby *et al.*, spectral library.

The temperatures of O stars exceed 30 000 K. Figure 1.1 shows that the strongest lines are those of HeII (once-ionized helium) and CIII (twice-ionized carbon); the Balmer lines of hydrogen are relatively weak because hydrogen is almost totally ionized. The spectra of B stars, which are cooler, have stronger hydrogen lines, together with lines of neutral helium, HeI. The A stars, with temperatures below 11 000 K, are cool enough that the hydrogen in their atmospheres is largely neutral; they have the strongest Balmer lines, and lines of singly ionized metals such as calcium. Note that the flux decreases sharply at wavelengths less than 3800 Å; this is called the *Balmer jump*. A similar *Paschen jump* appears at wavelengths that are  $3^2/2^2$  times longer, at around 8550 Å.

In F stars, the hydrogen lines are weaker than in A stars, and lines of neutral metals begin to appear. G stars, like the Sun, are cooler than about 6000 K. The most prominent absorption features are the 'H and K' lines of singly ionized calcium (CaII), and the G band of CH at 4300 Å. These were named in 1815 by Joseph Fraunhofer, who discovered the strong absorption lines in the Sun's spectrum, and labelled them from A to K in order from red to blue. Lines of neutral metals, such as the pair of D lines of neutral sodium (NaI) at 5890 Å and 5896 Å, are stronger than in hotter stars.

In K stars, we see mainly lines of neutral metals, and molecules such as TiO, titanium oxide. The spectrum of the M star, cooler than about 4000 K, shows deep absorption bands of TiO and of VO, vanadium oxide, as well as lines of neutral metals. This is not because M stars are rich in titanium, but because these molecules absorb red light very efficiently, and the atmosphere is cool enough that they do not break apart. L stars have surface temperatures below about 2200 K, and most of the titanium and vanadium in their atmospheres is condensed onto dust grains. Hence bands of TiO and VO are much weaker than in M stars; lines of neutral metals such as cesium and rubidium appear, while the sodium D lines become very strong and broad. The coolest objects of this class may not be stars at all, but *brown dwarfs*, with masses so low that their cores are not hot enough to fuse hydrogen into helium.

The spectrum of a galaxy is *composite*, including the light from a mixture of stars with different temperatures. The hotter stars give out most of the blue light, and the lines observed in the blue part of the spectrum of a galaxy such as the Milky Way are typically those of A, F, or G stars. O and B stars are rare and so do not contribute much of the visible light, unless a galaxy has had a recent burst of star formation. In the red part of the spectrum, we see lines from the cooler K stars, which produce most of the galaxy's red light. Thus the blue part of the spectrum of a galaxy such as the Milky Way shows the Balmer lines of hydrogen in absorption, while TiO bands are present in the red region.

It is much easier to measure the strength of spectral lines relative to the flux at nearby wavelengths than to determine  $F_\lambda(\lambda)$  over a large range in wavelength. Absorption and scattering by dust in interstellar space, and by the Earth's atmosphere, affects the blue light of stars more than the red; blue and red light also propagate differently through the telescope and the spectrograph. In practice, stellar temperatures are often estimated by comparing the observed depths of absorption lines in their spectra with the predictions of a *model stellar atmosphere*. This is a computer calculation of the way light propagates through a stellar atmosphere with a given temperature and composition; it is calibrated against stars for which  $F_\lambda$  has been measured carefully.

The lines in stellar spectra also give us information about the surface gravity. Figure 1.2 shows the spectra of three stars, all classified as A stars because the overall strength of their absorption lines is similar. However, the Balmer lines of the A dwarf are broader than those in the giant and supergiant stars, because atoms in its photosphere are more closely crowded together: this is known as the *Stark*

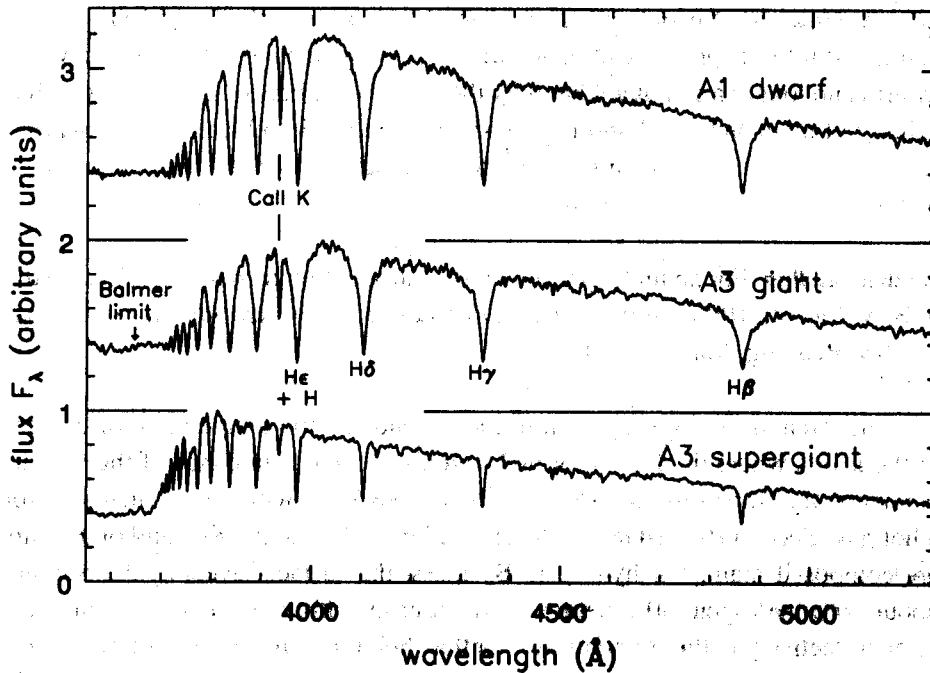


Figure 1.2 Spectra of an A1 dwarf, an A3 giant, and an A3 supergiant: the most luminous star has the narrowest spectral lines – G. Jacoby *et al.*, spectral library.

*effect.* If we use a model atmosphere to calculate the surface gravity of the star, and we also know its mass, we can then find its radius. For most stars, the surface gravity is within a factor of three of that in the Sun; these stars form the *main sequence* and are known as *dwarfs*, even though the hottest of them are very large and luminous.

All main-sequence stars are burning hydrogen into helium in their cores. For any particular spectral type, these stars have nearly the same mass and luminosity, because they have nearly identical structures: the hottest stars are the most massive, the most luminous, and the largest. The radii of main-sequence stars range between  $0.1R_{\odot}$  and about  $25R_{\odot}$ . Very roughly,

$$R \sim R_{\odot} \left( \frac{M}{M_{\odot}} \right)^{0.7} \quad \text{and} \quad L \sim L_{\odot} \left( \frac{M}{M_{\odot}} \right)^{\alpha}, \quad (1.6)$$

where  $\alpha \approx 5$  for  $M \lesssim M_{\odot}$ , and  $\alpha \approx 3.9$  for  $M_{\odot} \lesssim M \lesssim 10M_{\odot}$ . For the most massive stars, with  $M \gtrsim 10M_{\odot}$ ,  $L \sim 50L_{\odot}(M/M_{\odot})^{2.2}$ .

*Giant* and *supergiant* stars have a lower surface gravity and are much more distended; the largest stars have radii exceeding  $1000R_{\odot}$ . Equation 1.3 tells us that they are much brighter than main-sequence stars of the same spectral type. Below, we will see that they represent later stages of a star's life. *White dwarfs* are not main-sequence stars, but they have a much higher surface gravity and smaller

radii; a white dwarf is only about the size of the Earth, with  $R \approx 0.01 R_{\odot}$ . If we define a star by its property of generating energy by nuclear fusion, then a white dwarf is no longer a star at all, but only the ashes or embers of a star's core; it has exhausted its nuclear fuel and is now slowly cooling into blackness. A *neutron star* is an even smaller stellar remnant; these are only about 20 km across, despite having a mass larger than the Sun's.

**Further reading:** For an undergraduate-level introduction to stars, see D.A. Ostlie & B.W. Carroll, 1996, *An Introduction to Modern Stellar Astrophysics* (Addison-Wesley, Reading, Massachusetts).

The strength of a given spectral line depends on the temperature of the star in the layers where the line is formed, and also on the abundance of the various elements. By comparing the strengths of various lines with those calculated for a hot gas, Cecilia Payne-Gaposchkin showed in 1925 that the Sun and other stars are composed mainly of hydrogen. By mass, the surface layers of the Sun are about 70% hydrogen, 30% helium, and about 2% other elements. Astronomers refer collectively to the elements heavier than helium as *heavy elements* or *metals*, even though substances such as carbon, nitrogen, and oxygen would not normally be called metals.

There is a good reason to distinguish hydrogen and helium from the rest of the elements. These atoms were created in the aftermath of the Big Bang, less than half an hour after the Universe as we now know it came into existence. At the end of this time, the neutrons and protons in the Universe had combined into a mixture of about 75% hydrogen, 25% helium, and a trace of lithium. Since then, the stars have burned hydrogen to form helium, and helium into heavier elements; see the next subsection. Figure 1.3 shows the abundance of the commonest elements in the Sun's photosphere. Even oxygen, the most plentiful of the heavy elements, is over 1000 times rarer than hydrogen. The 'metals' are found in almost, but not exactly, the same proportions in all stars; the small differences can tell us a lot about the history of the material that went into making a star; see Section 4.2.

The fraction by mass of the heavy elements is denoted  $Z$ : the Sun's abundance  $Z_{\odot} \approx 0.02$ , while the most metal-poor stars in our Galaxy have less than 1/10 000 of this amount. If we want to specify the fraction of a particular element, such as oxygen, in a star, we often give its abundance relative to that in the Sun. We use a logarithmic scale:

$$[A/B] \equiv \log_{10} \left\{ \frac{(\text{number of A atoms/number of B atoms})_{\star}}{(\text{number of A atoms/number of B atoms})_{\odot}} \right\}, \quad (1.7)$$

where  $\star$  refers to the star and we again use  $\odot$  for the Sun. Thus, in a star with  $[\text{Fe}/\text{H}] = -2$ , iron is 1% as abundant as in the Sun. A warning:  $[\text{Fe}/\text{H}]$  is often used for a star's average heavy element abundance relative to the Sun; it does not always refer to measured iron content.