Intro. Modern Stellar Astrophy., Ostlie, 1996



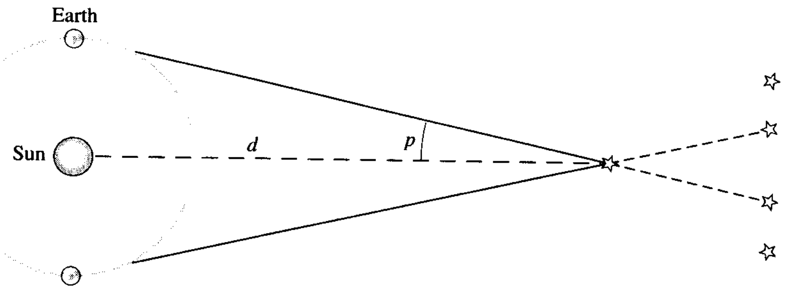


**Figure 3.2** Stellar parallax: $d = 1/p''$ pc.

$\alpha$ Centauri, and has a parallax angle of $0.77''$. If Earth's orbit around the Sun were represented by a dime, then Proxima Centauri would be located 1.5 miles away!) In fact, this cyclic change in a star's position is so difficult to detect that it was not until 1838 that it was first measured, by Friedrich Wilhelm Bessel (1784–1846), a German mathematician and astronomer.[1] Using spacecraft high above Earth's distorting atmosphere, parallax angles approaching $0.001''$ have been measured, corresponding to a distance of 1000 pc $\equiv$ 1 kiloparsec (kpc). This distance is still quite small compared to the 8 kpc distance to the center of our Milky Way Galaxy, so stellar parallax is useful only for surveying the local neighborhood of the Sun.

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**Example 3.1** In 1838, after 4 years of observing 61 Cygni, Bessel announced his measurement of a parallax angle of $0.316''$ for that star. This corresponds to a distance of

$$d = \frac{1}{p''}\text{ pc} = \frac{1}{0.316}\text{ pc} = 3.16\text{ pc} = 10.3\text{ ly},$$

within 10% of the modern value 11.1 ly. 61 Cygni is one of the Sun's nearest neighbors.

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## 3.2 The Magnitude Scale

Nearly all of the information astronomers have received about the universe beyond our solar system has come from the careful study of the light emitted by stars, galaxies, and interstellar clouds of gas and dust. Our modern

[1]Tycho Brahe had searched for stellar parallax 250 years earlier, but his instruments were too imprecise to find it. Tycho concluded that Earth does not move through space, and he was thus unable to accept Copernicus's model of a heliocentric solar system.

understanding of the universe has been made possible by the quantitative measurement of the intensity and polarization of light in every part of the electromagnetic spectrum.

The Greek astronomer Hipparchus was one of the first skywatchers to catalog the stars that he saw. In addition to compiling a list of the positions of some 850 stars, Hipparchus invented a numerical scale to describe how bright each star appeared in the sky. He assigned an **apparent magnitude** $m = 1$ to the brightest stars in the sky, and he gave the dimmest stars visible to the naked eye an apparent magnitude of $m = 6$. Note that a smaller apparent magnitude means a brighter-appearing star.

Since Hipparchus's time, astronomers have extended and refined his apparent magnitude scale. In the nineteenth century, it was thought that the human eye responded to the difference in the *logarithms* of the brightness of two luminous objects. This theory led to a scale in which a difference of one magnitude between two stars implies a constant *ratio* between their brightness. By the modern definition, a difference of 5 magnitudes corresponds exactly to a factor of 100 in brightness, so a difference of one magnitude corresponds exactly to a brightness ratio of $100^{1/5} \approx 2.512$. Thus a first magnitude star appears 2.512 times brighter than a second magnitude star, $2.512^2 = 6.310$ times brighter than a third magnitude star, and 100 times brighter than a sixth magnitude star.

Using sensitive instruments called **photometers**, astronomers can measure the apparent magnitude of an object with an accuracy of $\pm 0.01$ magnitude, and *differences* in magnitudes with an accuracy of $\pm 0.002$ magnitude. Hipparchus's scale has been extended in both directions, from $m = -26.81$ for the Sun to approximately $m = 29$ for the faintest object detectable. The total range of more than 55 magnitudes corresponds to over $100^{55/5} = (10^2)^{11} = 10^{22}$ for the ratio of the apparent brightness of the Sun to that of the faintest star or galaxy yet observed.

The "brightness" of a star is actually measured in terms of the **radiant flux** $F$ received from the star. The radiant flux is the total amount of light energy of all wavelengths that crosses a unit area oriented perpendicular to the direction of the light's travel in unit time; that is, it is the number of ergs of starlight energy arriving per second at one square centimeter of a detector aimed at the star.[2] Of course, the radiant flux received from an object depends on both its intrinsic **luminosity** (energy emitted per second) and its distance from the observer. The same star, if located farther from Earth, would appear less bright in the sky.

[2] 1 erg $= 10^{-7}$ joule.

Imagine a star of luminosity $L$ surrounded by a huge spherical shell of radius $r$. Then, assuming that no light is absorbed during its journey out to the shell, the radiant flux, $F$, measured at distance $r$ is related to the star's luminosity by

$$F = \frac{L}{4\pi r^2}. \tag{3.2}$$

Since $L$ does not depend on $r$, the radiant flux is inversely proportional to the square of the distance from the star. This is the well-known **inverse square law** for light.[3]

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**Example 3.2** The luminosity of the Sun is $L_\odot = 3.826 \times 10^{33}$ erg s$^{-1}$. At a distance of 1 AU $= 1.496 \times 10^{13}$ cm, Earth receives a radiant flux above its absorbing atmosphere of

$$F = \frac{L}{4\pi r^2} = 1.360 \times 10^6 \text{ erg s}^{-1} \text{ cm}^{-2}.$$

This value of the solar flux is known as the **solar constant**. At a distance of 10 pc $= 2.063 \times 10^6$ AU, an observer would measure the radiant flux to be only $(1/2.063 \times 10^6)^2$ as large. That is, the radiant flux from the Sun would be $3.196 \times 10^{-7}$ erg s$^{-1}$ cm$^{-2}$ at a distance of 10 pc.

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Using the inverse square law, astronomers can assign an **absolute magnitude**, $M$, to each star.[4] This is defined to be the apparent magnitude a star would have *if* it were located at a distance of 10 pc. Recall that a difference of 5 magnitudes between the apparent magnitudes of two stars corresponds to the smaller-magnitude star being 100 times brighter than the larger-magnitude star. This allows us to specify their flux ratio as

$$\frac{F_2}{F_1} = 100^{(m_1 - m_2)/5}. \tag{3.3}$$

Taking the logarithm of both sides leads to the alternate form:

$$m_1 - m_2 = -2.5 \log_{10}\left(\frac{F_1}{F_2}\right). \tag{3.4}$$

The connection between a star's apparent and absolute magnitudes and its distance may be found by combining Eqs. (3.2) and (3.3):

$$100^{(m-M)/5} = \frac{F_{10}}{F} = \left(\frac{d}{10 \text{ pc}}\right)^2,$$

[3]If the star is moving with a speed near that of light, the inverse square law must be modified slightly.

[4]The magnitudes discussed hereafter are actually *bolometric* magnitudes, measured over all wavelengths of light; see page 82.

where $F_{10}$ is the flux that would be received if the star were at a distance of 10 pc, and $d$ is the star's distance, measured in *parsecs*. Solving for $d$ gives

$$d = 10^{(m-M+5)/5} \text{ pc}. \tag{3.5}$$

The quantity $m - M$ is therefore a measure of the distance to a star and is called the star's **distance modulus**:

$$m - M = 5\log_{10}(d) - 5 = 5\log_{10}\left(\frac{d}{10 \text{ pc}}\right). \tag{3.6}$$

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**Example 3.3** The apparent magnitude of the Sun is $m_{\text{Sun}} = -26.81$, and its distance is 1 AU $= 4.848 \times 10^{-6}$ pc. Equation (3.6) shows that the absolute magnitude of the Sun is

$$M_{\text{Sun}} = m_{\text{Sun}} - 5\log_{10}(d) + 5 = 4.76,$$

as already given. The Sun's distance modulus is thus $m_{\text{Sun}} - M_{\text{Sun}} = -31.57$.[5]

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For two stars at the same distance, Eq. (3.2) shows that the ratio of their radiant fluxes is equal to the ratio of their luminosities. Thus Eq. (3.3) for absolute magnitudes becomes

$$100^{(M_1-M_2)/5} = \frac{L_2}{L_1}. \tag{3.7}$$

Letting one of these stars be the Sun reveals the direct relation between a star's absolute magnitude and its luminosity:

$$M = M_{\text{Sun}} - 2.5\log_{10}\left(\frac{L}{L_\odot}\right), \tag{3.8}$$

where the absolute magnitude and luminosity of the Sun are $M_{\text{Sun}} = 4.76$ and $L_\odot = 3.826 \times 10^{33}$ erg s$^{-1}$, respectively. It is left as an exercise for the reader to show that a star's apparent magnitude $m$ is related to the radiant flux $F$ received from the star by

$$m = M_{\text{Sun}} - 2.5\log_{10}\left(\frac{F}{F_{10,\odot}}\right), \tag{3.9}$$

[5]The magnitudes $m$ and $M$ for the Sun have a "Sun" subscript (instead of "$\odot$") to avoid confusion with $M_\odot$, the standard symbol for the Sun's mass.

where $F_{10,\odot}$ is the radiant flux received from the Sun at a distance of 10 pc (see Example 3.2).

The inverse square law for light, Eq. (3.2), relates the intrinsic properties of a star (luminosity $L$ and absolute magnitude $M$) to the quantities measured at a distance from that star (radiant flux $F$ and apparent magnitude $m$). At first glance, it may seem that astronomers must start with the measurable quantities $F$ and $m$ and then use the distance to the star (if known) to determine the star's intrinsic properties. However, if the star belongs to an important class of objects known as *pulsating variable stars*, its intrinsic luminosity $L$ and absolute magnitude $M$ can be determined *without* any knowledge of its distance. Equation (3.5) then gives the distance to the variable star. As will be discussed in Section 14.1, these stars act as beacons that illuminate the fundamental distance scale of the universe.

## 3.3 The Wave Nature of Light

Much of the history of physics is concerned with the evolution of our ideas about the nature of light. The speed of light was first measured with some accuracy in 1675, by the Danish astronomer Ole Roemer (1644–1710). Roemer observed the moons of Jupiter as they passed into the giant planet's shadow, and he was able to calculate when future eclipses of the moons should occur by using Kepler's laws. However, Roemer discovered that when Earth was moving closer to Jupiter, the eclipses occurred earlier than expected. Similarly, when Earth was moving away from Jupiter, the eclipses occurred behind schedule. Roemer realized that the discrepancy was caused by the differing amounts of time it took for light to travel the changing distance between the two planets, and he concluded that 22 minutes was required for light to cross the diameter of Earth's orbit.[6] The resulting value of $2.2 \times 10^{10}$ cm s$^{-1}$ was close to the modern value of the speed of light. In 1983 the speed of light *in vacuo* was recognized as a fundamental constant of nature whose value is, *by definition*, $c = 2.99792458 \times 10^{10}$ cm s$^{-1}$.

Even the fundamental nature of light has long been debated. Isaac Newton, for example, believed that light must consist of a rectilinear stream of particles, because only such a stream could account for the sharpness of shadows. Christian Huygens (1629–1695), a contemporary of Newton, advanced the idea that light must consist of waves. According to Huygens, light is described by the usual quantities appropriate for a wave. The distance between two successive wave crests is the **wavelength** $\lambda$, and the number of waves per second that

[6]We now know that it takes light about 16.5 minutes to travel 2 AU.