ASTRO 358 (49520): HOMEWORK 3

Assigned on Th Apr 3, 2008. Due by noon on Th Apr 10, 2008 in class

Instructions: In order to get full credit, you must show the method that you used to derive the answer. The number of points for each question is indicated in brackets, and the total score is 100 points.

1. For this question use the rotation curve in Figure 1.

(a) Using the plotted axes below Fig. 1, sketch the epicyclic frequency κ as a function of radius r over the solid body and flat part of the rotation curve, and label the peak value of κ in each region, in units of km s^{-1} pc^{-1}. Leave the transition region blank. [**15 pts**]

(b) Assume that within the central 350 pc radius of the galaxy there is a purely gaseous disk, which has a gas surface density $\Sigma_{\rm gas}$ of 2000 $M_\odot$ pc^{-2}, a gas velocity dispersion σ of 30 km s^{-1}, and the epicyclic frequency κ that you calculated in part (a). Determine whether the gas in this region is unstable to gravitational instabilities by estimating the Toomre Q parameter. In this problem, you can assume that the disk is axisymmetric and infinitely thin. [**20 pts**]

2. An Hα flux of 1.6×10^{-13} J s^{-1} m^{-2} is received from a galaxy in the Virgo cluster at a distance of 20 Mpc. What is the star formation rate of this galaxy in $M_\odot$ yr^{-1}? [**15 pts**]

3. For this question use Figures 2 and 3, which show the age of the Universe and the angular diameter distance $D_{\rm A}$ as a function of cosmological redshift z, for a cosmology with $\Omega_{\rm M} = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_{\rm k} = 0$, and $H_0 = 70$ km s^{-1} Mpc^{-1}.

(a) The Hα line has a rest wavelength of 656.285 nm. The spectrum of a distant galaxy $G1$ shows this line redshifted to a wavelength λ of 3281.425 nm. Calculate the redshift z of the galaxy $G1$. [**10 pts**]

(b) Assume that the above redshift is due purely due to the cosmological expansion of the Universe. What is the lookback time corresponding to this redshift? Expression your answer as a fraction of the present age of the Universe. [**10 pts**]

(c) The galaxy $G1$ has a disk that shows an exponential radial surface brightness profile with a scale length $R_{\rm s} \sim 0.5''$. Estimate the radius which encloses half of the total light of the galaxy, expressing your answer in kpc. [**15 pts**]

(d) Assume that the surface brightness of a source dims at the rate of $(1+z)^{-4}$. If a galaxy with an intrinsic surface brightness of 25 magnitude per $arcsec^2$ were observed at the same redshift as galaxy $G1$, what would be its observed surface brightness in magnitude per $arcsec^2$. [**15 pts**]

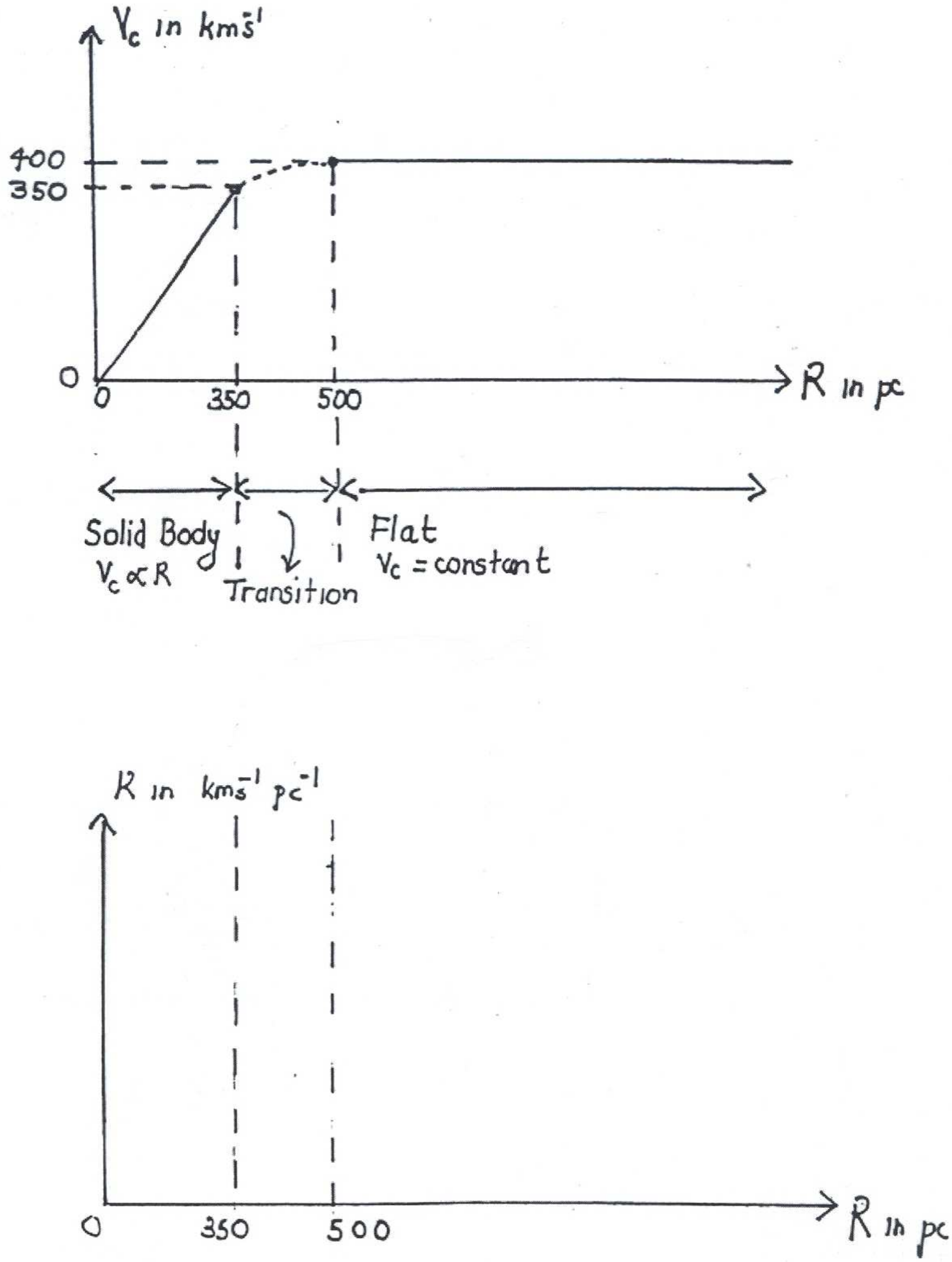

Fig. 1 – The circular speed $V_{\rm c}(R)$ is plotted as a function of galactocentric radius R in a disk galaxy. The rotation curve shows 3 regions: a solid body part where $V_{\rm c}$ is proportional to radius out to $R = 350$ pc, a flat part at $R \geq 500$ pc where $V_{\rm c}$ is independent of R , and a transition region in between.

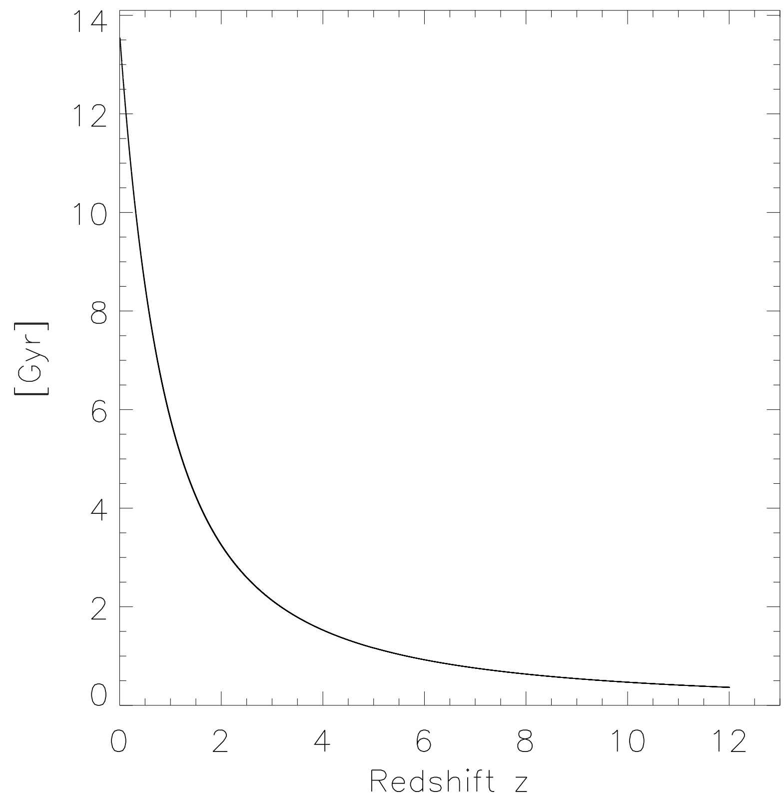

Fig. 2 – The age of the Universe is plotted *versus* redshift z for a cosmological model with $\Omega_{\rm M}$ = 0.3, $\Omega_\Lambda = 0.7$, $\Omega_{\rm k} = 0$, and $H_0 = 70$ km s^{-1} Mpc^{-1}. Assume an age of 13.7 Gyr at $z \sim 0$.

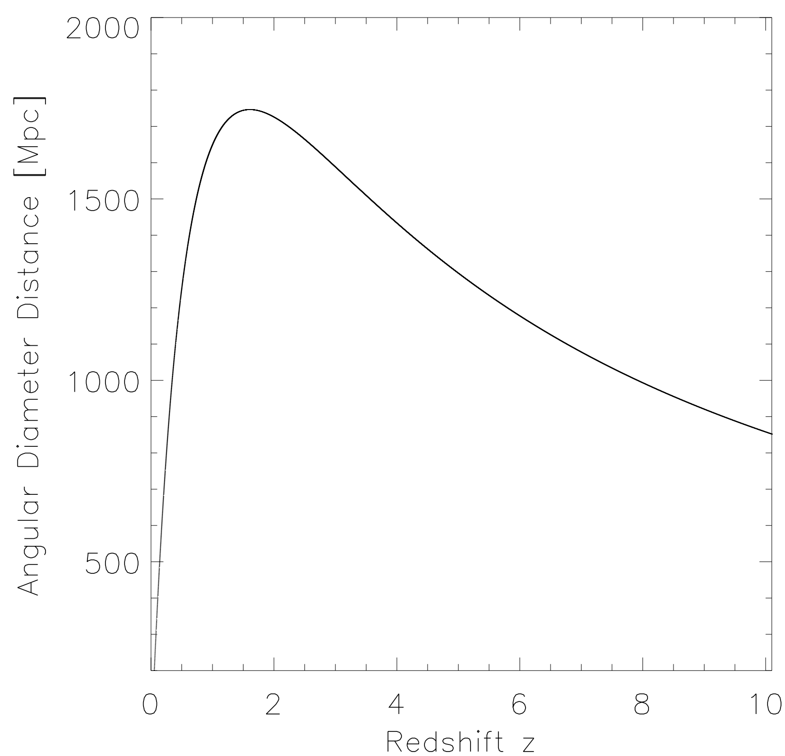

Fig. 3 – The angular diameter distance $D_{\rm A}$ is plotted *versus* redshift z for a cosmological model with $\Omega_{\rm M} = 0.3$, $\Omega_\Lambda = 0.7$, $\Omega_{\rm k} = 0$, and $H_0 = 70$ km s^{-1} Mpc^{-1}.

Values of physical constants

Wien's constant $W = 2.9 \times 10^{-3}$ m K
σ = Stefan-Boltzmann constant = 5.7×10^{-8} J s^{-1} m^{-2} K^{-4}
G = Gravitational constant = 6.7×10^{-11} N m^2 kg^{-2} = 4.5×10^{-3} (km s^{-1})^2 pc $M_\odot{}^{-1}$
k_B = Boltzmann constant = 1.38×10^{-23} J K^{-1}
Hubble's constant H_0 = 70 km s^{-1} Mpc^{-1}
Planck's constant $h = 6.6 \times 10^{-34}$ J s
Proton mass = 1.67×10^{-27} kg
1 $M_\odot = 2 \times 10^{30}$ kg
1 parsec (pc) = 3×10^{16} m
1 Joule = 10^7 erg

END OF ASSIGNMENT