ASTRO 301 (50405): Laws, Formulas, and Constants

We will discuss the mathematical formulas below and the underlying concepts in class. These formulas and constants will be provided to you during the exams.

- *The equivalence of mass and energy*: If a mass m could be *entirely* converted into energy, then the amount of energy E produced would be

$$E = mc^2 \tag{1}$$

where c = speed of light = 3×10^8 m s^{-1}. In other words, the total amount of energy E *stored* in a mass m is given by $E = mc^2$. In practice, the energy released from a mass m during fusion or other reactions is (fmc^2), where f denotes the efficiency factor with which mass is converted into energy, and is well below 100 percent.

- The flux F received at a distance d from an object of luminosity L is directly proportional to the luminosity L and is inversely proportional to the square of the distance

$$F = \frac{L}{4\pi d^2} \tag{2}$$

- *Wien's law* : A star (or blackbody) emits the maximum flux in its continuum spectrum at a wavelength λ_{peak} that is inversely proportional to its surface temperature T:

$$\lambda_{\text{peak}} = \frac{W}{T}, \tag{3}$$

where W = Wien's constant = 2.9×10^{-3} m K.

- *Stefan-Boltzmann law* : The total flux F_{surf} emitted at the surface of a star (or blackbody) over *all* wavelengths is proportional to the fourth power of its surface temperature T:

$$F_{\text{surf}} = \sigma T^4 \tag{4}$$

where σ = Stefan-Boltzmann constant = 5.7×10^{-8} J s^{-1} m^{-2} K^{-4}

- The energy E of a photon is inversely proportional to its wavelength λ and directly proportional to its frequency f:

$$E = \frac{hc}{\lambda} = hf \tag{5}$$

where h = Planck's constant = 6.6×10^{-34} J s ; c = speed of light = 3×10^8 m s^{-1}.

- Hubble's law states that on large scales, the expansion of the Universe causes galaxies separated by a large distance d to move away from each other at a speed v given by:

$$v = H_0 d \tag{6}$$

$$[v \text{ in km s}^{-1}] = [H_0 \text{ in km s}^{-1} \text{ Mly}^{-1}][d \text{ in Mly}] \tag{7}$$

where H_0 = Hubble's constant = 70 km s^{-1} Mpc^{-1} = 21.5 km s^{-1} Mly^{-1}.

- The Doppler redshift [blueshift] of a wave is the fractional increase [decrease] in its observed wavelength due to the relative motion of the emitting source away [toward] an observer.

$$\text{Doppler shift} = \frac{(\lambda_{\text{obs}} - \lambda_{\text{rest}})}{\lambda_{\text{rest}}} \tag{8}$$

where

λ_{rest} = wavelength measured when the source is at rest w.r.t. the observer

λ_{obs} c= wavelength measured when the source is moving w.r.t. the observer at relative speed v.

In the special case where the relative speed v of the source is well below the speed of light c, we can relate the Doppler shift to v by

$$\text{Doppler shift} = \frac{(\lambda_{\text{obs}} - \lambda_{\text{rest}})}{\lambda_{\text{rest}}} = \frac{v}{c} \tag{9}$$

where negative values of v denote the relative motion of the source *toward* the observer.