ASTRO 301 (48310): Laws, Formulas, and Constants

We will discuss the mathematical formulas below and the underlying concepts in class. These formulas and constants will be provided to you during the exams. Use the document 'ASTRO 301 (48310): GUIDELINES AND TIPS' for tips on how to study formulas, concepts, and laws without being bogged down by the math.

- *The equivalence of mass and energy*: If a mass m could be *entirely* converted into energy, then the amount of energy E produced would be

$$E = mc^2 \tag{1}$$

where c = speed of light = 3×10^8 m s^{-1}. In other words, the total amount of energy E *stored* in a mass m is given by $E = mc^2$. In practice, the energy released from a mass m during fusion or other reactions is (fmc^2), where f denotes the efficiency factor with which mass is converted into energy, and is well below 100 percent.

- The flux F received at a distance d from an object of luminosity L is directly proportional to the luminosity L and is inversely proportional to the square of the distance

$$F = \frac{L}{4\pi d^2} \tag{2}$$

- *Wien's law* : A star (or blackbody) emits the maximum flux in its continuum spectrum at a wavelength λ_{peak} that is inversely proportional to its surface temperature T:

$$\lambda_{\mathrm{peak}} = \frac{W}{T}, \tag{3}$$

where W = Wien's constant = 2.9×10^{-3} m K.

- *Stefan-Boltzmann law* : The total flux F_{surf} emitted at the surface of a star (or blackbody) over *all* wavelengths is proportional to the fourth power of its surface temperature T:

$$F_{\mathrm{surf}} = \sigma T^4 \tag{4}$$

where σ = Stefan-Boltzmann constant = 5.7×10^{-8} J s^{-1} m^{-2} K^{-4}

- The energy E of a photon is inversely proportional to its wavelength λ and directly proportional to its frequency f:

$$E = \frac{hc}{\lambda} = hf \tag{5}$$

where h = Planck's constant = 6.6×10^{-34} J s ; c = speed of light = 3×10^8 m s^{-1}.

- Hubble's law states that on large scales, the expansion of the Universe causes galaxies separated by a large distance d to move away from each other at a speed v given by:

$$v = H_0 d \tag{6}$$

$$[v \text{ in km s}^{-1}] = [H_0 \text{ in km s}^{-1} \text{ Mly}^{-1}][d \text{ in Mly}] \tag{7}$$

where H_0 = Hubble's constant = 70 km s^{-1} Mpc^{-1} = 21.5 km s^{-1} Mly^{-1}.

- The Doppler redshift [blueshift] of a wave is the fractional increase [decrease] in its observed wavelength due to the relative motion of the emitting source away [toward] an observer.

$$\text{Doppler shift} = \frac{(\lambda_{\text{obs}} - \lambda_{\text{rest}})}{\lambda_{\text{rest}}} \tag{8}$$

where

λ_{rest} = wavelength measured when the source is at rest w.r.t. the observer

λ_{obs} c= wavelength measured when the source is moving w.r.t. the observer at relative speed v.

In the special case where the relative speed v of the source is well below the speed of light c, we can relate the Doppler shift to v by

$$\text{Doppler shift} = \frac{(\lambda_{\text{obs}} - \lambda_{\text{rest}})}{\lambda_{\text{rest}}} = \frac{v}{c} \tag{9}$$

where negative values of v denote the relative motion of the source *toward* the observer.