Note: Please use cgs units (not mks) in all homework, and re-express answers in astronomical measures such as parsecs or solar masses when appropriate to visualize your answers. Full credit requires a statement of the principles involved, a logical and efficient procedure, and a correct numerical answer. Show the steps in your work; partial credit will be given.

2.1

(a) A photon with a wavelength of 417 Å photoionizes a neutral hydrogen atom in the ground state. What is the frequency of the photon in Hz and in Ryd, and its energy in eV? What is the kinetic energy of the the ejected photoelectron? Repeat for an atom in the n=2 level.

(b) A ground state hydrogen atom is exposed to a monochromatic collimated flux of radiation at a frequency of 3 Ryd. The flux is $\pi F = 10^{-1}$ erg s$^{-1}$ cm$^{-2}$. What is the photon flux (photons cm$^{-2}$ s$^{-1}$ )? What is the photoionization cross section of H$^0$ at this frequency? What is the photoionization probability per second, $\Gamma_{pi}$?

(c) A small cloud (optically thin) with a density of neutral hydrogen atoms $n(\mathrm{H}^0) =$ 100 cm$^{-3}$ is subjected to the ionizing flux of part (b). What is the ionization rate of neutrals in the gas (cm$^{-3}$ s$^{-1}$)?

(d) Suppose the gas of part (c) contains, in addition to the H$^0$ atoms, an ionized hydrogen density $N(H^+) = N_p = 100$ cm$^{-3}$. The electron temperature is $T = 15,000$ K and the electron density is $N_e = N_p$. What is the recombination coefficient $\alpha_A(T)$? What is the recombination rate for protons becoming H$^0$ (cm$^{-3}$ s$^{-1}$)?

(e) For the conditions of part (d), is the stated degree of ionization higher or lower than the steady state ionization equilibrium that would prevail for the given gas density and ionizing flux?

2.1 A cloud of hydrogen gas with a density $N = 10^9$ cm$^{-3}$ and temperature $T = 15,000$ K is located $10^{15}$ cm from an ionizing source consisting of a compact X-ray source orbiting an O star. The ionizing luminosity of the star and the luminosity of the X-ray source are each $L_U = L_X = 10^{38}$ erg s$^{-1}$. Make the approximation that the stellar ionizing luminosity is all at frequency $\nu = 1.5\nu_H$ and the X-ray luminosity is all at $h\nu = 1$ keV.

(a) Calculate the ionizing photon luminosities (photons per second) of the star and the X-ray source, $Q_U$ and $Q_X$. What is the probability per second $\Gamma_U$ that a neutral hydrogen atom is ionized by the stellar radiation at the front face of the cloud (toward the star)? Compare this with $\Gamma_X$ due to the X-ray flux. (Assume $a_\nu \propto \nu^{-3}$).

(b) What is the degree of ionization of hydrogen at the front face of the cloud? Give results for the case when only the UV photons are present, when only the X-rays are present, and when both are present. Express your answer in terms of the ionization fractions $1 - x = X(\mathrm{H}^0) \equiv N(\mathrm{H}^0)/N(H)$ and $x = X(\mathrm{H}^+)$.

(c) Give a simple estimate of the depth in the cloud to which the gas remains highly ionized. (Hint: Use the "photon counting" argument. Which photons should be included?)

(d) What is the optical depth for $\nu = 1.5\nu_H$ at the point where the gas is 50 percent ionized?

(e) What is the absorption length for the X-rays in the neutral gas?