FRI Astronomy Lab #8: "Bonus Lab"

Goal: In contrast to the last lab, this lab is entirely theoretical. While at the observatory we took data on pulsating stars and measured their frequencies of pulsation. When we use these observed frequencies to infer aspects of a star's structure, we call it "asteroseismology." This is the ultimate goal of many of our observations.

The purpose of this lab is to illustrate asteroseismology in a simpler context. Instead of a star we will examine the oscillation frequencies of a string. The string is assumed to be the usual "perfect" string, except that we have placed beads of different mass and width at given locations. These beads shift the eigenfrequencies from their ideal values and allow us to derive the properties of the beads from the frequencies *alone*.

Assignment: You will use the application at `modetrap.py` to match *by eye* several pulsation spectra that you are given below. Qualitative understanding is what the goal is, but I expect you to be able to obtain numbers quite similar to the exact ones for at least the simpler cases.

Background

The vibrating string is a canonical problem in physics. The equation (which we will derive later in this course) results from the application of $F = ma$ to a continuous medium, the string. Letting ψ be the vertical displacement of the string, we have

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2},$$

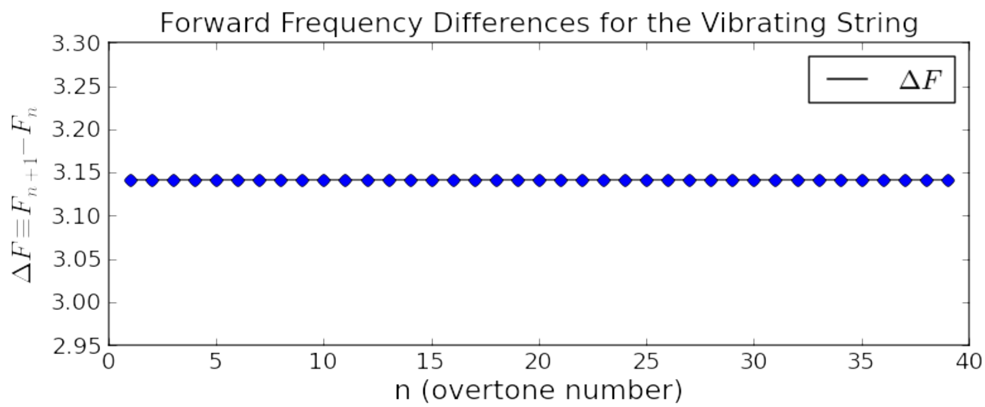
where $c^2 \equiv T/\mu$, T is the tension (a force) and μ is the mass per unit length of the string. Assuming the string has a length L and that its ends are fixed, then we find that the frequencies of oscillation are given by

$$f_n = \pi n c/L, \;\; n = 1, 2, 3 \ldots$$

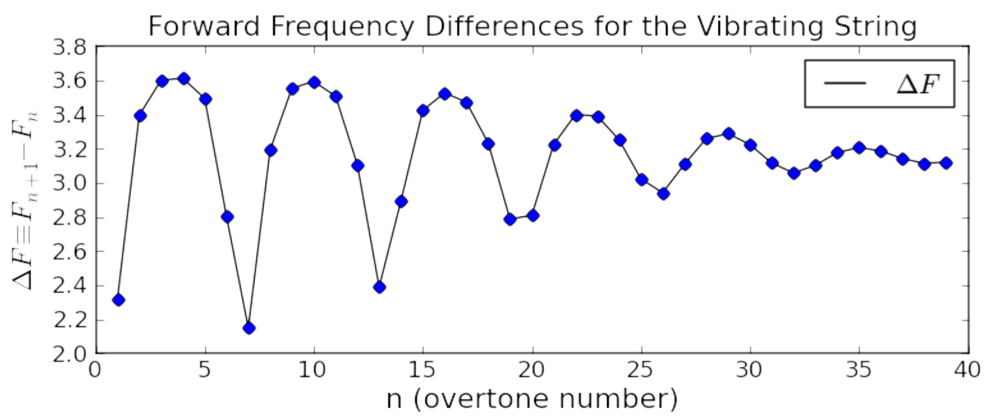
Thus, an ideal string has the special property that

$$\begin{aligned} \Delta f_n &\equiv f_{n+1} - f_n \\ &\equiv \pi c/L, \end{aligned}$$

i.e., the eigenfrequencies are evenly spaced. If we made the rather boring plot of Δf_n versus n for this case we would find



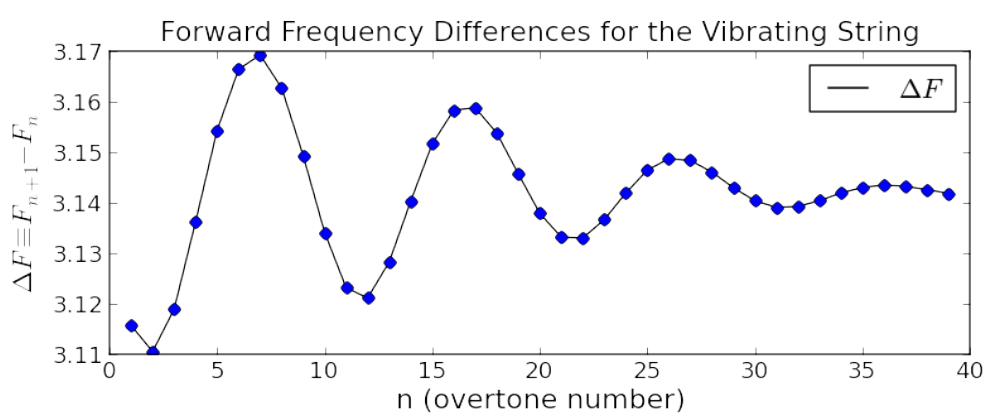
where we have assumed that $c/L = 1$ hz. However if I put one bead of a certain mass and width at a given location then the form of this plot is now



For the above plot I have used $(0.16, 2.0, 0.03)$ for (location, mass, width), respectively. Note that the location and width are measured in terms of L, the length of the string, so these quantities are always between 0 and 1. The mass can have any positive value.

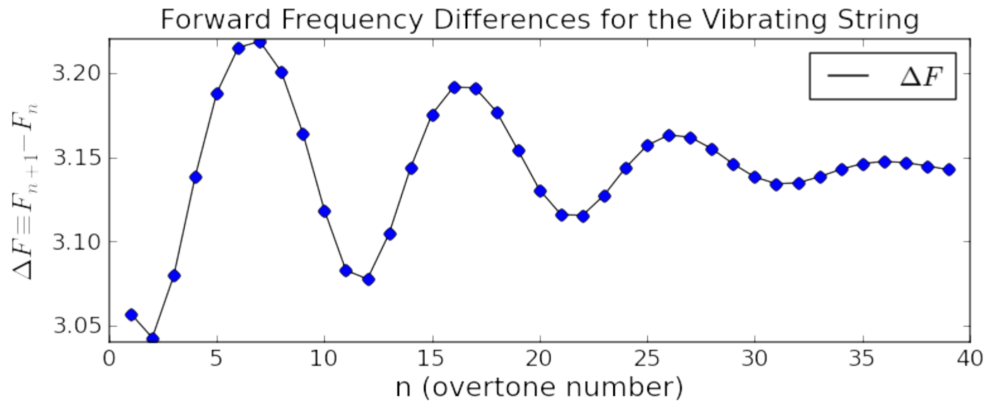
Instructions

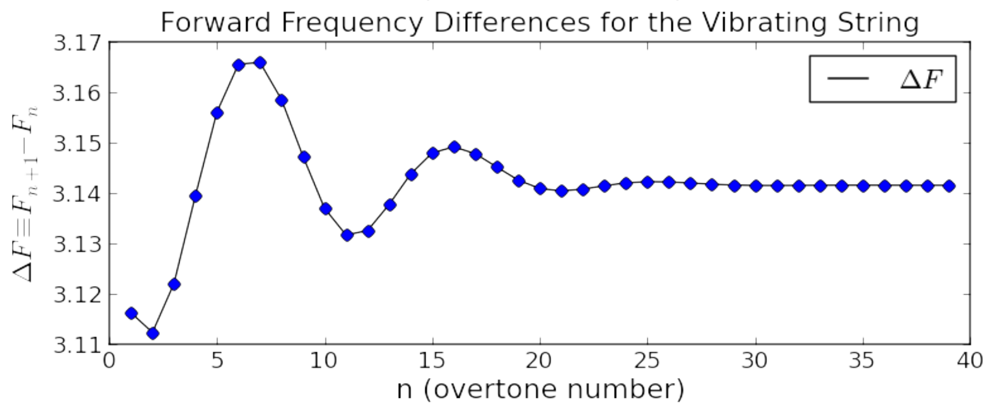
1. Using `modetrap.py` match *by eye* the Δf plot given below [hint: there is only one bead]:

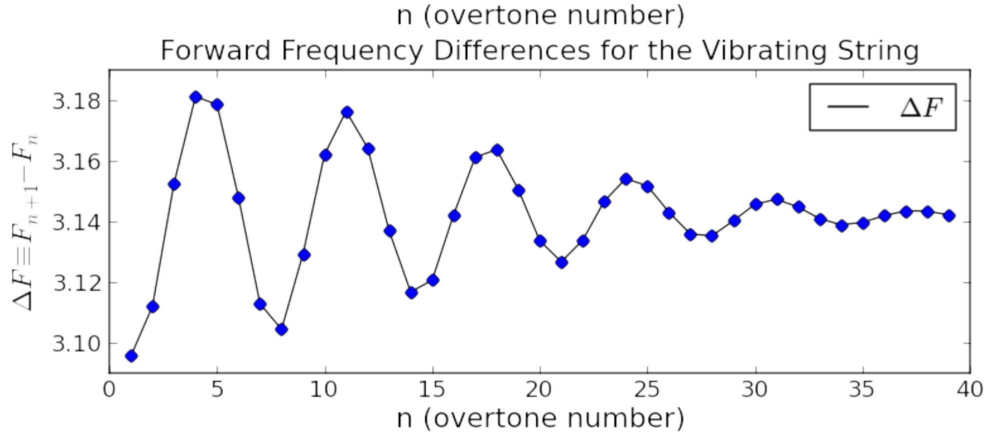


Open a Word document and paste in your best fit plot along with the values of bead location, mass, and width [do this for the other parts of the lab as well]. It is important to get close to the right answer so you may wish to consult with a mentor before moving on to the next item. Note: you don't need to search in increments finer than 0.01, i.e., two decimal places are sufficient accuracy for all values.

2. The following three plots were generated by changing only one parameter from the sdolution to problem 1. For each plot indicate which parameter changed and what its new value is:

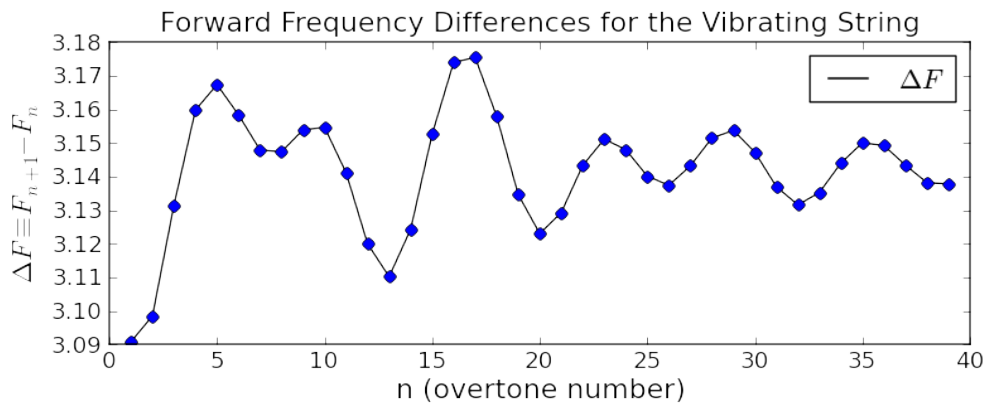






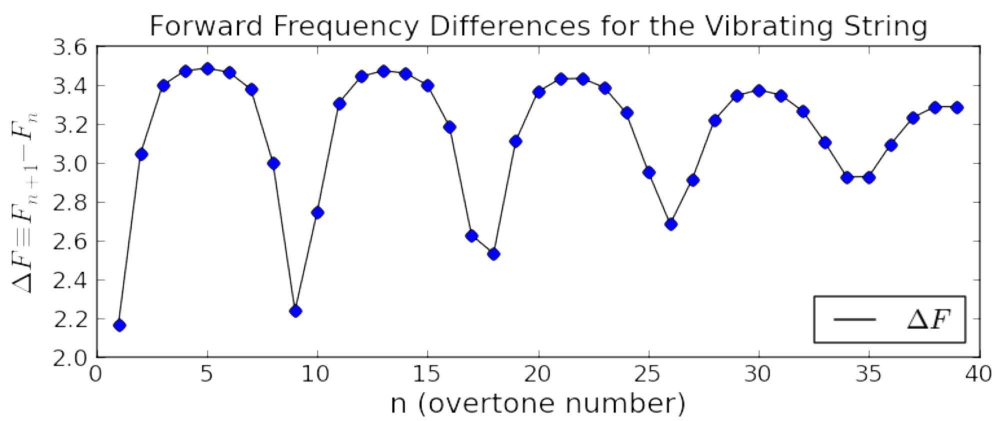
3. From the prior question, give a qualitative/semi-quantitative description of the effect of changing a) the mass of the bead, b) the width of the bead, and c) the location of the bead. You may also find it helpful to consider what happens when (one at a time) these quantities are set to zero.

4. Now it gets hard. Assume we have two beads. One of them has the parameters you found in step 1. The other one has parameters that are unknown. Try to derive the parameters describing the second bead based on the plot below. Paste your solution into the Word document.

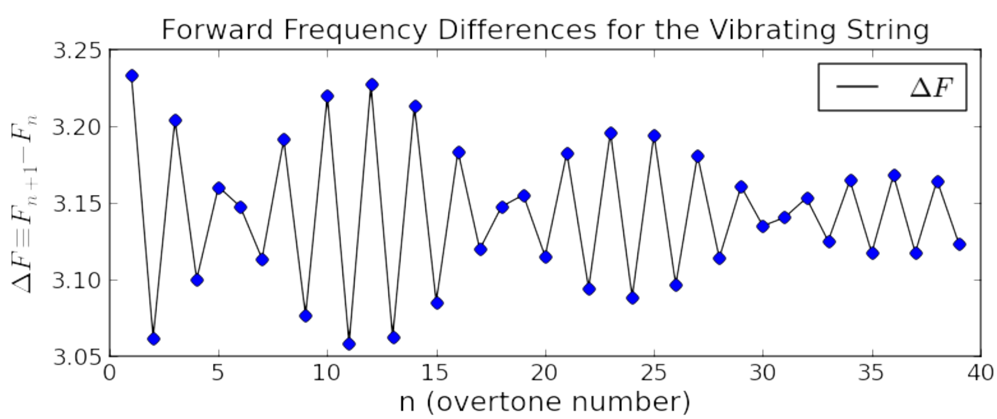


Finally, see if you can find solutions (either one or two beads) for the following diagrams:

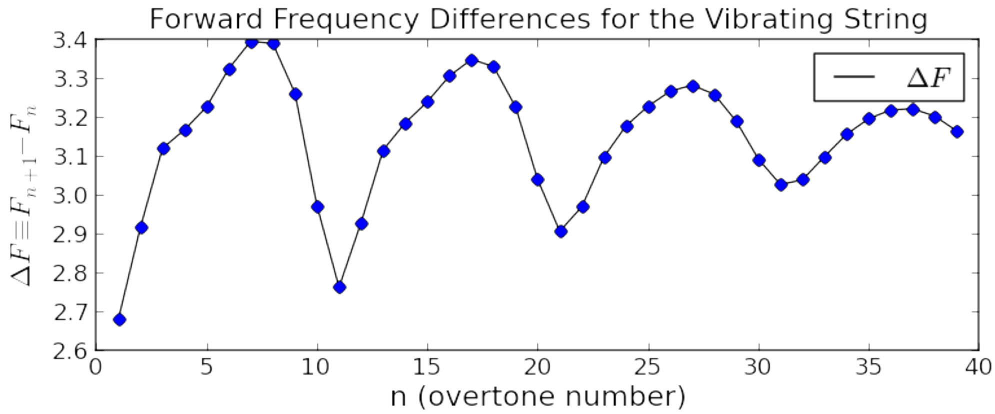
5.



6.



7.



Don't forget to *bask*…