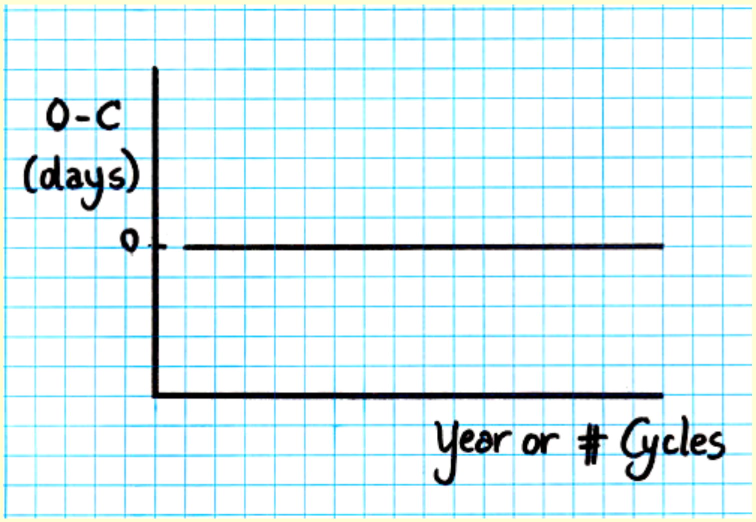
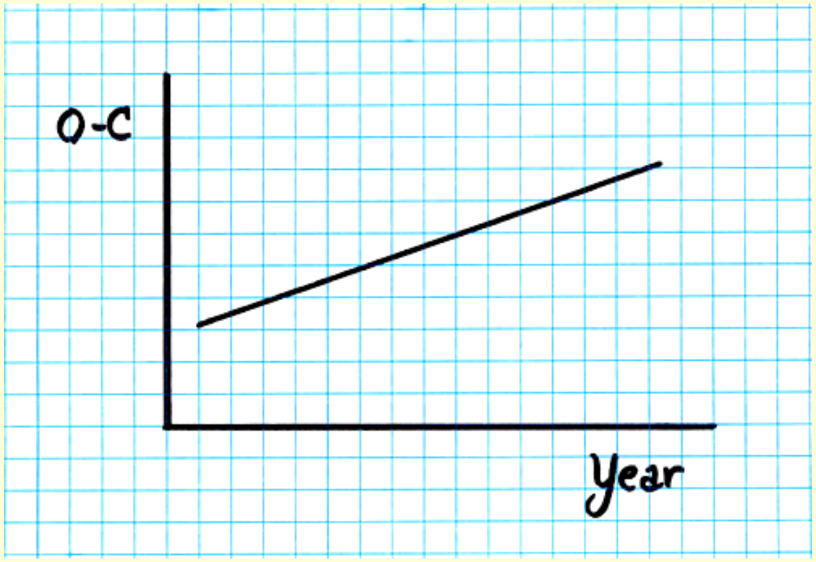
The O-C diagram

- O-C: Observed time – Calculated time

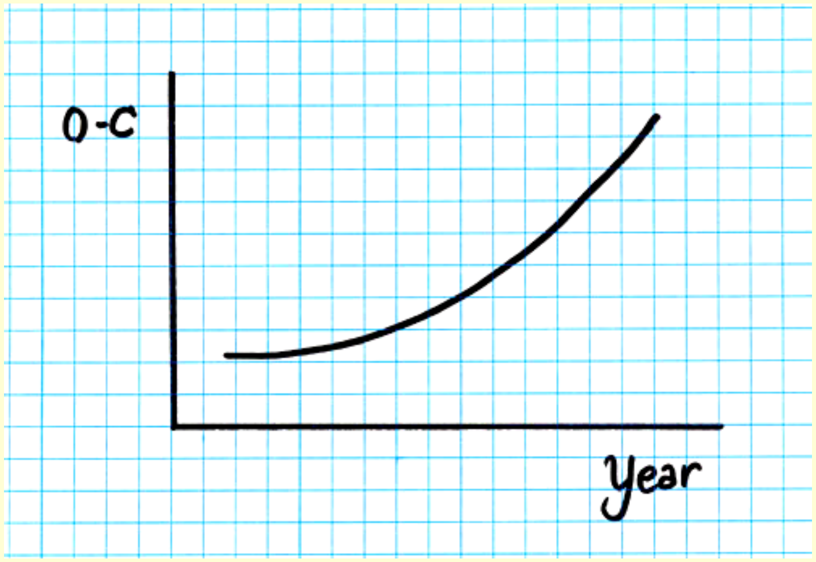


Period is correct and constant.

The O-C diagram

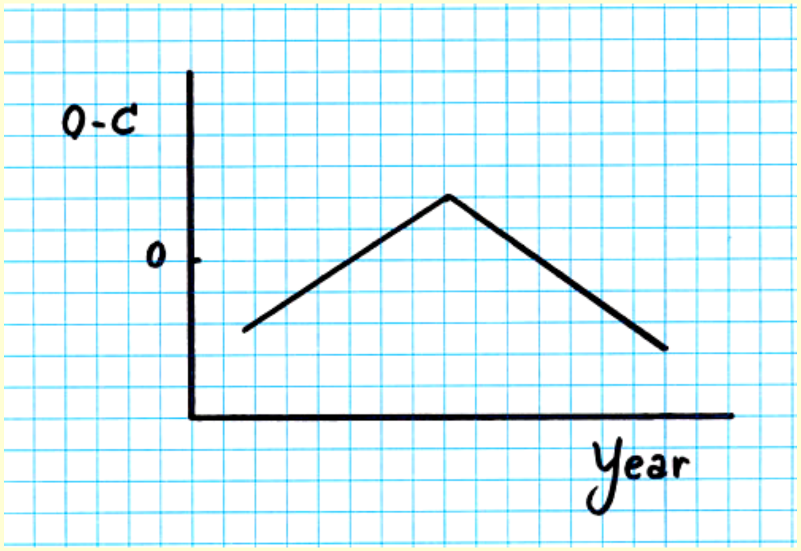


Period is incorrect but constant.



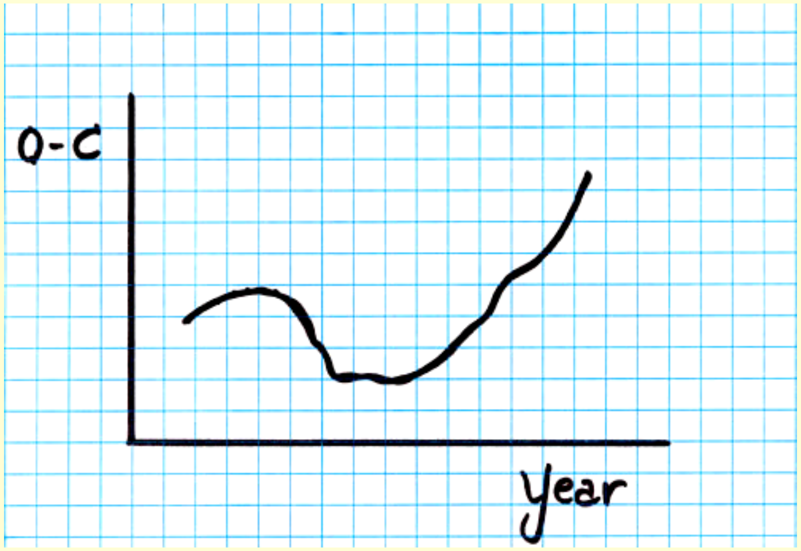
Period is changing (increasing), e.g., due to evolution.

The O-C diagram



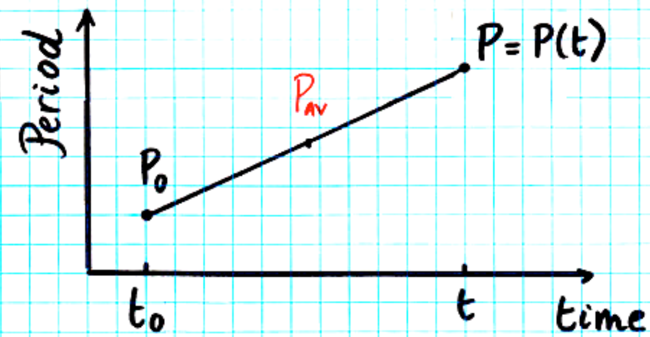
Abrupt change from one constant period to another,

e.g., in the RR Lyrae star SZ Hya.



Who knows ???

The relation between period changes and O-C



$$P = P_0 + \frac{dP}{dt}(t - t_0)$$

after N cycles

$$P_{AV} = (P_0 + P)/2$$

$$P_{AV} = P_0 + \frac{1}{2}\frac{dP}{dt}(t - t_0)$$

$$t - t_0 = N P_{AV}$$

$$P_{AV} = P_0 + \frac{1}{2}\frac{dP}{dt} N P_{AV}$$

$$P_{AV}\left(1 - \frac{1}{2}\frac{dP}{dt} N\right) = P_0$$

$$P_{AV} = \frac{P_0}{\left(1 - \frac{1}{2}\frac{dP}{dt} N\right)}$$

The relation between period changes and O-C

But we don't know P_0, P_{AV}, ... We only know the trial period P_V

$$(O-C) = NP_{AV} - NP_V$$

$$= \frac{NP_0}{1 - \frac{1}{2}\frac{dP}{dt}N} - NP_V$$

Series expansion, $\frac{1}{2}\frac{dP}{dt}N \ll 1$

$$(O-C) = NP_0 + \frac{1}{2}N^2 P_0 \frac{dP}{dt} - NP_V$$

$$(O-C) = N(P_0 - P_V) + N^2\left(\frac{1}{2}P_0\frac{dP}{dt}\right)$$

↳ quadratic function

The relation between period changes and O-C

(O-C) : quadratic function ; insert measured data

$$(O-C) = A_0 + A_1 N + A_2 N^2$$

A_0 = constant zero point error

$A_1 = P_0 - P_V$

→ Start Period $P_0 = P_V + A_1$

$A_2 \rightarrow \frac{dP}{dt}$

The relation between period changes and O-C

$$(O-C) = NP_0 + \frac{1}{2} N^2 P_0 \frac{dP}{dt} - NP_V$$

$$(O-C) = N(P_0 - P_V) + N^2 \left(\frac{1}{2} P_0 \frac{dP}{dt}\right)$$

↳ quadratic function

If the adopted period P_V is correct : $P_V = P_0$, then

$$(O-C) = \frac{1}{2} N^2 P_0 \frac{dP}{dt}$$

$$= \frac{1}{2} t^2 \left(\frac{1}{P_0} \frac{dP}{dt}\right)$$

$N = \frac{t}{P_0}$

O-C and period changes

Some remarks concerning Period Changes:

(1.) (o-c) in d (days) $= \ldots + \frac{1}{2} N^2 P_0 \frac{dP}{dt}$

$$\cong \ldots + \frac{1}{2} \underbrace{\left(\frac{1}{P}\frac{dP}{dt}\right)}_{d^{-1}} t^2$$

NB: $\frac{1}{P}\frac{dP}{dt}$ often expressed in yr^{-1} (thus 365.25 x larger!)

(2.) In publications we often find:

HJD (max) $= T_0 + P_0 . N + \alpha N^2$ (T_0 and P_0 in d),

so $\frac{1}{2P}\frac{dP}{dt} . t^2 = \alpha N^2$ with $t/P = N$

$\frac{1}{P}\frac{dP}{dt} = \frac{2\alpha}{P^2}$ (again in d^{-1}, not yr^{-1}) or $\alpha = \frac{1}{2} P \frac{dP}{dt}$ (in d)

O-C and period changes

Remarks (continued):

3. Often HJD (max) $= T_0 + P.N + \frac{\beta}{2} N^2$

$$\text{then } \beta = 2\alpha = P \frac{dP}{dt}$$

4. $\frac{d(\ln P)}{dt} = \frac{1}{P} \frac{dP}{dt}$

Example: BL Cam

$$HJD(max) = 2443125,8042 + 0,0391 . E + 6,153 . 10^{-13} . E^2$$

$$\alpha = 6,153 . 10^{-13} d \,, \quad P = 0,0391\, d$$

$$\left(\frac{1}{P} \frac{dP}{dt}\right) = \frac{2\alpha}{P^2} = 8.05 \times 10^{-10}\, d^{-1} \text{ or } 2.94 \times 10^{-7} yr^{-1}$$

$$\frac{dP}{dt} = 3,15 \times 10^{-11}\, d/d$$

O-C and period changes: some examples

Example: $\frac{dP}{dt} = 10^{-11}$, time span: 10 years

$$(O-C) \text{ in days} = \frac{1}{2}\left(10 \times 365{,}25\right)^2 \times 10^{-11} / P_0$$

$$= 6{,}67 \times 10^{-5} / P_0$$

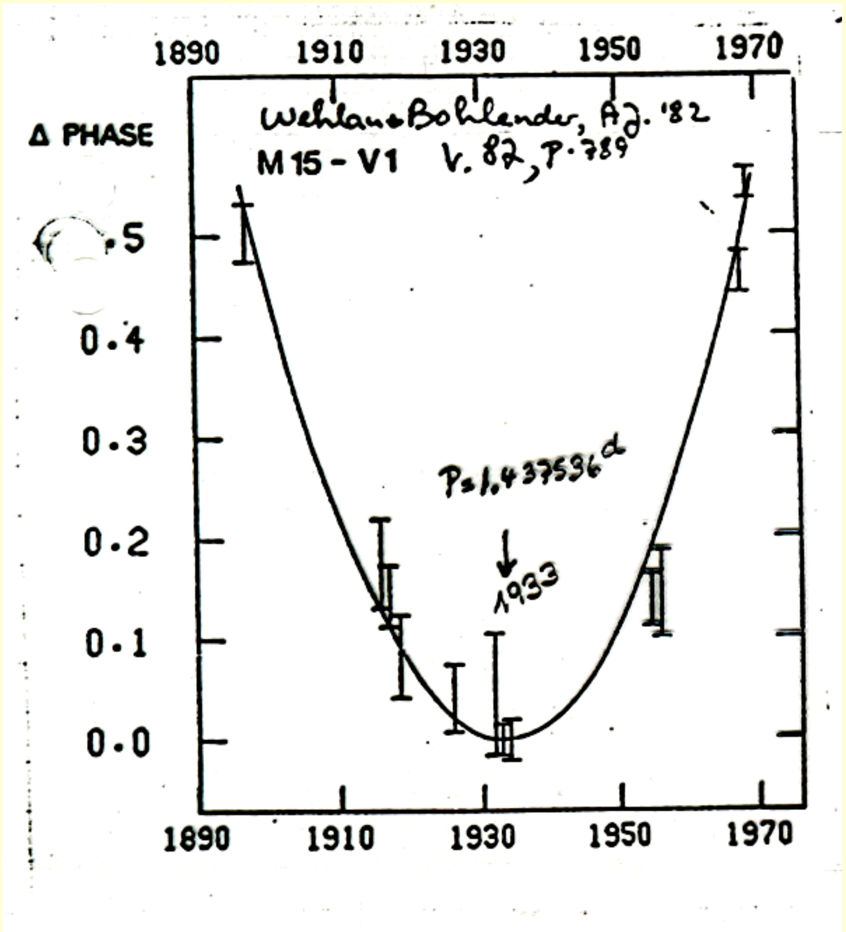
Cepheid: $P = 10\,d \rightarrow (O-C) = 6{,}67 \times 10^{-6}\,d = 0{,}58\,s$

δ Scuti Star: $P = 0{,}1\,d \rightarrow (O-C) = 6{,}67 \times 10^{-4}\,d = 57{,}6\,s$

Pulsar: $P = 0{,}03\,s = 3{,}472 \times 10^{-7}\,d$

$\rightarrow (O-C) = 0{,}0192\,d = 27{,}6\,\text{min}$!

O-C and period changes: some examples



Example: Period Changes in cluster BL Her Stars

BL Her Stars: Pop. II Cepheids

20 stars known with P between 1.13 - 7.90 d

(Wehlau & Bohlender, AJ 1982)

1932-1970: $\Delta\phi \sim 0.55 \Rightarrow (O-C) = 0.8\ d$

Cycles: ca. 9600

$P_0 = 1.437536\ d$

$$(O-C) = \frac{1}{2} N^2 P_0 \frac{dP}{dt}$$

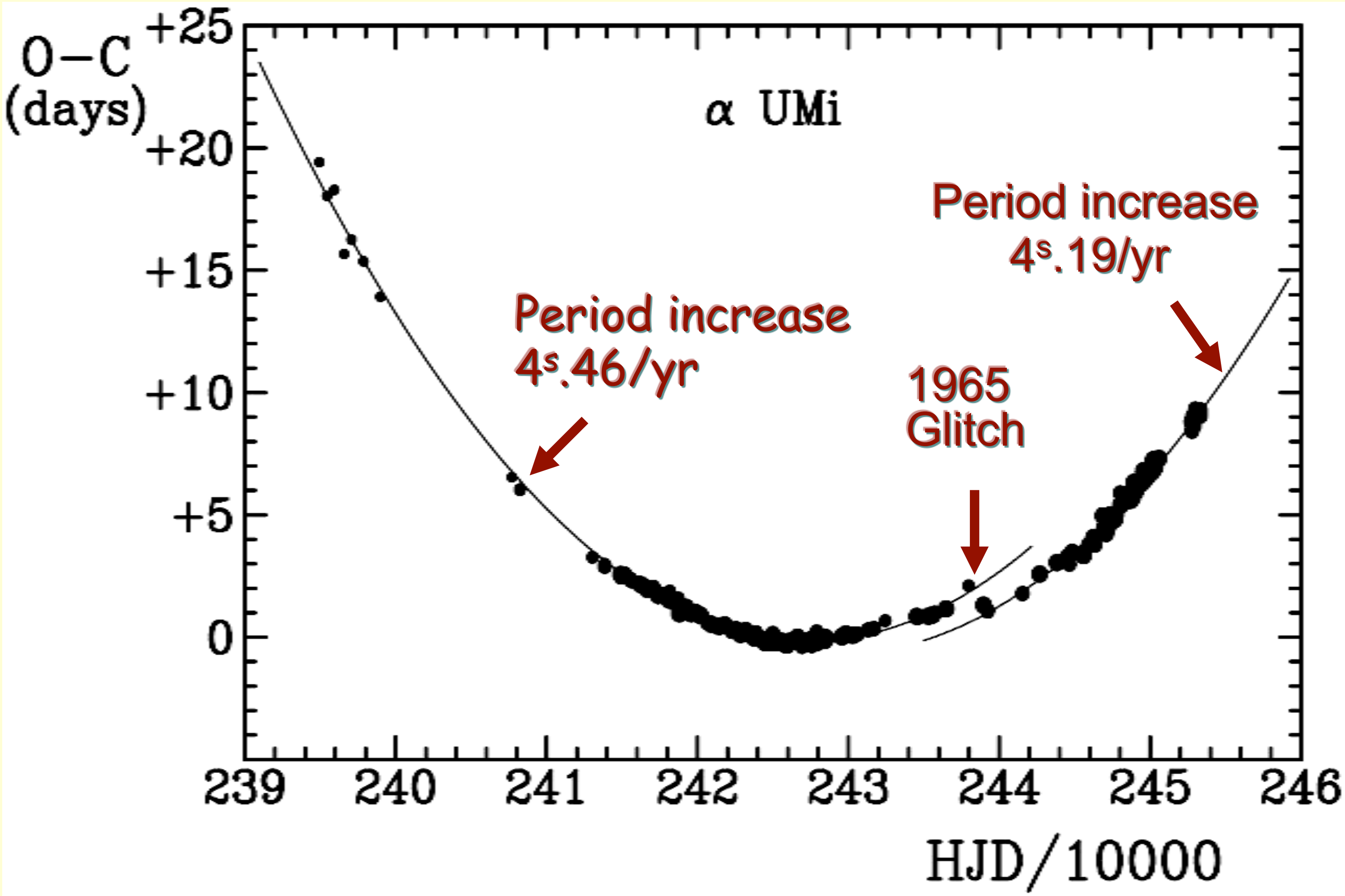
$$0.8 = \frac{1}{2} (9600)^2\ 1.437536 \frac{dP}{dt}$$

$$\Rightarrow \frac{dP}{dt} = 1{,}2 \times 10^{-8}\ d/d$$

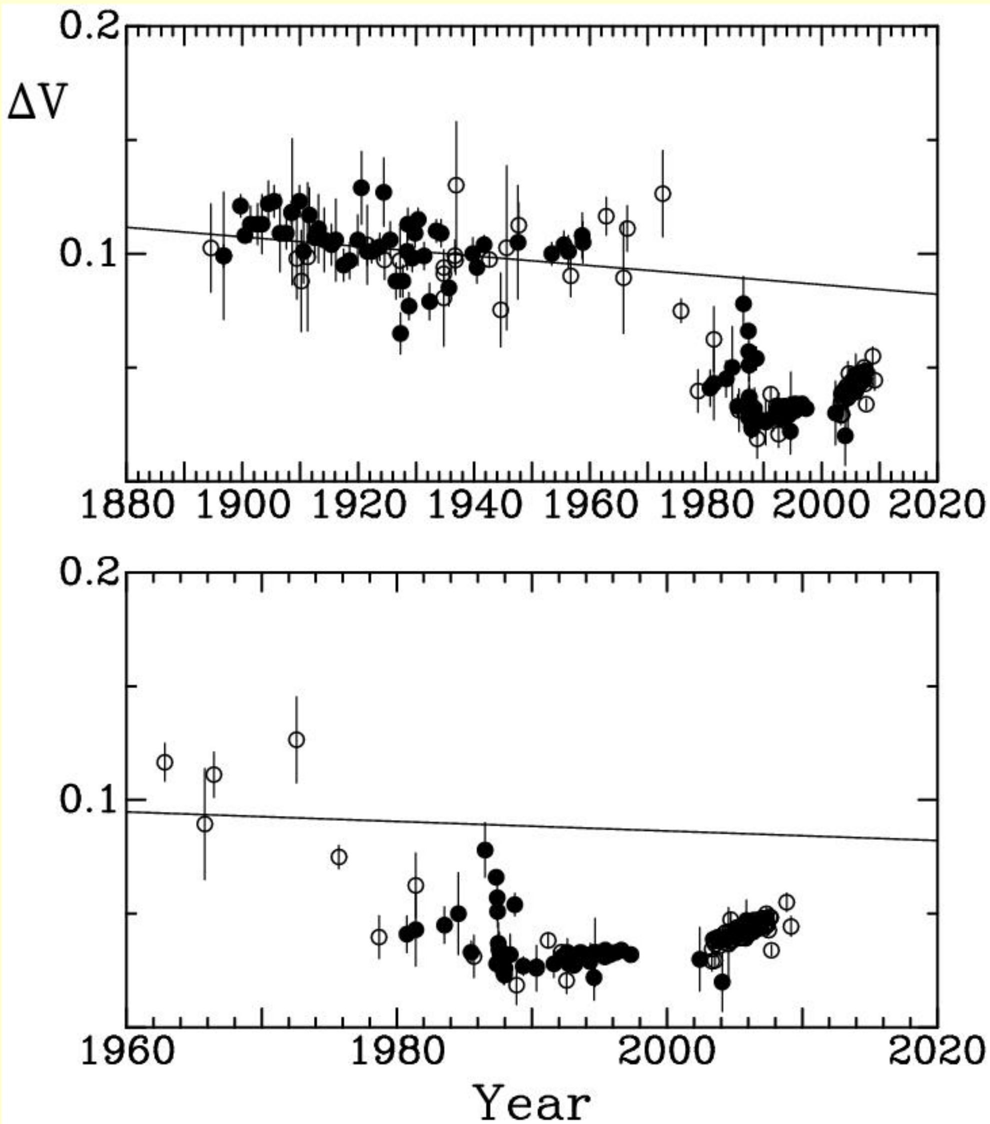
or $4{,}4 \times 10^{-6}\ d/yr$

Note: We started at the bottom of the curve!

Polaris - Period always increasing except for an unusual “glitch” around 1965

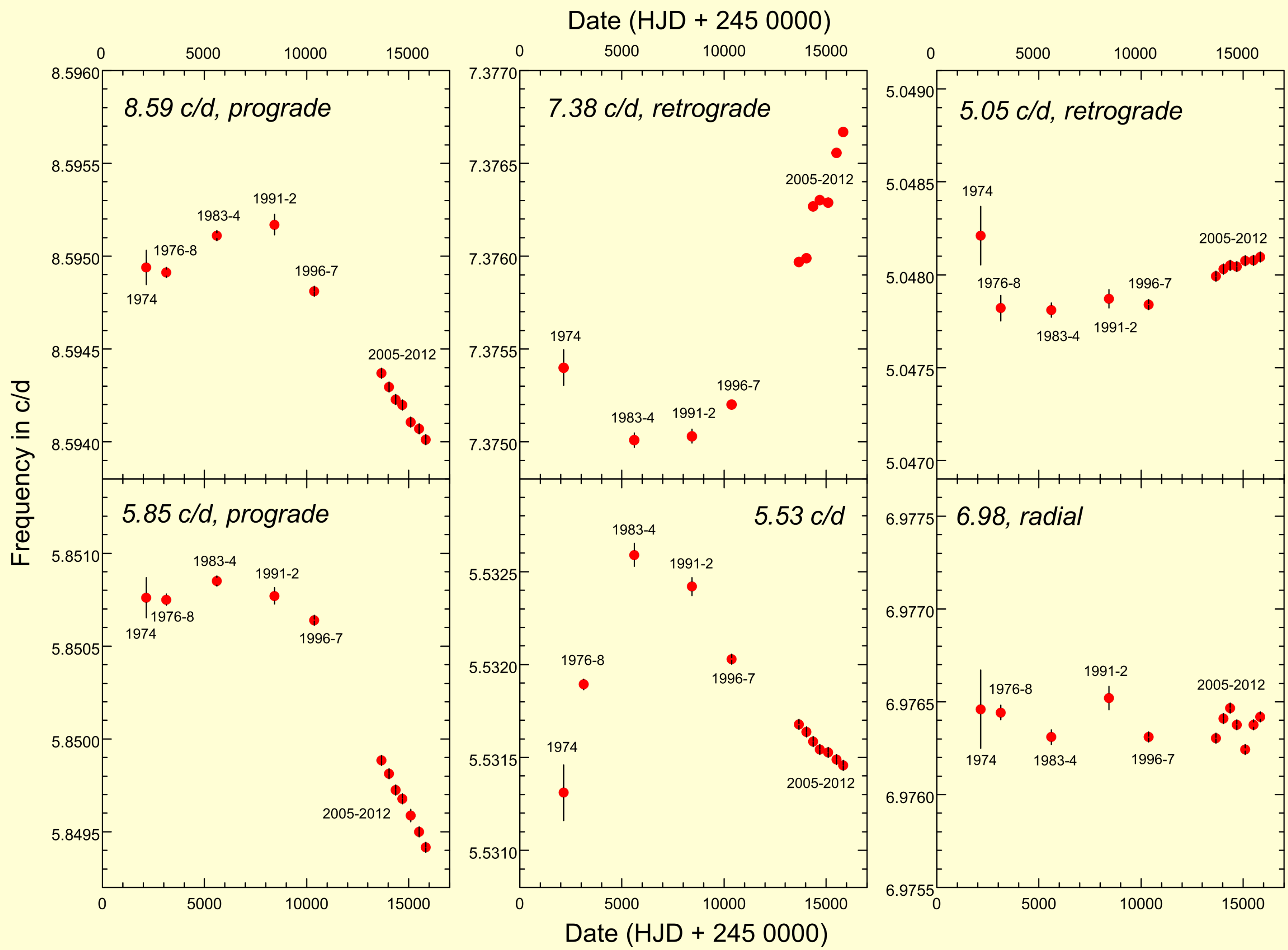


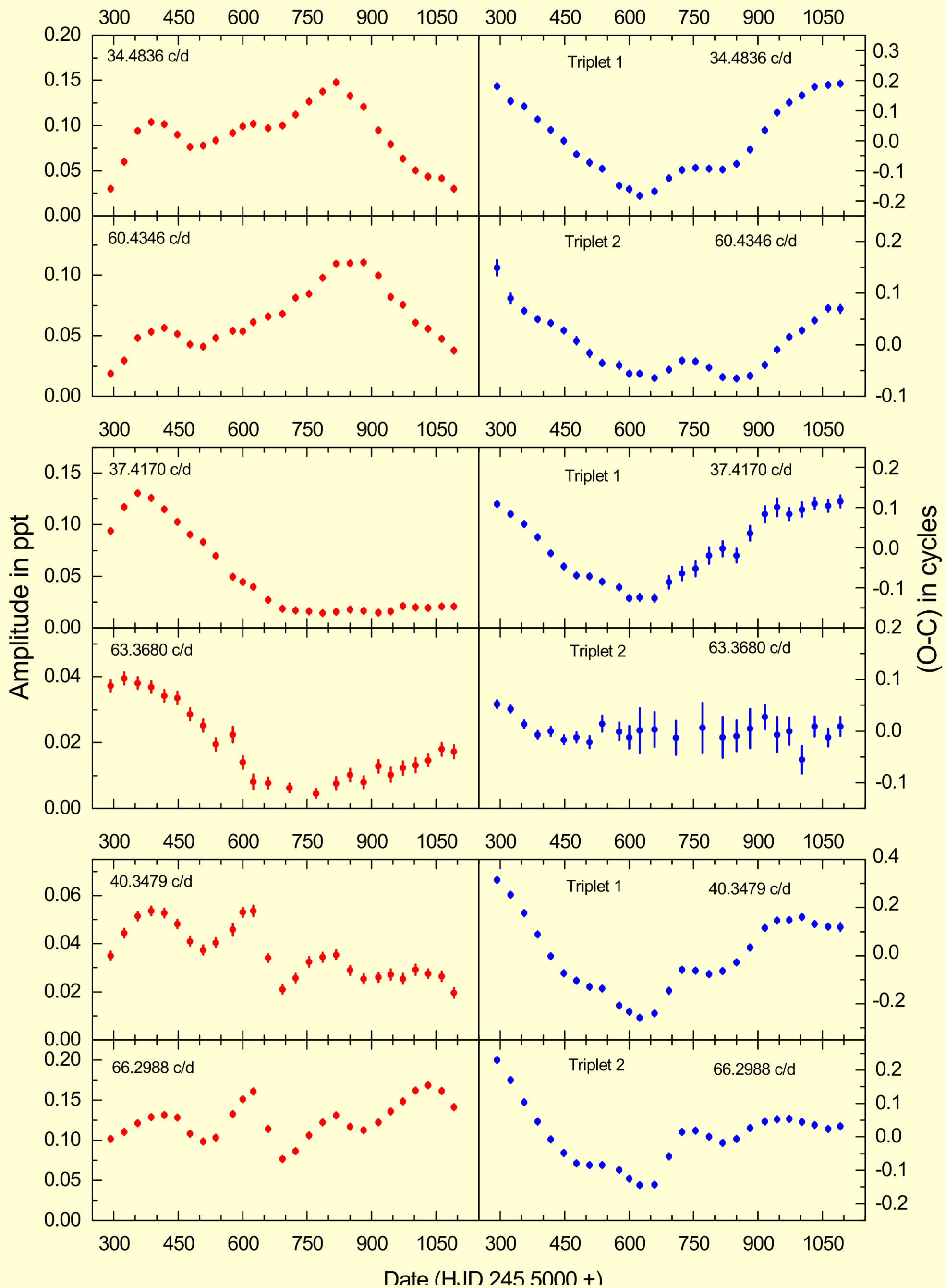
Polaris – amplitudes near "glitch" around 1965



Amplitudes from V_R
– filled circles
Pe, pg, visual photometry
- open circles
(See Turner 2009)

Frequency changes of the Delta Scuti star 4 CVn over 40 years

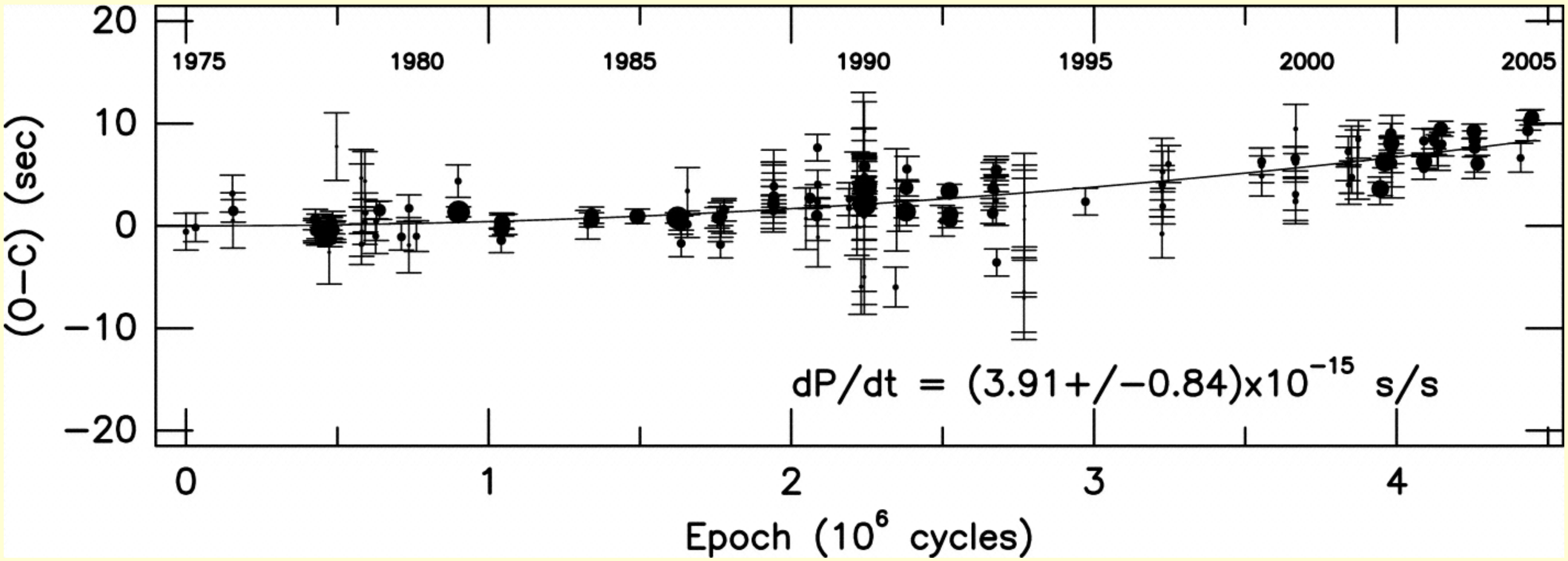




O-C changes of the Delta Scuti star KIC 8054146

O-C and period changes: some examples

G117-B15A: the most stable optical clock known
(Kepler et al. 2005, ApJ 634, 1311)



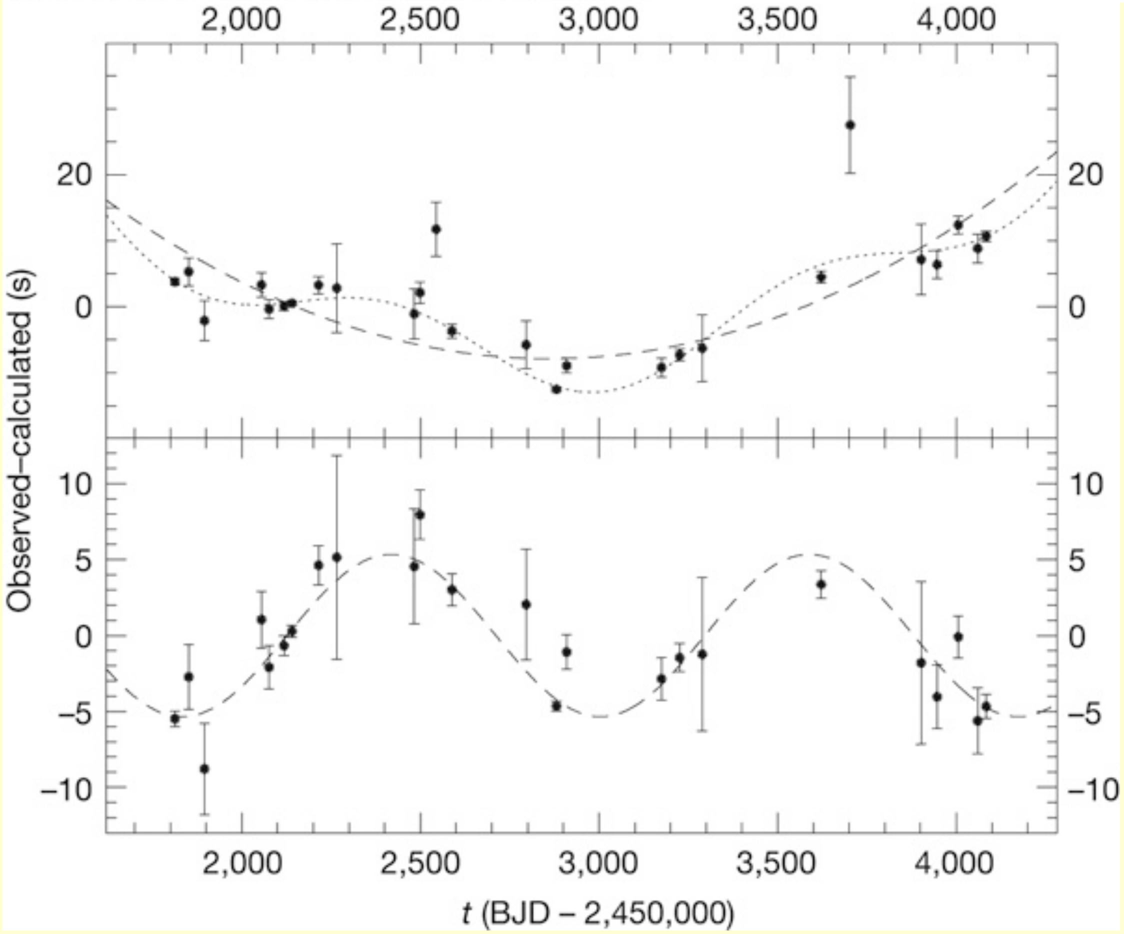
Note: period change due to proper motion (Pajdosz 1995, A&A 295, L17)

O-C and period changes: some examples

A giant planet orbiting the 'extreme horizontal branch' star V 391 Pegasi

R. Silvotti[1], S. Schuh[2], R. Janulis[3], J.-E. Solheim[4], S. Bernabei[5], R. Østensen[6], T. D. Oswalt[7], I. Bruni[5], R. Gualandi[5], A. Bonanno[8], G. Vauclair[9], M. Reed[10], C.-W. Chen[11], E. Leibowitz[12], M. Paparo[13], A. Baran[14], S. Charpinet[9], N. Dolez[9], S. Kawaler[15], D. Kurtz[16], P. Moskalik[17], R. Riddle[18] & S. Zola[14,19]

Nature 449, 189 (2007)



The pulsating subdwarf V391 Peg

Planet: 3 Jupiter masses, 1.7 AU, 3.2 yr period