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THE EFFECT OF ^{3}He DIFFUSION ON THE PULSATIONAL SPECTRA OF DBV MODELS

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ABSTRACT

We consider envelopes of DB white dwarfs that are not composed of pure ^{4}He but rather a mixture of ^{3}He and ^{4}He. Given this assumption, the same diffusive processes that produce a relatively pure H layer overlying an He layer in the DA's should work to produce an ^{3}He layer overlying an ^{4}He layer in the DB's. We examine the relevant timescales for diffusion in these objects and compare them with the relevant evolutionary timescales in the context of the DBV white dwarfs. We then explore the consequences that ^{3}He separation has on the pulsational spectra of DBV models. Since GD 358 is the best-studied member of this class of variables, we examine fits to its observed pulsation spectrum. We find that the inclusion of an ^{3}He layer results in a modest improvement in a direct fit to the periods, while a fit to the period *spacings* is significantly improved.

Subject headings: dense matter — stars: evolution — stars: oscillations — white dwarfs

1. ASTROPHYSICAL CONTEXT

The two greatest successes of white dwarf asteroseismology have come from the analysis and interpretation of Whole Earth Telescope data on the objects GD 358 and PG 1159 (Winget et al. 1991, 1994). The temporal spectra of both objects show well-defined multiplet structures as well as many consecutive radial orders of the same l. This allows us to estimate immediately the rotational frequency Ω and the mean period spacing $\langle \Delta P \rangle_l$, with the latter quantity putting strong constraints on the mass of the best-fitting model. Furthermore, the variations from equidistant period spacing, i.e., "mode trapping," give us information (at least in the case of GD 358) about the radial structure of the star, such as the thickness of its surface He layer.

In a detailed analysis of the frequency distribution in GD 358, Bradley & Winget (1994) found evidence of a chemical transition zone located at a mass depth of $1.5 \times 10^{-6} M_\star$, which they interpreted as the C/He boundary. An He layer thickness of $\sim 10^{-6} M_\star$ is thinner than expected on naive evolutionary grounds as well as thinner than suggested for the DA white dwarfs by the asteroseismological work of Clemens (1993). While the fit of Bradley & Winget (1994) explained the trapping features in the neighborhood of 700 s reasonably well, it did not do as well for periods near 500 s. They found evidence that these periods near 500 s might be better fitted if an additional chemical transition zone at a depth of $\sim 10^{-2} M_\star$ is assumed.

Independent of the above considerations, if the theory of diffusion of chemical species in stars is correct, then at least a partial separation of different isotopes of the same element must occur in white dwarfs, as was pointed out to one of us by D. Clayton (1988, private communication). At the time, there were no known observational consequences; this has since changed with the advent of white dwarf asteroseismology.

While the Galactic number ratio of ^{3}He to ^{4}He has been measured to be of order 10^{-4} in many astrophysical environments (Galli et al. 1995; Prantzos 1996), this does not necessarily hold for the stellar cores that are presumably the white dwarf progenitors. In fact, standard evolutionary theory would suggest quite low levels of ^{3}He in these objects (Galli et al. 1995). Our view is that given the theoretical uncertainties in the late stages of stellar evolution, we should not rule out any possibilities, and indeed we should seek to make as many independent measurements as possible.

These considerations led to two independent motivations for considering ^{3}He diffusion (Winget 1998). First, it is a process that may be a generic feature of white dwarf cooling and, as such, should be considered as part of our asteroseismological analyses. Second, it might allow us to place the mode-trapping data for GD 358 in a different context: this star could have a C/^{4}He transition zone at $\sim 10^{-2} M_\star$ and a thinner ^{4}He/^{3}He transition zone at $\sim 10^{-6} M_\star$. If this is the case, then the DBV's and the DAV's would again have the same order-of-magnitude He layer thicknesses. To determine the plausibility of these hypotheses, we first examine the relevant diffusion timescales and then the effect that such a layering structure has on our fits of GD 358.

2. DIFFUSION

2.1. *Timescales*

The process of diffusion is of major importance since we are aware of no other effect that could lead to the spatial separation of ^{3}He and ^{4}He. We therefore examine the relevant timescales for such diffusive processes to see if it is plausible for a significant fraction of the ^{3}He to have separated from ^{4}He in the elapsed evolutionary times for these objects.

Fortunately for us, Fontaine & Michaud (1979) have already examined the related problem of C diffusion in a background of normal He, i.e., ^{4}He. In their analysis, they treated C as a trace element. This is an excellent approximation for our case as well since we naively expect that the ^{3}He is only about one part in 10^4 of the ^{4}He.

Applying their equations for this case for a 0.612 $M_\odot$ model, we obtain the result shown in Figure 1. We see that if we assume a $T_{\rm eff}$ of 25,000 K for a typical DBV, then diffusive equilibrium between ^{3}He and ^{4}He should prevail down to approximately the $10^{-4} M_\star$ mass point. Thus, if the ^{4}He layer is

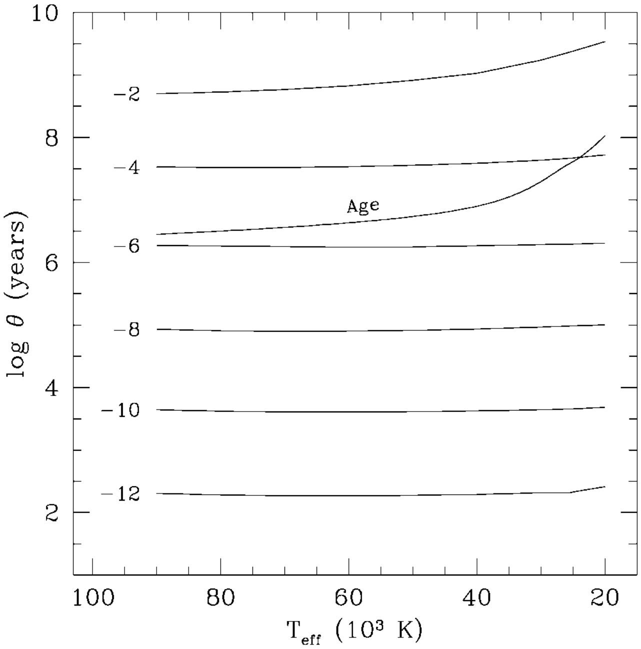

FIG. 1.—Timescale θ for diffusion of ^{3}He in a background of ^{4}He as a function of $T_{\rm eff}$ at different mass depths between -12 and -2. The curve labeled "Age" gives the age of the model as a function of $T_{\rm eff}$; below the line, the profile should have reached diffusive equilibrium, and above it, little diffusion should have occurred.

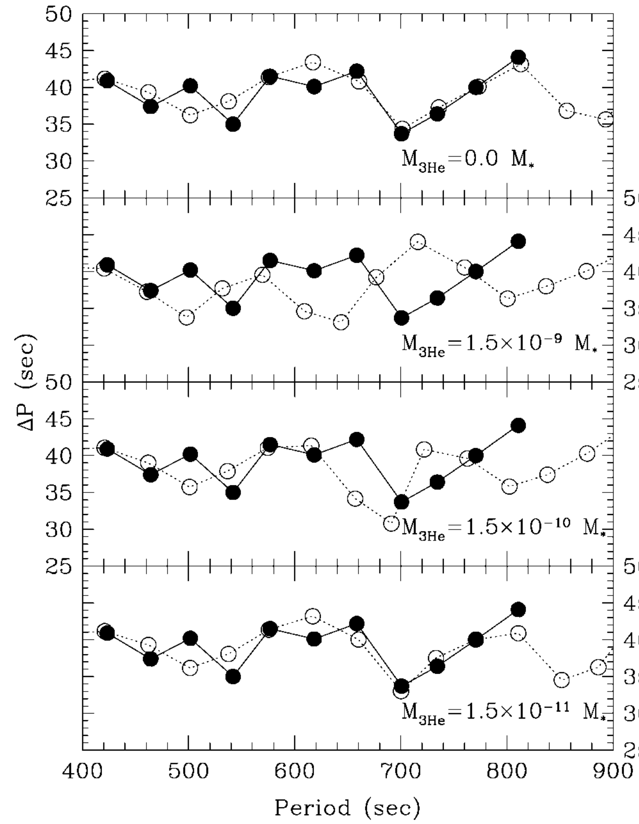

FIG. 2.—Comparison of the observed trapping (*filled circles, solid lines*) with carbon core models (*open circles, dotted lines*). The top panel is the best-fit model of Bradley & Winget (1994), which of course has $M_{^3{\rm He}} = 0.0$. The lower panels show the mode-trapping structure for nonzero values of $M_{^3{\rm He}}$, as indicated in each panel.

as thick or thicker than this, and the ^{3}He abundance is $\sim 10^{-4}$ that of the ^{4}He abundance, we would expect an ^{3}He layer of thickness $\sim 10^{-8}$.

One clear prediction of this is that the spectra of DB's in this temperature range should show more or less pure ^{3}He, at least for ^{4}He layers that are thicker than $\sim 10^{-6}$. This is currently in the process of being tested using high-resolution spectroscopic observations (D. Koester 2000, private communication).

We mention as an aside that the case of ^{3}He may be unique in that it is a trace isotope that is *lighter* than the dominant species, so that it can diffuse upward and produce a thin but not asteroseismologically negligible layer. For trace isotopes that sink, there should be no such signature.

2.2. *Profiles*

In this preliminary exploration, we will only consider ^{4}He/^{3}He zones with equilibrium profiles, although the profiles could in fact be less sharp than this. The treatment of the chemical profiles in the transition zones in our present and previous work is based on the work of Arcoragi & Fontaine (1980). Essentially, we use equation (A6) of Arcoragi & Fontaine (1980), which assumes that an equilibrium distribution has been reached and that one of the elements may be treated as a trace element. This is certainly a valid assumption for the initial stages of ^{3}He diffusion, given the expected abundance ratio of ^{3}He to ^{4}He.

3. EQUATION OF STATE

To include the effects of an ^{3}He layer, we have made a relatively simple modification to the envelope routines in our evolutionary code. Since we are interested in modeling a DB, we are free to use the array space normally reserved for the H profile and use it for the ^{3}He profile. To this end, we have replaced the H equation of state (EOS) and opacities with those appropriate for ^{3}He. In doing this, we have taken the ^{3}He EOS and opacities to be equal to those of ^{4}He at $\frac{3}{4}$ the density (to correct for the isotopic mass ratio). This approximation should be more than sufficient for our purposes. A comparison of the region of period formation for the cases of an ^{4}He/H and an ^{4}He/^{3}He envelope is given in Montgomery & Winget (1999).

4. PREVIOUS BEST FITS

We now wish to examine the effect that an ^{3}He layer could have on the pulsation frequencies. First, in the top panel of Figure 2, we show one of the best-fit models for the star GD 358 from Bradley & Winget (1994). The filled circles connected by solid lines show the observed mode-trapping structure, and the open circles connected by dotted lines show the results from the best-fit model. This model has $M_\star/M_\odot = 0.61$, $T_{\rm eff} = 24{,}044$ K, and $M_{^4{\rm He}}/M_\star = 1.5 \times 10^{-6}$. The lower three panels show the effect a thin layer of ^{3}He has on the mode-trapping structure. If the layer is as thick as $M_{^3{\rm He}}/M_\star = 1.5 \times 10^{-10}$, corresponding to $N(^3{\rm He})/N(^4{\rm He}) = 10^{-4}$, then we see that the mode-trapping structure in the vicinity of 700 s is significantly altered. Unfortunately, this is the region in which the fit was already quite good, whereas in the region around 500 s, where the fit was poor, there is very little change. However, this analysis clearly demonstrates that the inclusion of an ^{3}He layer can have a measurable effect on the calculated mode-trapping structure.

TABLE 1
RESULTS OF FITS USING A GENETIC ALGORITHM

Model	T_{eff} (K)	$M_\star/M_\odot$	$\log(M_{^4He}/M_\star)$	$\log(M_{^3He}/M_\star)$	$\sigma(P)$	$\sigma(\Delta P)$
C/thick4He	23,100	0.655	−2.74	…	2.30	2.65
C/thick4He/3He	23,200	0.605	−2.74	−6.02	1.99	2.29
C/thin4He	23,800	0.600	−5.76	…	2.71	2.53
C/thin4He/3He	24,400	0.580	−5.71	−10.09	1.95	2.66
O/C/4He[a]	22,600	0.650	−2.74	…	1.50	1.92
O/C/4He/3He[a]	22,300	0.630	−2.79	−7.49	1.29	1.32
Inferred errors	500	0.015	0.06	0.12	…	…

[a] These fits were performed using $f = 1/[\sigma(P) + \sigma(\Delta P)]$ as the fitness criterion; for the other models, $f = 1/\sigma(P)$ was used.

5. NEW FITS USING A GENETIC ALGORITHM

In an effort to test asteroseismologically the viability of the hypothesis of an ^{3}He layer, we performed an extensive set of calculations. Using a genetic algorithm (Metcalfe 1999; Metcalfe, Nather, & Winget 2000), the parameters $M_\star$, T_{eff}, $M_{^4He}$, and $M_{^3He}$ were varied to produce pulsational models whose $l = 1$ and $m = 0$ periods were compared with those inferred from the observations. The region of parameter space explored is given by

$$0.45\ M_\odot < M_\star < 0.95\ M_\odot,$$

$$20{,}000\ \mathrm{K} < T_{eff} < 30{,}000\ \mathrm{K},$$

$$\sim 10^{-7} M_\star < M_{^4He} < 10^{-2} M_\star,$$

$$10^{-5} M_{^4He} < M_{^3He} < 10^{-3} M_{^4He}.$$

The best-fit models from these runs are listed in Table 1. The column labeled $\sigma(P)$ gives the residuals (standard deviation) of the calculated and observed periods, and the column labeled $\sigma(\Delta P)$ gives the residuals of the calculated and observed period *spacings*. For the upper four models, we have taken the fitness criterion to be $1/\sigma(P)$, whereas for the lower two models, we took it to be $1/[\sigma(P) + \sigma(\Delta P)]$; thus, for these last two entries, we are fitting not just the periods but the spacings between consecutive periods. We have chosen this criterion for the models with a more realistic core composition based on our suspicion that the period *spacings* will be a more sensitive diagnostic of the ^{3}He layer than the periods themselves will be.

As a reference to previous fits, the fit of Bradley & Winget (1994) for a C core model has $\sigma(P) \sim 2.3$ s. Thus, the present best fits, both with and without an ^{3}He layer, having period residuals of ∼1.5 and 1.3 s, respectively, represent a significant improvement over previous fits.

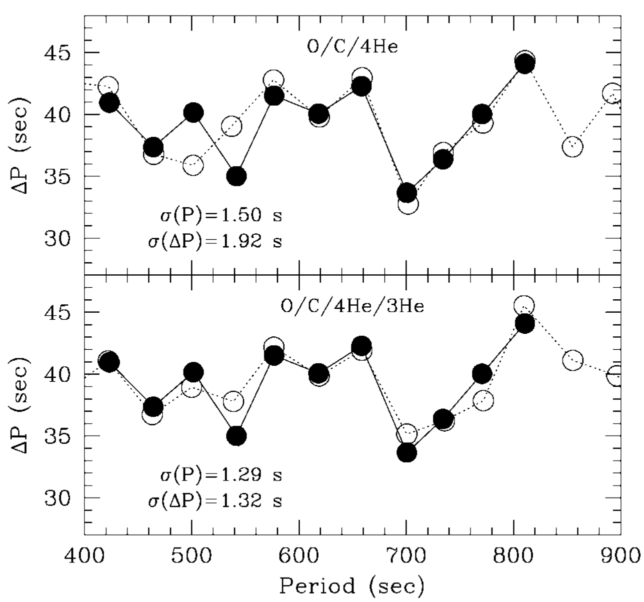

FIG. 3.—*Upper panel*: Best-fit model without ^{3}He. *Lower panel*: Best-fit model with ^{3}He. As can be seen, these fits have much smaller residuals than the previous fit given in the upper panel of Fig. 2.

6. DISCUSSION

We see from Table 1 that the fits with O/C cores are significantly better than those with pure carbon cores, independent of whether an ^{3}He layer is present or not, a result previously found by Metcalfe et al. (2000). In Figure 3, we display the mode-trapping diagrams for these two fits. Both fits reproduce the periods quite well, with the ^{3}He model reproducing the period *spacings* much better (∼1.3 s compared with ∼1.9 s). This indicates that the period spacing may be a better diagnostic for the fine structure produced by the ^{4}He/^{3}He transition zone than just the periods themselves.

We now seek to understand the relative importance of the O/C and ^{4}He/^{3}He transition zones. In the asymptotic limit of high radial overtones and large periods, the frequency of a given mode is given by a simple radial integral of the Brunt-Väisälä frequency. Using a "period formation" diagram, we can show the relative weight that a given region has in determining a mode's period.

In Figure 4, we show such a diagram for the case of our best-fit ^{3}He model. The three peaks that are labeled correspond to the O/C, C/^{4}He, and ^{4}He/^{3}He transition zones. Using the equilibrium diffusion coefficients for the ^{4}He/^{3}He transition zone, we see that it is of relatively minor importance in determining the mode frequencies. In contrast, the O/C and C/^{4}He transition zones are both quite pronounced and should significantly affect the periods of the modes calculated in the models; this is borne out by the major improvement in the standard deviation of the periods, $\sigma(P)$, with the inclusion of an O/C chemical profile. The period *spacings*, on the other hand, are a differential quantity, and therefore they are sensitive to even small deviations in the background structure. It is therefore not surprising that the inclusion of an ^{3}He layer results in a major reduction in the residuals of the period spacings.

We now attempt to quantify the statistical significance of the improvement of the fit when the parameter corresponding to the ^{3}He layer thickness is added. Following Koen & Laney (2000), we apply the Bayesian information criterion for $N =$ 11 data points. We find that the addition of a parameter should be accompanied by a decrease in the residuals of at least ∼10%

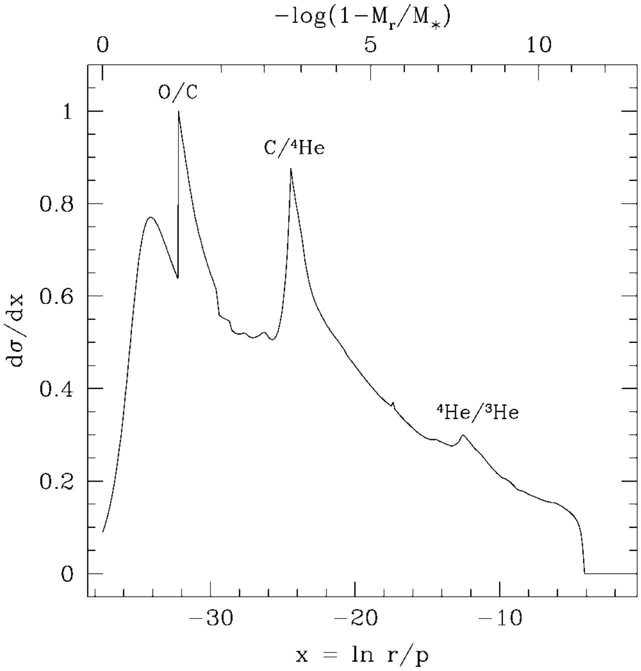

FIG. 4.—Relative contribution of a region to the frequency of a mode σ as a function of $x = \ln(r/p)$, where r is the radius and p is the pressure, both in cgs units. The features corresponding to the O/C, C/^{4}He, and ^{4}He/^{3}He transition zones have been labeled.

in order to be considered statistically significant. We note that this is a necessary but certainly not sufficient condition for the validity of adding a parameter.

Examining the best-fit O/C core models, we see that the standard deviation of the fit to the period spacings improved from 1.92 to 1.32 s with the addition of ^{3}He. This is a decrease of 30% and should therefore be considered statistically significant.

Finally, we again note that the models with O/C cores yield markedly lower residuals, regardless of which criterion is used (periods or period *spacings*). This result has recently been obtained by Metcalfe et al. (2000), who report the results of such fits in an extensive parameter space of white dwarf models.

7. CONCLUSIONS

From this preliminary analysis, we have shown that diffusion theory applied to white dwarfs predicts that any ^{3}He initially present in the DBV's down to a depth of $\sim 10^{-4} M_\star$ should have diffused upward to produce a surface layer of ^{3}He. In addition to being detectable spectroscopically (D. Koester 2000, private communication), such an ^{3}He layer can significantly affect the asteroseismological fits and therefore needs to be included in such analyses.

We find that although the inclusion of an ^{3}He layer results in only a marginal improvement to the fits to the periods, the fit to the period *spacings* is significantly improved. This is because the period spacings are more sensitive to the fine structure than an ^{3}He/^{4}He transition zone produces than the periods themselves are.

Finally, we find that an O/C core (essentially, the transition in the core from an O/C mixture to pure C) fits the observed pulsational spectrum of GD 358 *much* better than a pure C core, in agreement with Metcalfe et al. (2000), who first obtained this result. This gives us the hope of someday being able to constrain the prior nuclear-burning history of GD 358 and other pulsating white dwarfs.

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REFERENCES

Arcoragi, J. P., & Fontaine, G. 1980, ApJ, 242, 1208
Bradley, P. A., & Winget, D. E. 1994, ApJ, 430, 850
Clemens, J. C. 1993, Ph.D. thesis, Univ. Texas at Austin
Fontaine, G., & Michaud, G. 1979, ApJ, 231, 826
Galli, D., Palla, F., Ferrini, F., & Penco, U. 1995, ApJ, 443, 536
Koen, C., & Laney, D. 2000, MNRAS, 311, 636
Metcalfe, T. S. 1999, AJ, 117, 2503
Metcalfe, T. S., Nather, R. E., & Winget, D. E. 2000, ApJ, 545, 974
Montgomery, M. H., & Winget, D. E. 1999, Baltic Astron., 9, 23
Prantzos, N. 1996, A&A, 310, 106
Winget, D. E. 1998, J. Phys.: Condensed Matter, 10, 11,247
Winget, D. E., et al. 1994, ApJ, 430, 839
———. 1991, ApJ, 378, 326