

# Cryogenic temperature-dependent refractive index measurements of N-BK7, BaLKN3, and SF15 for NOTES PDI

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## ABSTRACT

In order to enable high quality lens designs using N-BK7, BaLKN3, and SF15 at cryogenic temperatures, we have measured the absolute refractive index of prisms of these three materials using the Cryogenic, High-Accuracy Refraction Measuring System (CHARMS) at NASA's Goddard Space Flight Center, as a function of both wavelength and temperature. For N-BK7, we report absolute refractive index and thermo-optic coefficient ( $dn/dT$ ) at temperatures ranging from 50 to 300 K at wavelengths from 0.45 to 2.7  $\mu\text{m}$ ; for BaLKN3 we cover temperatures ranging from 40 to 300 K and wavelengths from 0.4 to 2.6  $\mu\text{m}$ ; for SF15 we cover temperatures ranging from 50 to 300 K and wavelengths from 0.45 to 2.6  $\mu\text{m}$ . We compare our measurements with others in the literature and provide temperature-dependent Sellmeier coefficients based on our data to allow accurate interpolation of index to other wavelengths and temperatures. While we generally find good agreement ( $\pm 2 \times 10^{-4}$  for N-BK7,  $< 1 \times 10^{-4}$  for the other materials) at room temperature between our measured values and those provided by the vendor, there is some variation between the datasheets provided with the prisms we measured and the catalog values published by the vendor. This underlines the importance of measuring the absolute refractive index of the material when precise knowledge of the refractive index is required.

**Keywords:** Refractive index, N-BK7, BaLKN3, SF15, cryogenic, infrared, refractometer, thermo-optic coefficient, CHARMS

## 1. INTRODUCTION

The Cryogenic High Accuracy Refraction Measuring System (CHARMS) has been recently developed at GSFC to make refractive index measurements in the absolute sense (in vacuum) down to temperatures as low as 15 K with unsurpassed accuracy using the method of minimum deviation refractometry.<sup>1,2,3</sup> For low index materials with only modest dispersion such as synthetic fused silica, CHARMS can measure absolute index with an uncertainty in the sixth decimal place of index. For other materials with higher indices and high dispersion, CHARMS can measure absolute index with an uncertainty of about one part in the fourth decimal place of index.

For this study one prismatic sample each of N-BK7, BaLKN3 and SF15 was purchased by NOTES, so no conclusions as to the interspecimen variability of refractive index for these materials can be drawn from the CHARMS measurements alone. The prisms were fabricated to fit in the custom CHARMS sample chamber - common prism dimensions are refracting face length and height of 38.1 mm and 28.6 mm, respectively. The apex angle of the prism for each material is designed so that the beam deviation angle for the highest index in the material's transparent range will equal the largest accessible deviation angle of the refractometer, 60°. Nominal apex angles for the prisms measured in this study are: N-BK7 - 59.0°, BaLKN3 - 59.0°, and SF15 - 56.0°.

## 2. PRESENTATION OF MEASURED INDEX DATA

Detailed descriptions of our data acquisition and reduction processes are documented elsewhere<sup>4</sup>, as are our calibration procedures<sup>5</sup>. In general we fit our raw measured data to a temperature dependent Sellmeier model of the form:

$$n^2(\lambda, T) - 1 = \sum_{i=1}^m \frac{S_i(T) \cdot \lambda^2}{\lambda^2 - \lambda_i^2(T)}$$

where  $S_i$  are the strengths of the resonance features in the material at wavelengths  $\lambda_i$ . When dealing with a wavelength interval between wavelengths of physical resonances in the material, the summation may be approximated by only a few terms,  $m$  - typically three<sup>6</sup>. In such an approximation, resonance strengths  $S_i$  and wavelengths  $\lambda_i$  no longer have direct

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physical significance but are rather parameters used to generate an adequately accurate fit to empirical data. If these parameters are assumed to be functions of T, one can generate a temperature-dependent Sellmeier model for  $n(\lambda, T)$ .

Historically, this modeling approach has been employed with significant success for a variety of materials despite a rather serious sparsity of available index measurements – to cover a wide range of temperatures and wavelengths – upon which to base a model. One solution to the shortcoming of lack of measured index data has been to appeal to room temperature refractive index data at several wavelengths to anchor the model and then to extrapolate index values for other temperatures using accurate measurements of the thermo-optic coefficient  $dn/dT$  at those temperatures, which are much easier to make than accurate measurements of the index itself at exotic temperatures<sup>6</sup>. This is of course a potentially dangerous assumption, depending on the sample material in question and the required accuracy of refractive index knowledge.

Meanwhile, with CHARMS, we have made direct measurements of index densely sampled over a wide range of wavelengths and temperatures to produce a model with residuals on the order of the uncertainties in our raw index measurements. For our models, we have found that 4th order temperature dependences in all three terms in each of  $S_i$  and  $\lambda_i$  work adequately well, as also found previously in the literature<sup>6</sup>. The Sellmeier equation consequently becomes:

$$n^2(\lambda, T) - 1 = \sum_{i=1}^3 \frac{S_i(T) \cdot \lambda^2}{\lambda^2 - \lambda_i^2(T)}$$

where,

$$S_i(T) = \sum_{j=0}^4 S_{ij} \cdot T^j$$

$$\lambda_i(T) = \sum_{j=0}^4 \lambda_{ij} \cdot T^j$$

These Sellmeier models are our best statistical representation of the measured data over the complete measured ranges of wavelength and temperature. All of the following tabulated values for the refractive index have been calculated using this Sellmeier model based on our measured data using the appropriate coefficients in Table 5 for N-BK7, Table 10 for BaLKN3, and Table 15 for SF15. Typically the residuals of the fits from measured values is less than the uncertainty respective measurements (see Table 1 for N-BK7, Table 6 for BaLKN3, and Table 11 for SF15 for measurement uncertainties).

## 2.1 N-BK7

Absolute refractive indices of N-BK7 were measured over the 0.45 to 2.7 microns wavelength range and over the temperature range from 50 to 300 K for a single test specimen. Our typical measurement uncertainties for this material are listed in Table 1. Indices are plotted in Figure 1, tabulated in Table 2 for selected temperatures and wavelengths. Spectral dispersion is plotted in Figure 2, tabulated in Table 3. Thermo-optic coefficient is plotted in Figure 3, tabulated in Table 4. Coefficients for the three term Sellmeier model with 4<sup>th</sup> order temperature dependence are given in Table 5.

Table 1: Uncertainty in absolute refractive index measurements of N-BK7 for selected wavelengths and temperatures

| wavelength   | 55 K    | 75 K    | 100 K   | 200 K   | 295 K   |
|--------------|---------|---------|---------|---------|---------|
| 0.50 microns | 5.5E-05 | 5.5E-05 | 5.5E-05 | 5.5E-05 | 5.5E-05 |
| 0.75 microns | 2.1E-05 | 2.0E-05 | 2.0E-05 | 2.0E-05 | 2.0E-05 |
| 1.00 microns | 1.6E-05 | 1.4E-05 | 1.4E-05 | 1.4E-05 | 1.4E-05 |
| 1.50 microns | 1.5E-05 | 1.3E-05 | 1.3E-05 | 1.3E-05 | 1.3E-05 |
| 2.50 microns | 1.9E-05 | 1.8E-05 | 1.8E-05 | 1.7E-05 | 1.7E-05 |

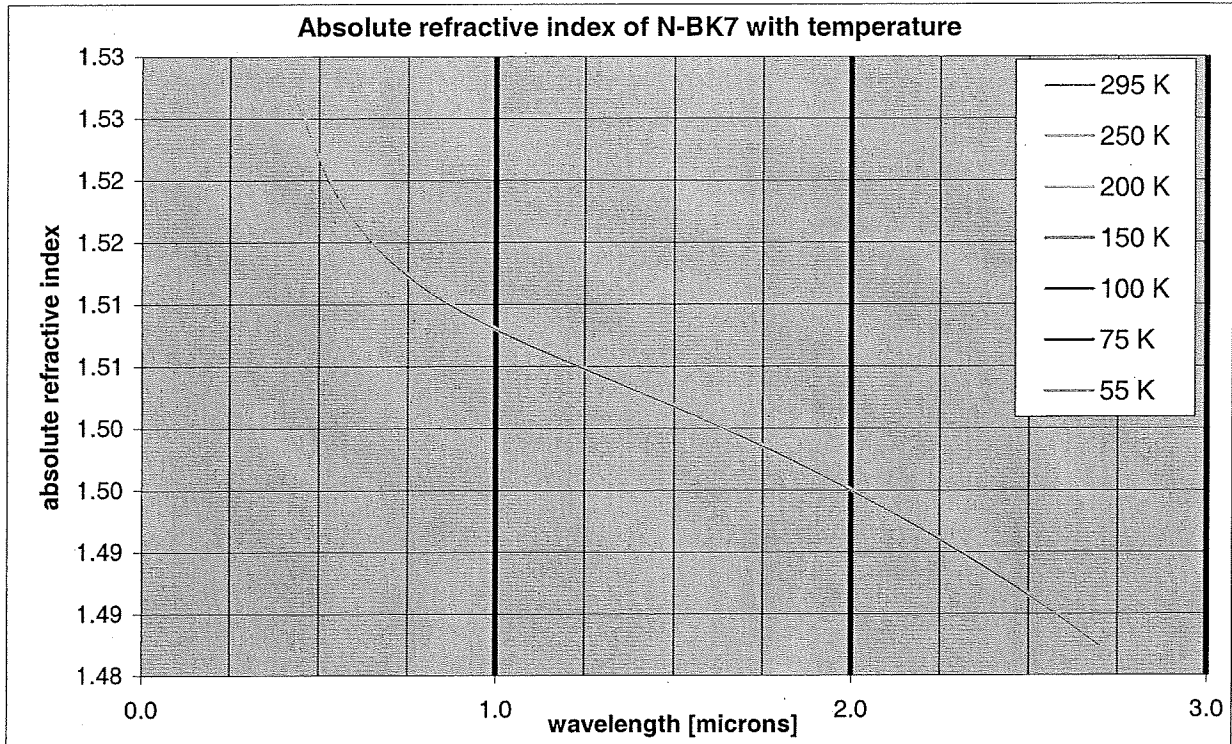


Figure 1: Measured absolute refractive index of N-BK7 as a function of wavelength for selected temperatures

Table 2: Measured absolute refractive index of N-BK7 for selected wavelengths and temperatures

| wavelength  | 60 K    | 70 K    | 80 K    | 90 K    | 100 K   | 150 K   | 200 K   | 250 K   | 295 K   |
|-------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.5 microns | 1.52196 | 1.52195 | 1.52194 | 1.52192 | 1.52191 | 1.52185 | 1.52184 | 1.52186 | 1.52195 |
| 0.6 microns | 1.51688 | 1.51686 | 1.51685 | 1.51684 | 1.51683 | 1.51676 | 1.51674 | 1.51675 | 1.51684 |
| 0.7 microns | 1.51366 | 1.51365 | 1.51364 | 1.51362 | 1.51361 | 1.51354 | 1.51351 | 1.51352 | 1.51359 |
| 0.8 microns | 1.51139 | 1.51137 | 1.51136 | 1.51135 | 1.51133 | 1.51126 | 1.51123 | 1.51123 | 1.51129 |
| 0.9 microns | 1.50961 | 1.50960 | 1.50959 | 1.50957 | 1.50956 | 1.50949 | 1.50945 | 1.50945 | 1.50950 |
| 1.0 microns | 1.50812 | 1.50811 | 1.50810 | 1.50808 | 1.50807 | 1.50799 | 1.50796 | 1.50796 | 1.50799 |
| 1.5 microns | 1.50191 | 1.50190 | 1.50188 | 1.50187 | 1.50185 | 1.50178 | 1.50174 | 1.50174 | 1.50176 |
| 2.0 microns | 1.49514 | 1.49512 | 1.49511 | 1.49509 | 1.49508 | 1.49500 | 1.49496 | 1.49497 | 1.49500 |
| 2.5 microns | 1.48661 | 1.48660 | 1.48658 | 1.48657 | 1.48655 | 1.48648 | 1.48645 | 1.48645 | 1.48649 |

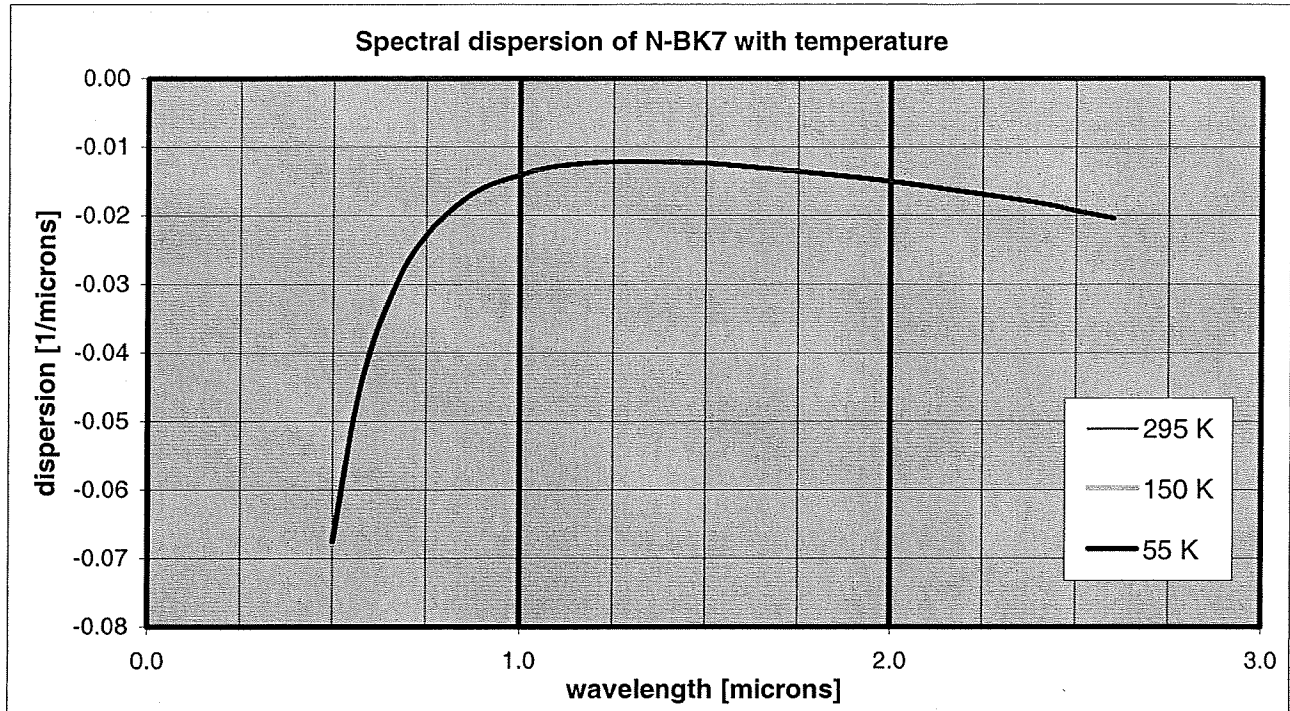


Figure 2: Measured spectral dispersion ( $dn/d\lambda$ ) of N-BK7 as a function of wavelength for selected temperatures

Table 3: Measured spectral dispersion ( $dn/d\lambda$ ) of N-BK7 for selected wavelengths and temperatures

| wavelength   | 60 K     | 70 K     | 80 K     | 90 K     | 100 K    | 150 K    | 200 K    | 250 K    | 295 K    |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.50 microns | -0.06755 | -0.06756 | -0.06757 | -0.06758 | -0.06760 | -0.06769 | -0.06783 | -0.06800 | -0.06821 |
| 0.60 microns | -0.03963 | -0.03963 | -0.03964 | -0.03964 | -0.03965 | -0.03970 | -0.03978 | -0.03987 | -0.03996 |
| 0.70 microns | -0.02664 | -0.02664 | -0.02664 | -0.02664 | -0.02665 | -0.02668 | -0.02672 | -0.02677 | -0.02684 |
| 0.80 microns | -0.01998 | -0.01998 | -0.01998 | -0.01999 | -0.01999 | -0.02000 | -0.02004 | -0.02007 | -0.02010 |
| 0.90 microns | -0.01602 | -0.01602 | -0.01602 | -0.01602 | -0.01602 | -0.01604 | -0.01605 | -0.01608 | -0.01611 |
| 1.00 microns | -0.01409 | -0.01409 | -0.01409 | -0.01410 | -0.01410 | -0.01411 | -0.01412 | -0.01414 | -0.01415 |
| 1.50 microns | -0.01233 | -0.01233 | -0.01233 | -0.01233 | -0.01233 | -0.01232 | -0.01232 | -0.01232 | -0.01232 |
| 2.00 microns | -0.01504 | -0.01504 | -0.01504 | -0.01503 | -0.01503 | -0.01504 | -0.01503 | -0.01502 | -0.01501 |
| 2.50 microns | -0.01930 | -0.01930 | -0.01929 | -0.01929 | -0.01929 | -0.01928 | -0.01927 | -0.01925 | -0.01923 |

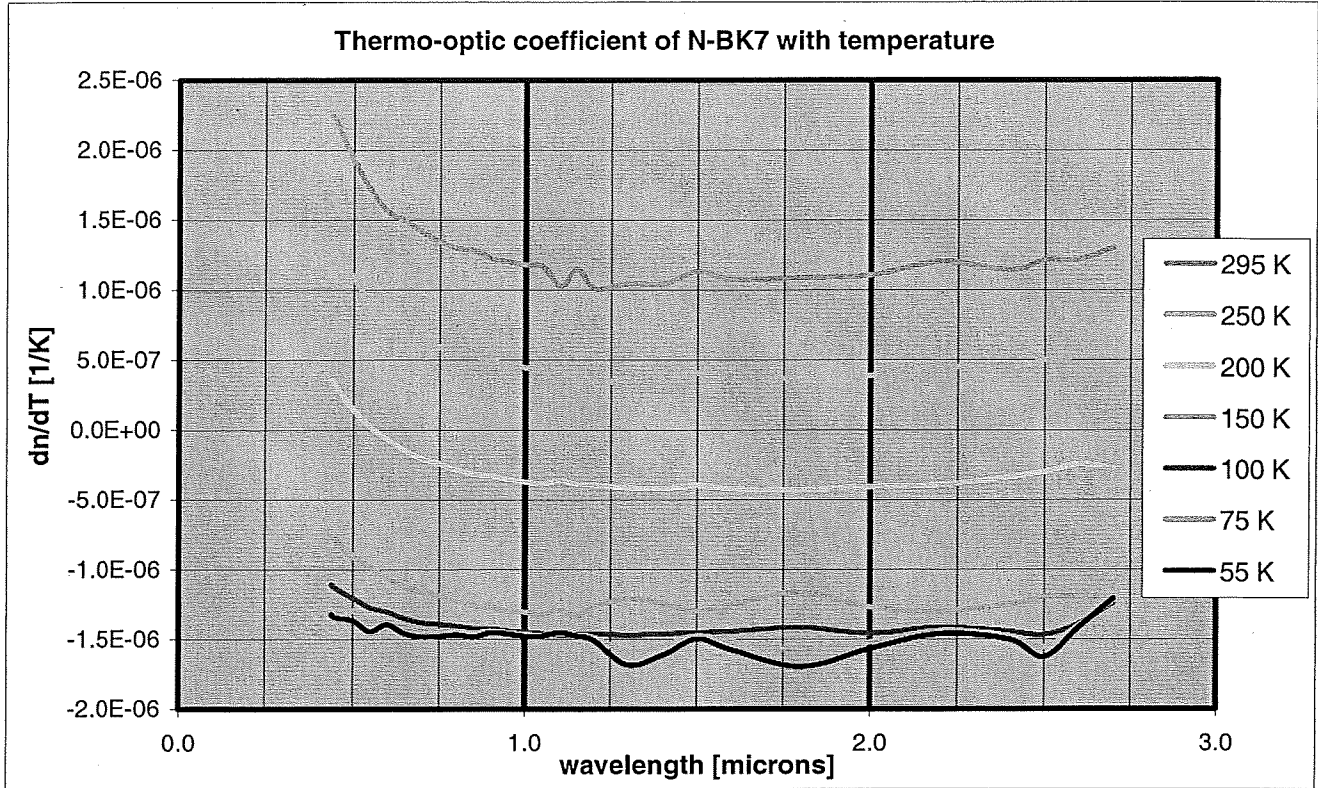


Figure 3: Measured thermo-optic coefficient (dn/dT) of N-BK7 as a function of wavelength for selected temperatures

Table 4: Measured thermo-optic coefficient (dn/dT) of N-BK7 for selected wavelengths and temperatures

| wavelength   | 60 K      | 70 K      | 80 K      | 90 K      | 100 K     | 150 K     | 200 K     | 250 K    | 295 K    |
|--------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|----------|
| 0.50 microns | -1.35E-06 | -1.31E-06 | -1.27E-06 | -1.24E-06 | -1.20E-06 | -9.04E-07 | 1.48E-07  | 1.08E-06 | 1.93E-06 |
| 0.60 microns | -1.38E-06 | -1.36E-06 | -1.34E-06 | -1.32E-06 | -1.30E-06 | -1.06E-06 | -6.48E-08 | 7.91E-07 | 1.56E-06 |
| 0.70 microns | -1.47E-06 | -1.45E-06 | -1.42E-06 | -1.40E-06 | -1.38E-06 | -1.16E-06 | -2.04E-07 | 6.48E-07 | 1.42E-06 |
| 0.80 microns | -1.46E-06 | -1.45E-06 | -1.43E-06 | -1.42E-06 | -1.40E-06 | -1.23E-06 | -2.85E-07 | 5.50E-07 | 1.30E-06 |
| 0.90 microns | -1.45E-06 | -1.44E-06 | -1.44E-06 | -1.43E-06 | -1.42E-06 | -1.27E-06 | -3.27E-07 | 4.92E-07 | 1.23E-06 |
| 1.00 microns | -1.47E-06 | -1.47E-06 | -1.46E-06 | -1.46E-06 | -1.45E-06 | -1.30E-06 | -3.71E-07 | 4.46E-07 | 1.18E-06 |
| 1.50 microns | -1.49E-06 | -1.48E-06 | -1.47E-06 | -1.46E-06 | -1.45E-06 | -1.30E-06 | -3.93E-07 | 4.09E-07 | 1.13E-06 |
| 2.00 microns | -1.56E-06 | -1.53E-06 | -1.51E-06 | -1.48E-06 | -1.46E-06 | -1.27E-06 | -4.05E-07 | 3.90E-07 | 1.11E-06 |
| 2.50 microns | -1.61E-06 | -1.57E-06 | -1.54E-06 | -1.50E-06 | -1.47E-06 | -1.19E-06 | -2.97E-07 | 5.00E-07 | 1.22E-06 |

Table 5: Coefficients for the temperature-dependent Sellmeier fit of the refractive index of N-BK7 measured by CHARMS. Average absolute residual of the fit from the measured data is  $1 \times 10^{-5}$ ; the measurement uncertainty for N-BK7 is listed in Table 1.

| Coefficients for the temperature dependent Sellmeier equation for N-BK7 |                |                |                |                |                |                |
|---|----------------|----------------|----------------|----------------|----------------|----------------|
| 50 K ≤ T ≤ 300 K; 0.45 μm ≤ λ ≤ 2.7 μm                                  |                |                |                |                |                |                |
|   | S <sub>1</sub> | S <sub>2</sub> | S <sub>3</sub> | λ <sub>1</sub> | λ <sub>2</sub> | λ <sub>3</sub> |
| Constant term   | 1.07036        | 0.202896       | 0.850659       | 8.20884E-02    | 0.141222       | 9.38638        |
| T term  | 1.75658E-03    | -1.75289E-03   | 1.03233E-03    | 4.35600E-05    | -5.41379E-05   | 5.24515E-03    |
| T <sup>2</sup> term   | -4.28401E-06   | 4.18934E-06    | -6.88935E-06   | 5.85421E-08    | 4.72890E-06    | -3.54202E-05   |
| T <sup>3</sup> term   | -1.65687E-11   | 4.52824E-10    | 2.56510E-08    | -4.13612E-10   | -2.73546E-08   | 1.32249E-07    |
| T <sup>4</sup> term   | 7.55467E-12    | -8.18792E-12   | -4.71564E-11   | 5.610590E-13   | 8.01380E-11    | -2.38407E-10   |

## 2.2 BaLKN3

Absolute refractive indices of BaLKN3 were measured over the 0.4 to 2.6 microns wavelength range and over the temperature range from 40 to 300 K for a single test specimen. Our typical measurement uncertainties for this material are listed in Table 6. Indices are plotted in Figure 4, tabulated in Table 7 for selected temperatures and wavelengths. Spectral dispersion is plotted in Figure 5, tabulated in Table 8. Thermo-optic coefficient is plotted in Figure 6, tabulated in Table 9. Coefficients for the three term Sellmeier model with 4<sup>th</sup> order temperature dependence are given in Table 10.

Table 6: Uncertainty in absolute refractive index measurements of BaLKN3 for selected wavelengths and temperatures

| wavelength   | 40 K    | 75 K    | 100 K   | 200 K   | 295 K   |
|--------------|---------|---------|---------|---------|---------|
| 0.50 microns | 5.9E-05 | 5.9E-05 | 5.9E-05 | 5.9E-05 | 5.9E-05 |
| 0.75 microns | 2.2E-05 | 2.1E-05 | 2.0E-05 | 2.0E-05 | 2.0E-05 |
| 1.00 microns | 1.5E-05 | 1.4E-05 | 1.4E-05 | 1.3E-05 | 1.3E-05 |
| 1.50 microns | 1.4E-05 | 1.3E-05 | 1.2E-05 | 1.1E-05 | 1.1E-05 |
| 2.50 microns | 1.7E-05 | 1.6E-05 | 1.6E-05 | 1.5E-05 | 1.5E-05 |

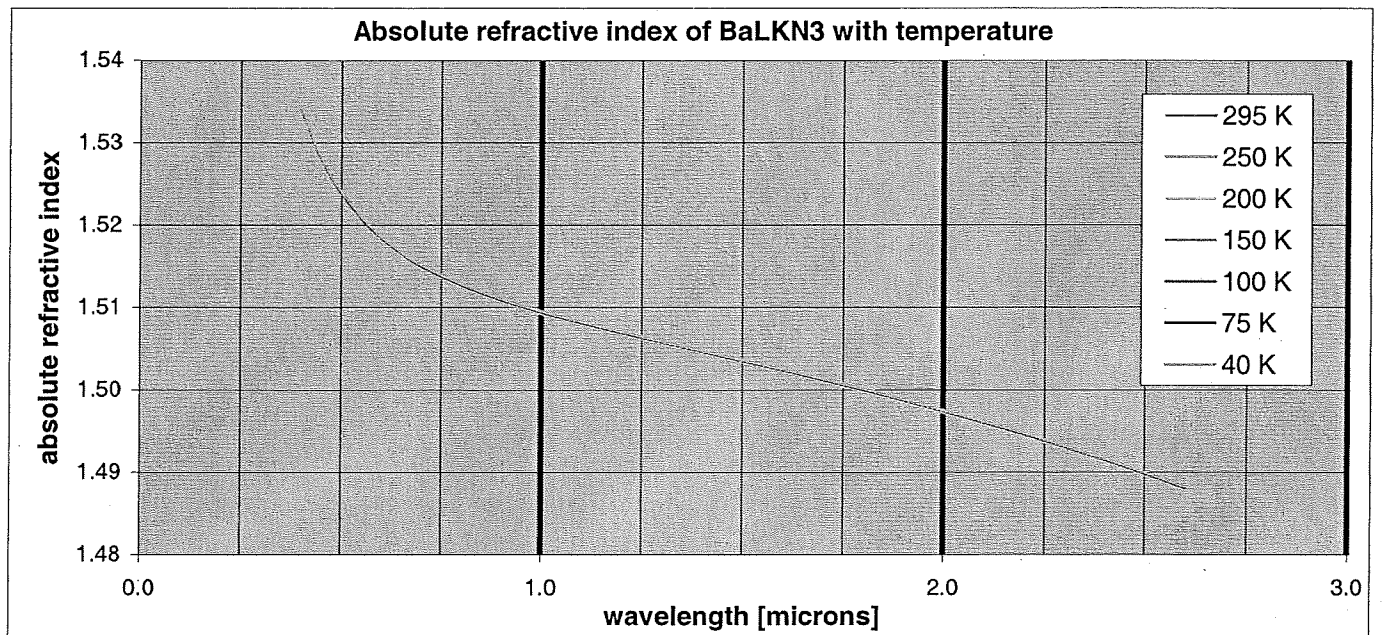


Figure 4: Measured absolute refractive index of BaLKN3 as a function of wavelength for selected temperatures

Table 7: Measured absolute refractive index of BaLKN3 for selected wavelengths and temperatures

| wavelength   | 40 K    | 50 K    | 60 K    | 70 K    | 80 K    | 90 K    | 100 K   | 150 K   | 200 K   | 250 K   | 295 K   |
|--------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.40 microns | 1.53422 | 1.53421 | 1.53420 | 1.53419 | 1.53417 | 1.53416 | 1.53414 | 1.53408 | 1.53407 | 1.53412 | 1.53419 |
| 0.50 microns | 1.52405 | 1.52404 | 1.52403 | 1.52401 | 1.52399 | 1.52398 | 1.52396 | 1.52388 | 1.52385 | 1.52385 | 1.52390 |
| 0.60 microns | 1.51860 | 1.51859 | 1.51858 | 1.51856 | 1.51854 | 1.51852 | 1.51851 | 1.51842 | 1.51837 | 1.51837 | 1.51840 |
| 0.70 microns | 1.51521 | 1.51520 | 1.51519 | 1.51517 | 1.51515 | 1.51513 | 1.51511 | 1.51502 | 1.51497 | 1.51495 | 1.51498 |
| 0.80 microns | 1.51285 | 1.51284 | 1.51282 | 1.51281 | 1.51279 | 1.51277 | 1.51275 | 1.51266 | 1.51260 | 1.51258 | 1.51260 |
| 0.90 microns | 1.51105 | 1.51104 | 1.51102 | 1.51100 | 1.51099 | 1.51097 | 1.51095 | 1.51085 | 1.51079 | 1.51077 | 1.51078 |
| 1.00 microns | 1.50957 | 1.50955 | 1.50954 | 1.50952 | 1.50950 | 1.50948 | 1.50946 | 1.50937 | 1.50930 | 1.50928 | 1.50929 |
| 1.50 microns | 1.50368 | 1.50367 | 1.50365 | 1.50363 | 1.50361 | 1.50359 | 1.50357 | 1.50348 | 1.50341 | 1.50338 | 1.50339 |
| 2.00 microns | 1.49756 | 1.49755 | 1.49753 | 1.49752 | 1.49750 | 1.49748 | 1.49746 | 1.49736 | 1.49729 | 1.49727 | 1.49728 |
| 2.50 microns | 1.49002 | 1.49000 | 1.48999 | 1.48997 | 1.48995 | 1.48993 | 1.48991 | 1.48982 | 1.48976 | 1.48974 | 1.48975 |

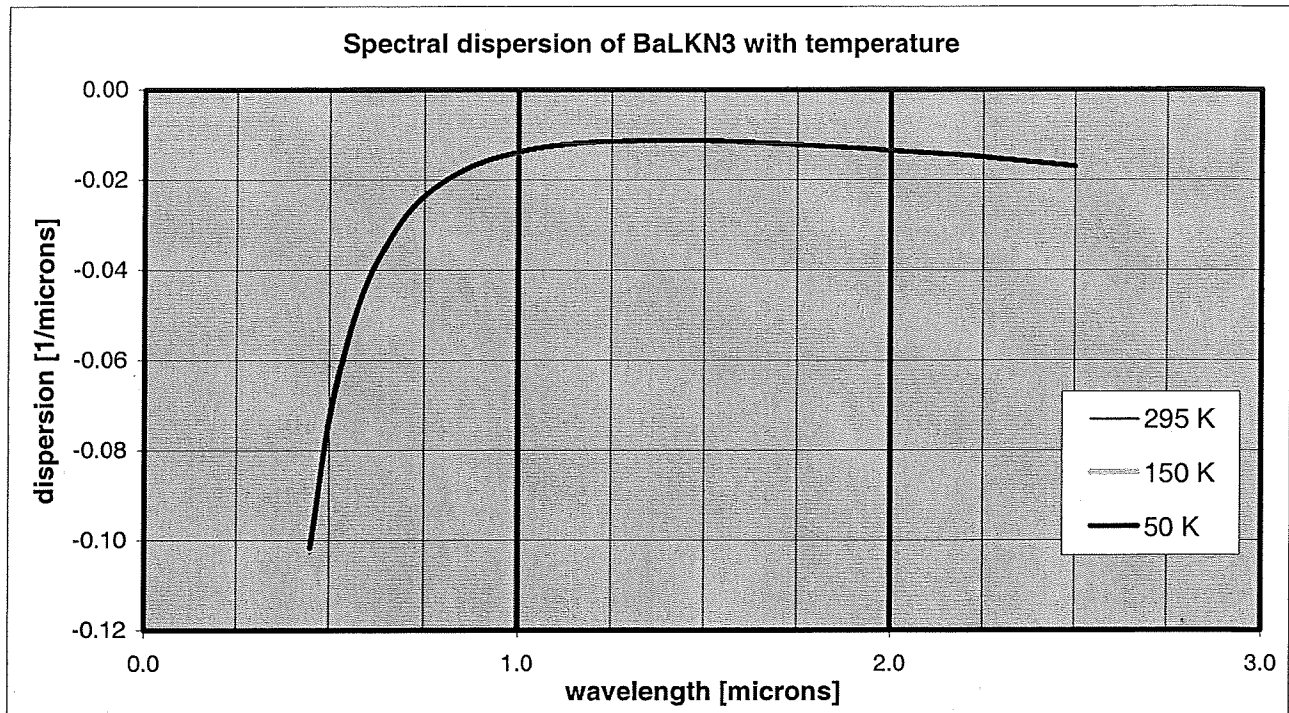


Figure 5: Measured spectral dispersion ( $dn/d\lambda$ ) of BaLKN3 as a function of wavelength for selected temperatures

Table 8: Measured spectral dispersion ( $dn/d\lambda$ ) of BaLKN3 for selected wavelengths and temperatures

| wavelength   | 40 K     | 50 K     | 60 K     | 70 K     | 80 K     | 90 K     | 100 K    | 150 K    | 200 K    | 250 K    | 295 K    |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.50 microns | -0.07292 | -0.07292 | -0.07293 | -0.07293 | -0.07294 | -0.07296 | -0.07297 | -0.07309 | -0.07327 | -0.07348 | -0.07369 |
| 0.60 microns | -0.04216 | -0.04217 | -0.04218 | -0.04219 | -0.04220 | -0.04220 | -0.04221 | -0.04226 | -0.04236 | -0.04247 | -0.04259 |
| 0.70 microns | -0.02790 | -0.02789 | -0.02789 | -0.02790 | -0.02790 | -0.02790 | -0.02791 | -0.02794 | -0.02800 | -0.02807 | -0.02813 |
| 0.80 microns | -0.02053 | -0.02052 | -0.02052 | -0.02052 | -0.02052 | -0.02052 | -0.02052 | -0.02055 | -0.02058 | -0.02062 | -0.02067 |
| 0.90 microns | -0.01612 | -0.01612 | -0.01612 | -0.01613 | -0.01613 | -0.01613 | -0.01613 | -0.01615 | -0.01617 | -0.01620 | -0.01623 |
| 1.00 microns | -0.01388 | -0.01388 | -0.01388 | -0.01388 | -0.01389 | -0.01389 | -0.01389 | -0.01390 | -0.01391 | -0.01393 | -0.01395 |
| 1.50 microns | -0.01129 | -0.01129 | -0.01130 | -0.01130 | -0.01130 | -0.01130 | -0.01130 | -0.01129 | -0.01133 | -0.01133 | -0.01128 |
| 2.00 microns | -0.01350 | -0.01350 | -0.01350 | -0.01350 | -0.01350 | -0.01350 | -0.01350 | -0.01349 | -0.01349 | -0.01348 | -0.01349 |
| 2.50 microns | -0.01700 | -0.01699 | -0.01698 | -0.01698 | -0.01698 | -0.01698 | -0.01698 | -0.01700 | -0.01697 | -0.01697 | -0.01697 |

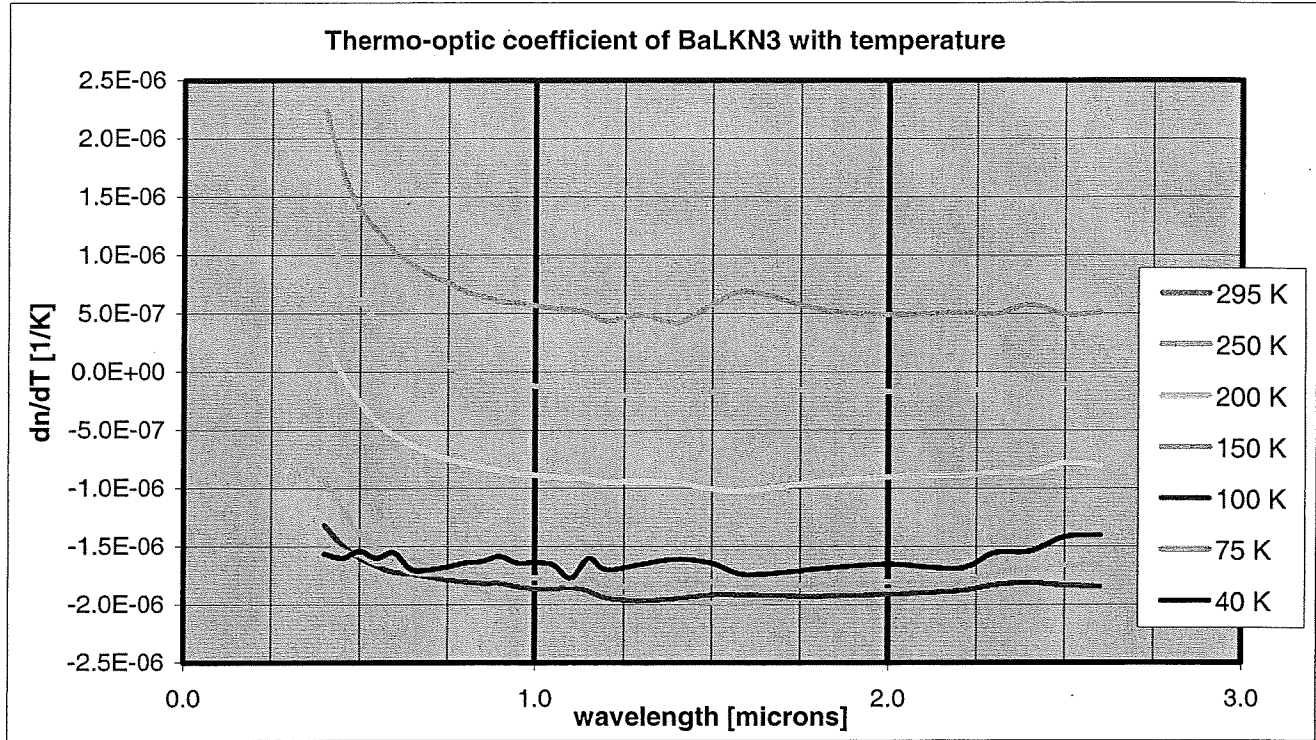


Figure 6: Measured thermo-optic coefficient (dn/dT) of BaLKN3 as a function of wavelength for selected temperatures

Table 9: Measured thermo-optic coefficient (dn/dT) of BaLKN3 for selected wavelengths and temperatures

| wavelength   | 40 K      | 50 K      | 60 K      | 70 K      | 80 K      | 90 K      | 100 K     | 150 K     | 200 K     | 250 K     | 295 K    |
|--------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|
| 0.40 microns | -1.56E-06 | -1.52E-06 | -1.48E-06 | -1.44E-06 | -1.40E-06 | -1.36E-06 | -1.31E-06 | -9.27E-07 | 2.83E-07  | 1.31E-06  | 2.24E-06 |
| 0.50 microns | -1.54E-06 | -1.55E-06 | -1.56E-06 | -1.57E-06 | -1.58E-06 | -1.59E-06 | -1.60E-06 | -1.38E-06 | -2.60E-07 | 6.06E-07  | 1.38E-06 |
| 0.60 microns | -1.55E-06 | -1.58E-06 | -1.61E-06 | -1.64E-06 | -1.66E-06 | -1.69E-06 | -1.72E-06 | -1.62E-06 | -5.44E-07 | 2.83E-07  | 1.03E-06 |
| 0.70 microns | -1.70E-06 | -1.71E-06 | -1.72E-06 | -1.73E-06 | -1.74E-06 | -1.75E-06 | -1.76E-06 | -1.65E-06 | -6.93E-07 | 1.08E-07  | 8.30E-07 |
| 0.80 microns | -1.64E-06 | -1.67E-06 | -1.70E-06 | -1.72E-06 | -1.75E-06 | -1.78E-06 | -1.80E-06 | -1.75E-06 | -7.84E-07 | -8.25E-09 | 6.90E-07 |
| 0.90 microns | -1.59E-06 | -1.63E-06 | -1.66E-06 | -1.70E-06 | -1.74E-06 | -1.78E-06 | -1.82E-06 | -1.81E-06 | -8.48E-07 | -8.33E-08 | 6.05E-07 |
| 1.00 microns | -1.64E-06 | -1.68E-06 | -1.71E-06 | -1.75E-06 | -1.79E-06 | -1.83E-06 | -1.86E-06 | -1.85E-06 | -8.87E-07 | -1.21E-07 | 5.69E-07 |
| 1.50 microns | -1.64E-06 | -1.69E-06 | -1.73E-06 | -1.78E-06 | -1.83E-06 | -1.87E-06 | -1.92E-06 | -1.99E-06 | -1.01E-06 | -1.71E-07 | 5.81E-07 |
| 2.00 microns | -1.65E-06 | -1.70E-06 | -1.74E-06 | -1.78E-06 | -1.83E-06 | -1.87E-06 | -1.91E-06 | -1.88E-06 | -9.02E-07 | -1.69E-07 | 4.91E-07 |
| 2.50 microns | -1.42E-06 | -1.49E-06 | -1.56E-06 | -1.63E-06 | -1.70E-06 | -1.77E-06 | -1.84E-06 | -1.82E-06 | -7.79E-07 | -1.08E-07 | 4.96E-07 |

Table 10: Coefficients for the temperature-dependent Sellmeier fit of the refractive index of BaLKN3 measured by CHARMS. Average absolute residual of the fit from the measured data is  $1 \times 10^{-5}$  and the measurement uncertainty for BaLKN3 is listed in Table 6.

| Coefficients for the temperature dependent Sellmeier equation for BaLKN3 |                |                |                |                |                |                |
|--|----------------|----------------|----------------|----------------|----------------|----------------|
| 40 K ≤ T ≤ 300 K; 0.4 μm ≤ λ ≤ 2.6 μm                                    |                |                |                |                |                |                |
|  | S <sub>1</sub> | S <sub>2</sub> | S <sub>3</sub> | λ <sub>1</sub> | λ <sub>2</sub> | λ <sub>3</sub> |
| Constant term  | 1.02149        | 0.254360       | 0.953229       | 7.75429E-02    | 0.146401       | 10.4822        |
| T term   | 1.09614E-03    | -1.09610E-03   | -1.15882E-03   | 4.95158E-05    | 1.35564E-04    | -5.99745E-03   |
| T <sup>2</sup> term  | -4.59805E-06   | 4.53092E-06    | 1.39465E-05    | -1.40035E-07   | -4.89553E-07   | 7.24685E-05    |
| T <sup>3</sup> term  | 6.64131E-09    | -6.35629E-09   | -5.48539E-08   | -1.08903E-10   | 6.14665E-10    | -2.84140E-07   |
| T <sup>4</sup> term  | 1.87385E-12    | -2.20840E-12   | 7.03221E-11    | 8.22667E-13    | 4.69066E-13    | 3.63353E-10    |



### 2.3 SF15

Absolute refractive indices of SF15 were measured over the 0.45 to 2.6 microns wavelength range and over the temperature range from 50 to 300 K for a single test specimen. Our typical measurement uncertainties for this material are listed in Table 11. Indices are plotted in Figure 7, tabulated in Table 12 for selected temperatures and wavelengths. Spectral dispersion is plotted in Figure 8, tabulated in Table 13. Thermo-optic coefficient is plotted in Figure 9, tabulated in Table 14. Coefficients for the three term Sellmeier model with 4<sup>th</sup> order temperature dependence are given in Table 15.

Table 11: Uncertainty in absolute refractive index measurements of SF15 for selected wavelengths and temperatures

| wavelength   | 50 K    | 75 K    | 100 K   | 200 K   | 295 K   |
|--------------|---------|---------|---------|---------|---------|
| 0.50 microns | 8.3E-05 | 8.3E-05 | 8.3E-05 | 8.4E-05 | 8.5E-05 |
| 0.75 microns | 3.4E-05 | 3.4E-05 | 3.4E-05 | 3.4E-05 | 3.4E-05 |
| 1.00 microns | 2.3E-05 | 2.2E-05 | 2.2E-05 | 2.2E-05 | 2.2E-05 |
| 1.50 microns | 1.5E-05 | 1.4E-05 | 1.4E-05 | 1.4E-05 | 1.4E-05 |
| 2.50 microns | 1.6E-05 | 1.5E-05 | 1.5E-05 | 1.5E-05 | 1.5E-05 |

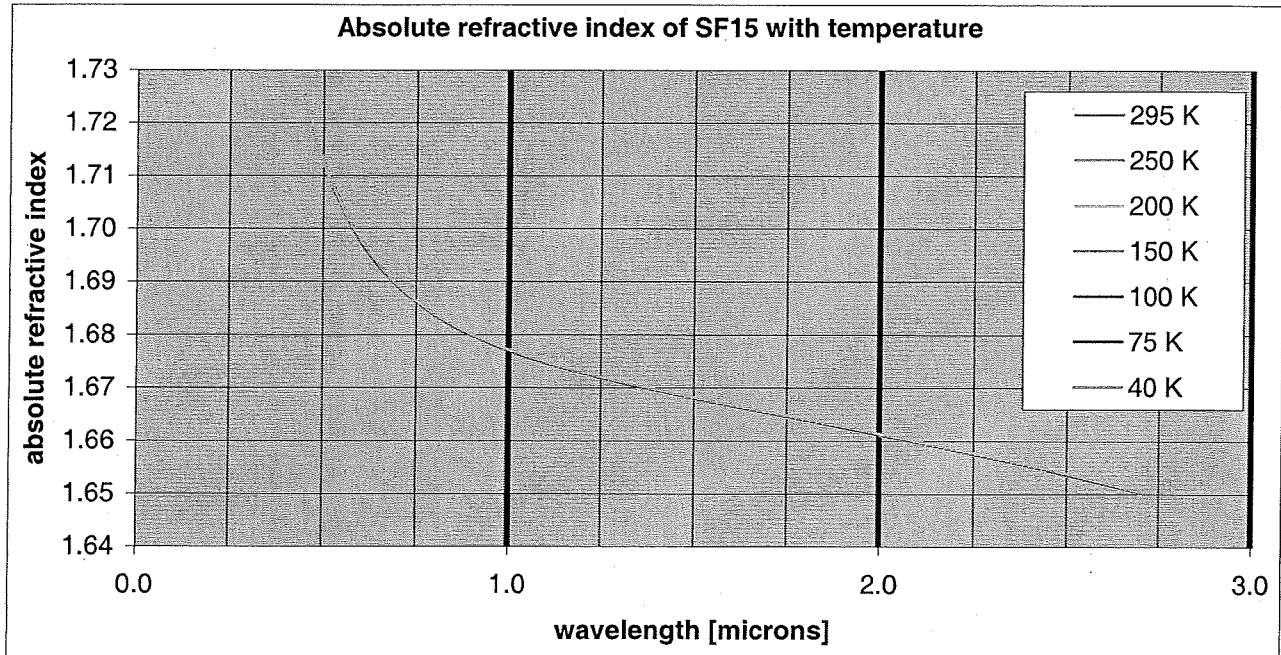


Figure 7: Measured absolute refractive index of SF15 as a function of wavelength for selected temperatures

Table 12: Measured absolute refractive index of SF15 for selected wavelengths and temperatures

| wavelength  | 40 K    | 50 K    | 60 K    | 70 K    | 80 K    | 90 K    | 100 K   | 150 K   | 200 K   | 250 K   | 295 K   |
|-------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.5 microns | 1.71251 | 1.71249 | 1.71248 | 1.71246 | 1.71245 | 1.71244 | 1.71242 | 1.71242 | 1.71250 | 1.71267 | 1.71289 |
| 0.6 microns | 1.69782 | 1.69780 | 1.69778 | 1.69776 | 1.69774 | 1.69772 | 1.69771 | 1.69766 | 1.69769 | 1.69779 | 1.69795 |
| 0.7 microns | 1.68937 | 1.68935 | 1.68933 | 1.68931 | 1.68929 | 1.68927 | 1.68925 | 1.68919 | 1.68918 | 1.68925 | 1.68937 |
| 0.8 microns | 1.68391 | 1.68389 | 1.68387 | 1.68385 | 1.68382 | 1.68380 | 1.68378 | 1.68371 | 1.68369 | 1.68374 | 1.68384 |
| 0.9 microns | 1.68007 | 1.68005 | 1.68003 | 1.68000 | 1.67998 | 1.67996 | 1.67994 | 1.67986 | 1.67984 | 1.67987 | 1.67996 |
| 1.0 microns | 1.67718 | 1.67715 | 1.67713 | 1.67711 | 1.67709 | 1.67706 | 1.67704 | 1.67696 | 1.67693 | 1.67696 | 1.67704 |
| 1.5 microns | 1.66823 | 1.66821 | 1.66818 | 1.66816 | 1.66813 | 1.66811 | 1.66809 | 1.66799 | 1.66794 | 1.66796 | 1.66802 |
| 2.0 microns | 1.66137 | 1.66134 | 1.66132 | 1.66130 | 1.66127 | 1.66125 | 1.66122 | 1.66112 | 1.66107 | 1.66108 | 1.66115 |
| 2.5 microns | 1.65383 | 1.65381 | 1.65378 | 1.65376 | 1.65373 | 1.65371 | 1.65368 | 1.65359 | 1.65354 | 1.65356 | 1.65363 |

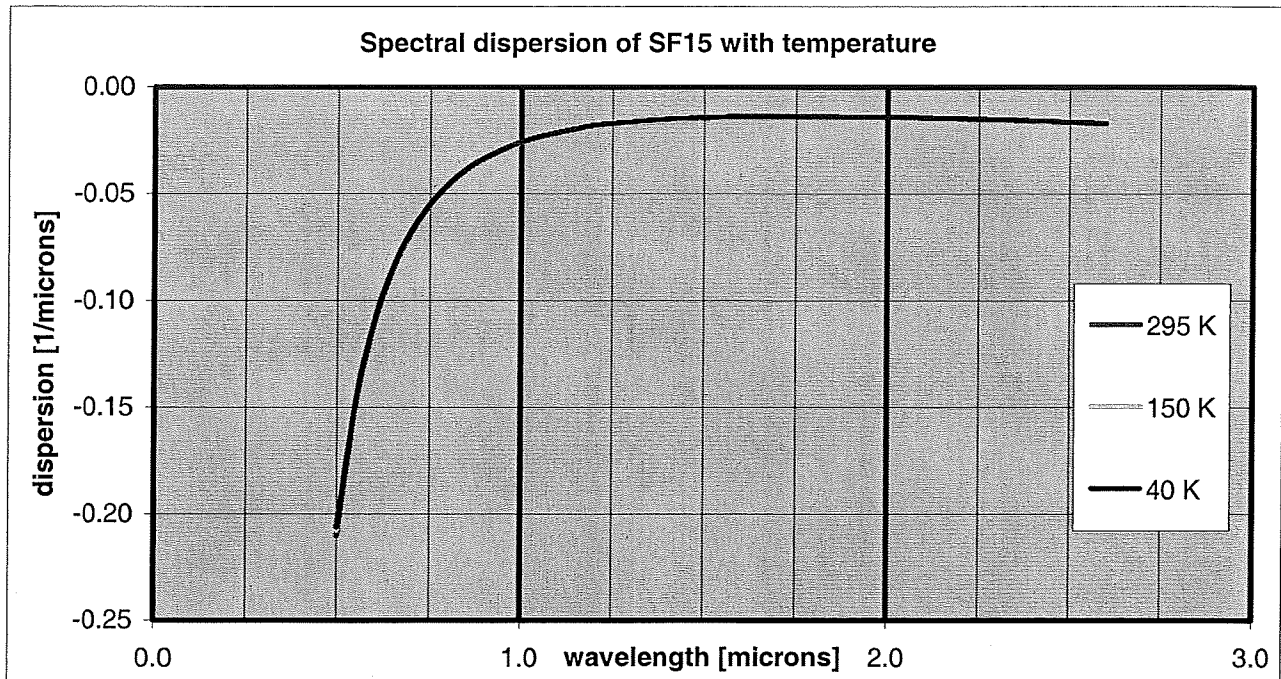


Figure 8: Measured spectral dispersion ( $dn/d\lambda$ ) of SF15 as a function of wavelength for selected temperatures

Table 13: Measured spectral dispersion ( $dn/d\lambda$ ) of SF15 for selected wavelengths and temperatures

| wavelength   | 40 K     | 50 K     | 60 K     | 70 K     | 80 K     | 90 K     | 100 K    | 150 K    | 200 K    | 250 K    | 295 K    |
|--------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.50 microns | -0.20621 | -0.20630 | -0.20638 | -0.20646 | -0.20653 | -0.20661 | -0.20669 | -0.20730 | -0.20819 | -0.20919 | -0.21012 |
| 0.60 microns | -0.10948 | -0.10951 | -0.10954 | -0.10956 | -0.10958 | -0.10961 | -0.10964 | -0.10988 | -0.11028 | -0.11075 | -0.11122 |
| 0.70 microns | -0.06710 | -0.06711 | -0.06713 | -0.06714 | -0.06715 | -0.06716 | -0.06718 | -0.06730 | -0.06750 | -0.06776 | -0.06802 |
| 0.80 microns | -0.04535 | -0.04536 | -0.04537 | -0.04537 | -0.04538 | -0.04539 | -0.04540 | -0.04547 | -0.04559 | -0.04574 | -0.04590 |
| 0.90 microns | -0.03307 | -0.03308 | -0.03308 | -0.03308 | -0.03309 | -0.03310 | -0.03310 | -0.03315 | -0.03323 | -0.03333 | -0.03343 |
| 1.00 microns | -0.02568 | -0.02569 | -0.02569 | -0.02569 | -0.02570 | -0.02570 | -0.02571 | -0.02574 | -0.02580 | -0.02586 | -0.02593 |
| 1.50 microns | -0.01421 | -0.01421 | -0.01421 | -0.01421 | -0.01421 | -0.01422 | -0.01422 | -0.01424 | -0.01426 | -0.01427 | -0.01428 |
| 2.00 microns | -0.01405 | -0.01405 | -0.01405 | -0.01405 | -0.01405 | -0.01406 | -0.01406 | -0.01406 | -0.01406 | -0.01405 | -0.01405 |
| 2.50 microns | -0.01641 | -0.01641 | -0.01642 | -0.01642 | -0.01641 | -0.01641 | -0.01641 | -0.01639 | -0.01636 | -0.01633 | -0.01633 |

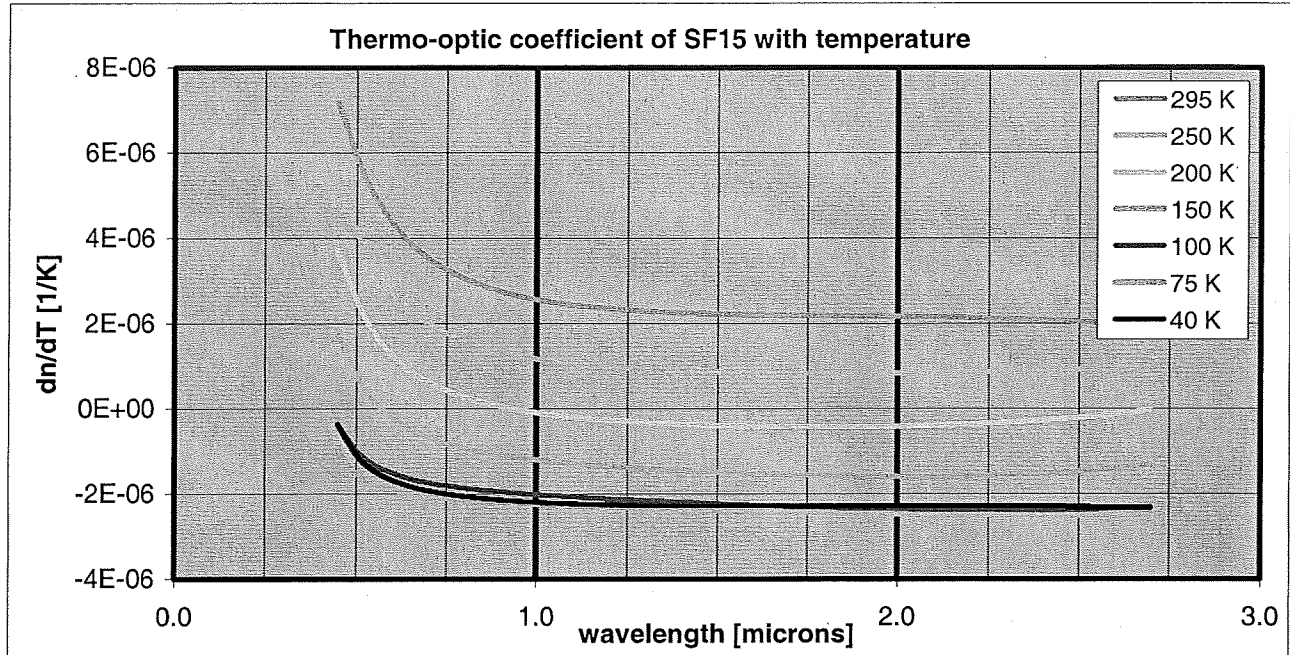


Figure 9: Measured thermo-optic coefficient ( $dn/dT$ ) of SF15 as a function of wavelength for selected temperatures

Table 14: Measured thermo-optic coefficient ( $dn/dT$ ) of SF15 for selected wavelengths and temperatures

| wavelength   | 40 K      | 50 K      | 60 K      | 70 K      | 80 K      | 90 K      | 100 K     | 150 K     | 200 K     | 250 K    | 295 K    |
|--------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|----------|
| 0.50 microns | -1.09E-06 | -1.39E-06 | -1.54E-06 | -1.56E-06 | -1.46E-06 | -1.27E-06 | -9.75E-07 | 7.68E-07  | 2.55E-06  | 4.14E-06 | 5.92E-06 |
| 0.60 microns | -1.67E-06 | -1.86E-06 | -1.94E-06 | -1.93E-06 | -1.85E-06 | -1.72E-06 | -1.50E-06 | -2.09E-07 | 1.26E-06  | 2.73E-06 | 4.36E-06 |
| 0.70 microns | -1.92E-06 | -2.04E-06 | -2.09E-06 | -2.08E-06 | -2.02E-06 | -1.91E-06 | -1.73E-06 | -6.68E-07 | 6.26E-07  | 2.01E-06 | 3.53E-06 |
| 0.80 microns | -2.06E-06 | -2.15E-06 | -2.18E-06 | -2.17E-06 | -2.11E-06 | -2.01E-06 | -1.86E-06 | -9.23E-07 | 2.72E-07  | 1.60E-06 | 3.05E-06 |
| 0.90 microns | -2.14E-06 | -2.21E-06 | -2.24E-06 | -2.22E-06 | -2.17E-06 | -2.08E-06 | -1.95E-06 | -1.08E-06 | 5.51E-08  | 1.34E-06 | 2.75E-06 |
| 1.00 microns | -2.20E-06 | -2.26E-06 | -2.28E-06 | -2.26E-06 | -2.21E-06 | -2.13E-06 | -2.01E-06 | -1.20E-06 | -8.90E-08 | 1.18E-06 | 2.56E-06 |
| 1.50 microns | -2.28E-06 | -2.34E-06 | -2.38E-06 | -2.39E-06 | -2.37E-06 | -2.32E-06 | -2.23E-06 | -1.49E-06 | -3.89E-07 | 8.62E-07 | 2.22E-06 |
| 2.00 microns | -2.27E-06 | -2.37E-06 | -2.43E-06 | -2.47E-06 | -2.47E-06 | -2.44E-06 | -2.36E-06 | -1.59E-06 | -4.10E-07 | 8.38E-07 | 2.17E-06 |
| 2.50 microns | -2.29E-06 | -2.39E-06 | -2.46E-06 | -2.50E-06 | -2.50E-06 | -2.46E-06 | -2.36E-06 | -1.46E-06 | -1.98E-07 | 9.31E-07 | 2.05E-06 |

Table 15: Coefficients for the temperature-dependent Sellmeier fit of the refractive index of SF15 measured by CHARMS. Average absolute residual of the fit from the measured data is  $1 \times 10^{-5}$  and the measurement uncertainty for SF15 is listed in Table 11.

| Coefficients for the temperature dependent Sellmeier equation for SF15       |                |                |                |              |              |              |
|--|----------------|----------------|----------------|--------------|--------------|--------------|
| 50 K $\leq$ T $\leq$ 300 K; 0.45 $\mu$ m $\leq$ $\lambda$ $\leq$ 2.6 $\mu$ m |                |                |                |              |              |              |
|  | S <sub>1</sub> | S <sub>2</sub> | S <sub>3</sub> | $\lambda_1$  | $\lambda_2$  | $\lambda_3$  |
| Constant term  | 1.54987        | 0.239639       | 1.00213        | -0.10935     | 0.234428     | 10.5886      |
| T term   | 6.36428E-04    | -6.45708E-04   | -1.35256E-03   | -5.12795E-05 | 1.22009E-04  | -6.57792E-03 |
| T <sup>2</sup> term  | -3.71495E-06   | 3.72173E-06    | 1.47878E-05    | 3.13053E-07  | -6.67055E-07 | 7.18922E-05  |
| T <sup>3</sup> term  | 3.70739E-09    | -3.66675E-09   | -3.17135E-08   | -3.50897E-10 | 7.61606E-10  | -1.53702E-07 |
| T <sup>4</sup> term  | -1.71512E-24   | -1.55595E-24   | 3.38150E-23    | -5.01524E-20 | -1.37513E-18 | -5.30573E-19 |

### 3. COMPARISON WITH LITERATURE VALUES

There is very little refractive index data with which to compare our CHARMS measurements for these materials at room temperature, and even less at cryogenics temperatures. In Figure 10 we have compared the fit generated based on CHARMS measured data with the dispersion relation published in the Schott optical glass catalog<sup>7</sup> for N-BK7 at room temperature (22° C). In addition, several data points were listed on the datasheet provided with the sample from Schott, and additional data points were obtained from a second datasheet provided by Schott which corresponds to a batch of N-BK7 produced in a different, but temporally adjacent, production run (batch # TNA08622). All values from Schott were measured in air and corrected by Schott to the standard conditions of 22° C and 760 mm Hg. We further corrected Schott's data to correspond to values of absolute refractive index by multiplying by a constant value of 1.00027 to compensate for the refractive index of air. Figure 10 shows a range of interspecimen variability on the order of approximately  $\pm 2 \times 10^{-4}$  with our measurements contained within that range.

The results for BaLKN3 are plotted in Figure 11. Here we show the deviation of our measurements from the datasheet provided with the sample prism, and also from another Schott datasheet provided by Q.Gong/Swales that corresponds to an unspecified batch of BaLKN3.

The results for SF15 are plotted in Figure 12. The deviation between the CHARMS measured values and those in the Schott glass catalog are on the order of our measurement uncertainty for this material. An additional datasheet provided by Q.Gong/Swales from an unspecified batch of SF15 confirms this agreement, but no datasheet was provided from the manufacturer for this batch of SF15.

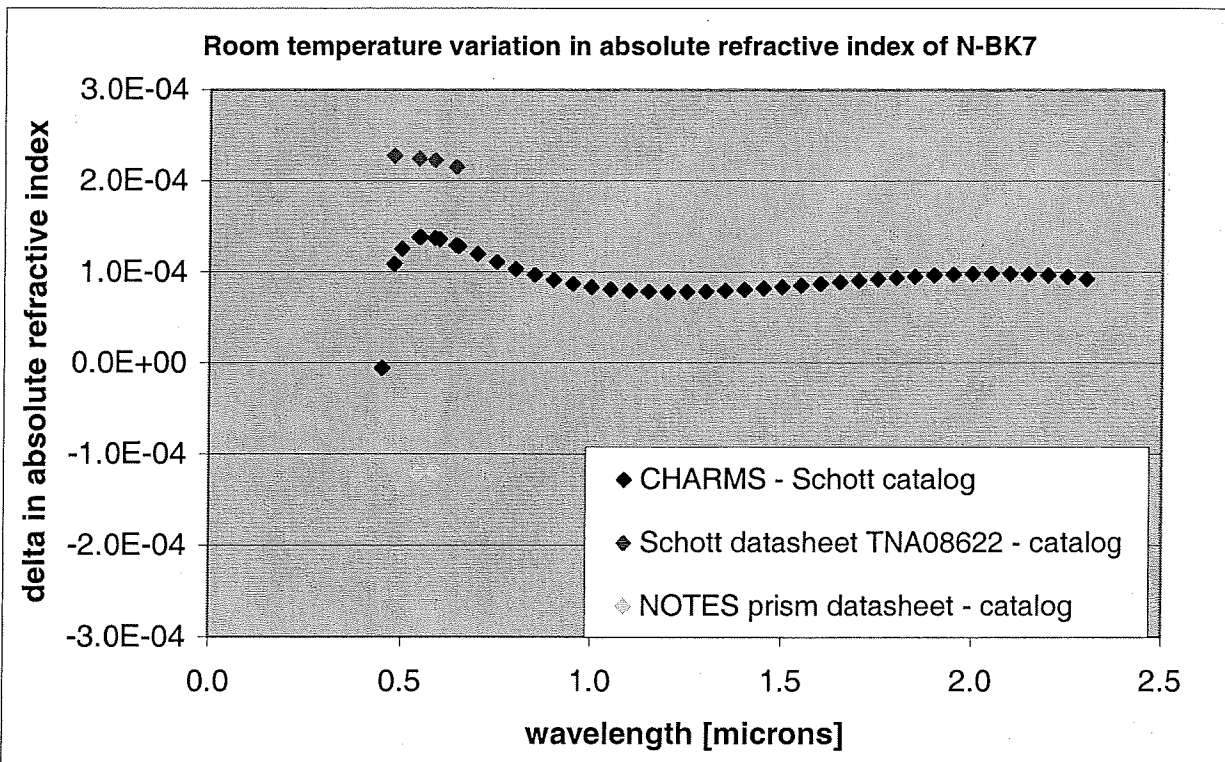


Figure 10: Comparison of measured and published values for the refractive index of N-BK7 at room temperature

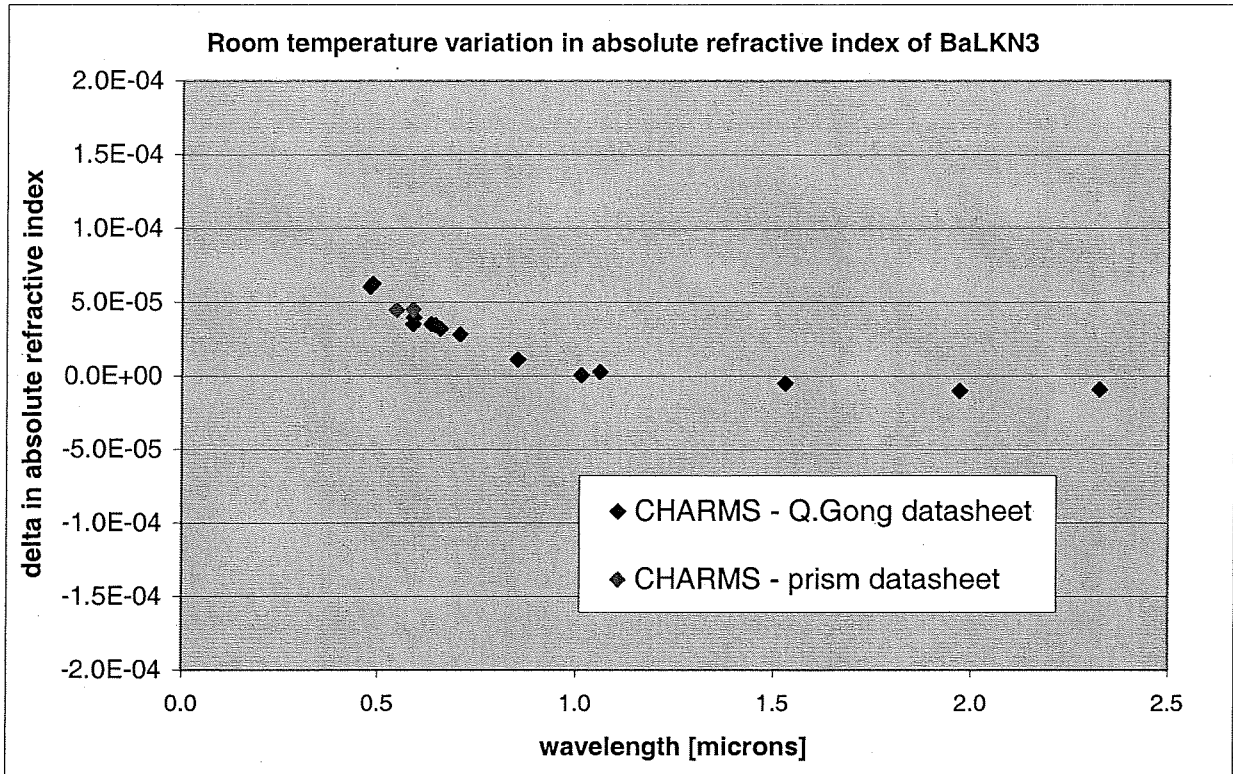


Figure 11: Comparison of measured and published values for the refractive index of BaLKN3 at room temperature

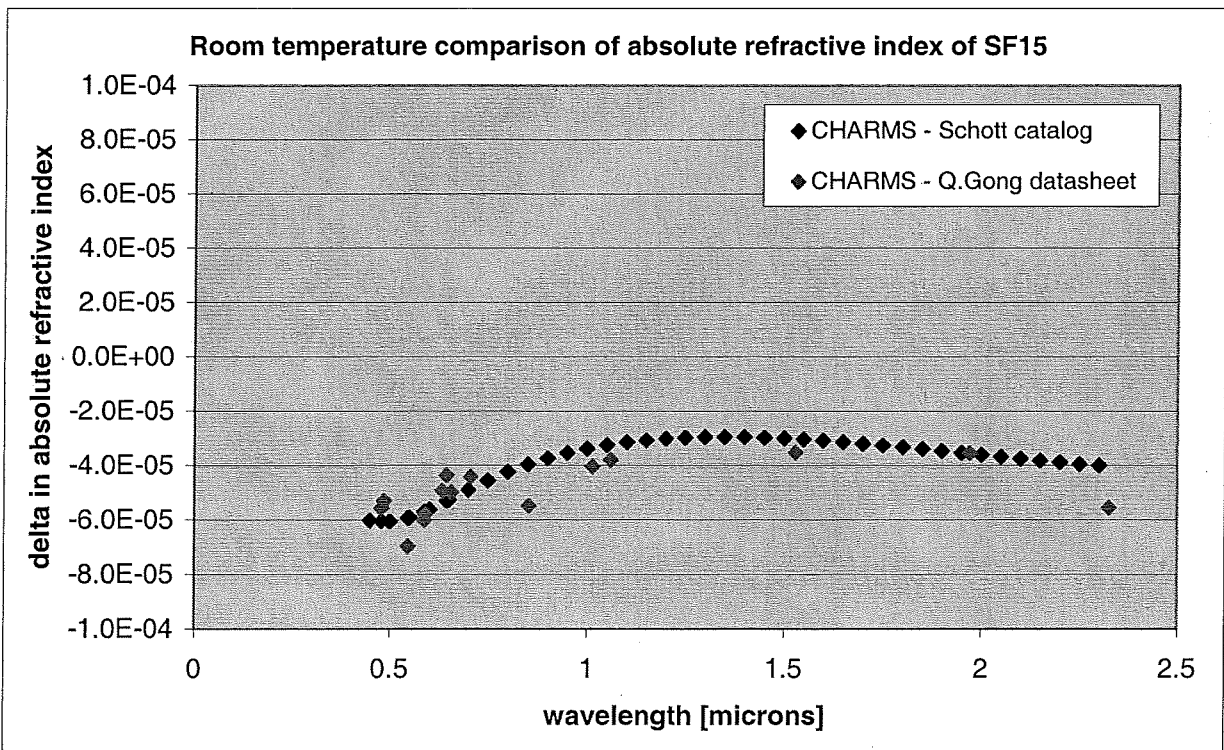


Figure 12: Comparison of measured and published values for the refractive index of SF15 at room temperature

#### 4. CONCLUSION

We have measured the absolute refractive indices of the materials N-BK7, BaLKN3, and SF15. We have found variability between multiple datasheets provided by Schott and the Schott optical glass catalog dispersion relation on the order of  $2 \times 10^{-4}$  for the material N-BK7. Even though the uncertainty in the CHARMS measurements described herein are (sometimes significantly) smaller than this variability, one must use caution in applying the measured CHARMS data to batches other than those measured when the required accuracy of knowledge of refractive index is less than  $2 \times 10^{-4}$ .

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