The Impact of Cold Dark Matter Variants on the Halos of the First Stars and Galaxies:
Angular Momentum and Vortex Creation in BEC Dark Matter

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1. INTRODUCTION

- The formation of the first stars and galaxies depended upon the internal structure of their host dark matter halos, especially the central regions, where baryonic cooling and condensation are likely to have occurred first.

- While the halos of standard, collisionless cold dark matter (CDM) are characterized by the singular NFW density profile, some forms of cold dark matter have been proposed which alter this universal behaviour on small scales by flattening the central profile. Among these are light bosonic particles, including axionic and other forms of CDM, that form a Bose-Einstein condensate (BEC) in which superfluidity can affect halo dynamics.
  -- In the strongly self-interacting regime, BEC halos can have profiles with constant density cores, in better agreement with observed rotation curves of dwarf and LSB galaxies (see [2]).

- We shall explore one aspect of the impact of this hypothesis on the structure of the halos underlying the first stars and galaxies, the effects of angular momentum.
  -- Laboratory BECs are known to develop quantum vortices when rotated with sufficient angular velocity (see figure below).

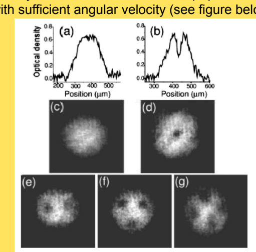


FIG. 7. Absorption images of a BEC stirred with a laser beam; in all images the condensate has about 10^5 atoms, the temperature is below 80 nK, and the rotation rate $\Omega/(2\pi)$ increases from 145 Hz for (a) and (c) to 168 Hz for (g). Images (a) and (b) show the optical density for images (c) and (d), with the clear appearance of the vortex core. Images (e)–(g) show states with two, three, and four vortices. From Madison *et al.*, 2000a.

  -- In cosmology, simulations of structure formation in the CDM model show that halos acquire angular momentum as they form, consistent with that expected from gravitational tidal torquing by the surrounding large-scale structure.
  -- Vortices could, in principle, then result if the CDM is a BEC.

- We shall address this point here by calculating the critical angular velocity for vortex creation in some simple models of BEC/CDM galactic halos and comparing it with the angular velocity of CDM halos (see [6]).

2. ROTATING, SELF-GRAVITATING BEC HALOS

- We describe self-gravitating BEC halos with varying degrees of rotational support by self-consistently coupling the Gross-Pitaevskii (GP) equation of motion for the complex macroscopic wave function $\psi(\mathbf{r},t)$, to the Poisson equation, where $|\psi|^2(\mathbf{r},t) = n$, the number density of particles of mass m,

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + (m\Phi + g|\psi|^2 - \mu)\psi, \quad \Delta\Phi = 4\pi G m|\psi|^2$$

- We assume that halos are comprised of N particles in the condensed state described by ψ, and so $\int|\psi|^2 = N$, which fixes the chemical potential μ

- BEC dark matter is *self-interacting* as described by an effective interaction potential $g|\psi|^4/2$ with coupling constant $g = 4\pi\hbar^2 a_S/m$ where a_s is the 2-body scattering length.

- In a frame rotating with velocity Ω, our system is stationary; the above equation of motion has an additional operator $V_{rot} = -\mathbf{\Omega}(\mathbf{r})\cdot\mathbf{L}$ on the rhs., where $\mathbf{L} = i\hbar\nabla\times\mathbf{r}$, and the respective variables and quantities are understood to be in the new frame.

- Such stationary systems can then be studied via the corresponding GP energy functional

$$\mathcal{E}[\psi] = \int_{\mathbb{R}^3}\left[\frac{\hbar^2}{2m}|\nabla\psi|^2 + \frac{m}{2}\Phi|\psi|^2 + \frac{g}{2}|\psi|^4 + i\hbar\psi^*\mathbf{\Omega}\cdot(\mathbf{r}\times\nabla\psi)\right]$$

3. VORTEX CREATION IN BEC HALOS AS MACLAURIN SPHEROIDS

- We use an energy argument to derive the critical angular velocity for vortex creation in a rotating, self-gravitating BEC halo by finding the angular velocity which lowers the energy in the presence of a vortex.

- We consider a d-quantized, straight vortex line along the rotation axis, with core radius s. For d=1, s can be approximated by the healing length $\xi^2 = \hbar^2/(2\rho g)$ with ρ = mn.

- The equilibrium matter distribution of the BEC halo is given, in general, by the static GP equation:

$$-\frac{\hbar^2}{2m}\frac{1}{|\psi|}\Delta|\psi| + m\Phi - \frac{m}{2}(\mathbf{\Omega}\times\mathbf{r})^2 + g|\psi|^2 - \mu = 0$$

quantum pressure — self-interaction

There are two limiting cases of interest:
i) weak coupling (e.g. axions) → quantum pressure dominates
→ cuspy 1/r central profile prohibited
ii) strong coupling → quantum pressure negligible → n = 1 polytrope (see e.g. [3]) → nonrotating case has flat density core

- For simplicity, we thus consider a uniformly rotating spheroid of constant density – the Maclaurin spheroid – an approximate solution of the static GP equation, a family of solutions is characterized by the angular rotation $(\Omega/\Omega_G)^2$, where $\Omega_G^2 = \pi\rho G$, or, equivalently, the eccentricity e.

4. RESULTS

- For a BEC/CDM halo, vortices form if $\Omega > \Omega_c$, where the **critical angular velocity** Ω_c in units of Ω_G is

$$\frac{\Omega_c}{\Omega_G} = \frac{\Omega_{QM}}{\Omega_G}\left[\ln\frac{a}{\xi} + \frac{25}{12} + \frac{\pi}{2}\left(\frac{\Omega_G}{\Omega_{QM}}\right)^2\left(\frac{A_1(e)}{6}\left(\frac{a}{\xi}\right)^{-4} + A_3(e)\frac{1-e^2}{9}\left(\frac{a}{\xi}\right)^{-2}\right)\right]$$

where the semi-major axis is a, A_1 and A_3 are functions of the eccentricity, and we define $\Omega_{QM} \equiv \hbar/(ma^2)$

(i.e. Ω for spheroid with $|\mathbf{L}| = N\hbar$)

- In the weak coupling limit (e.g. axions) as g → 0, Ω_c → infinity and, hence, **no vortices form,** contrary to recent statements in the literature [5].

- In general, we must determine whether a given set of BEC parameters (m,g) makes the spheroid rotation velocity for a given e equal the critical value above which a vortex forms, $\Omega = \Omega_c$. We observe that

$$\frac{\Omega_{QM}}{\Omega_G} \equiv \frac{m_H}{m} \qquad a/\xi = (g/g_{min})^{1/2}$$

where $m_H \equiv \frac{\hbar}{a^2(\pi G m\rho)^{1/2}}$ and $g_{min} \equiv \frac{\hbar^2}{2\rho a^2}$

the latter of which depend only on halo parameters.

- The solution for a given e is then a curve in the $(m/m_H, g/g_{min})$-plane, independent of other halo parameters.

REFERENCES

[1] Barnes J., Efstathiou G., 1987, ApJ 319, 575
[2] Böhmer C.G., Harko T., 2007, JCAP 06, 025
[3] Goodman J., 2000, New Astronomy 5, no.2, 103
[4] Madison K.W., Chevy F., Wohlleben W., Dalibard J., 2000, PRL 84, 806
[5] Sikivie P., Yang Q., 2009, Phys.Rev.Lett. 103, 111301
[6] Rindler-Daller T., Shapiro P.R., 2009, arXiv:0912.2897

Q: What values of e correspond to CDM halos ?
A:
- Cosmological N-body simulations of the CDM universe show that halos form with net angular momentum such that $\lambda = L|W|^{1/2}/(GM^{5/2})$, which expresses their degree of rotational support, has values in the range 0.01 to 0.1, with median value 0.05 (e.g. [1]).
  For our Maclaurin spheroid halos, $\lambda^2 \sim \mathbf{t} = T/|W|$, where T = rotational K.E., W = gravitational P.E.

  λ = (0.01, 0.05, 0.1) corresponds to e = (0.051, 0.249, 0.464) or **t** = (0.00035, 0.0085, 0.0321) , respectively.

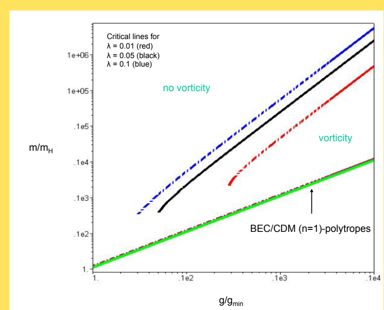
  For e = 0.249, say, $\Omega/\Omega_G \sim 0.18$, which fixes Ω for a given CDM halo mass M and volume V,
  e.g. for Milky-Way-sized halo, M = 10^{12} M_{solar}, R = 100 kpc,
  Ω = 1.06 x 10^{-17} Hz

  - We calculate the critical lines $\Omega_c = \Omega$ such that the CDM halo rotation velocity equals the critical value above which a vortex forms, for the three specified values of e for which λ = (0.01, 0.05, 0.1) = (red, black, blue), see figure below.

  - For each critical line, no vortex is allowed for parameters in the space above the line (higher λ → higher curves).

  - For a Milky-Way-sized halo, say, $m_H \sim 10^{-58}$ for m in grams.

Dimensionless BEC particle mass m/m_H vs. coupling strength g/g_{min}: critical lines $\Omega = \Omega_c$



Q: Where do BEC/CDM halos lie on this plane ?
A:
- For BEC halos, the condition of gravitational equilibrium restricts the values of m and g to another curve in the $(m/m_H, g/g_{min})$-plane: in the strongly-coupled regime, non-rotating BEC halos are just (n=1)-polytropes, for which the size R is related to the BEC parameters according to

  $R = \pi[\hbar^2 a_s/(Gm^3)]^{1/2}$ (see [2]). The resulting

  relationship between m/m_H and g/g_{min} is plotted in the above figure as well. For CDM halo λ-values, the degree of rotational support is small enough that this relationship should still be a good approximation. There is almost no sensitivity to e (respective curves lie on top of each other).

- Since the BEC parameters which satisfy the above relationship (green curve) are all below our critical curves for which $\Omega = \Omega_c$, BEC/CDM halos, in general, *will* typically form vortices.

5. CONCLUSIONS

- We find that quantum vortices are favoured in halos which form from strongly-coupled BEC/CDM particles.
  Vortex creation causes the central density to drop (for illustration see the image above).

- If BEC halos form with substantially lower central densities than do standard CDM halos, the ability of baryons there to cool radiatively and condense to become self-gravitating, a precondition for star formation, will be suppressed.

Acknowledgments

This work was supported by the DFG under research unit FG 960; NSF grant AST 0708176, NASA grant NNX07AH09G and Chandra grant SAO TM8-9009X.