Life of a Star

An Interview with a White Dwarf, Sol

Reflection Problems

PAGE: Hang on, let's talk about that balance.

SOL: Before I could collapse, I had to satisfy a few conditions:
Conservation of energy – the kinetic and potential energies balance

$$2E_K + E_P = 0$$

When I started to collapse, the mass and gravity made an energy potential greater than the kinetic energy of the atoms.

$$2E_K < E_P$$

PAGE: $\mathbf{E_K}$ is the average kinetic energy of the atoms in the nebula gas, **3/2 NkT**. And the mass created a potential energy $\mathbf{E_P}$. So the size or radius **r** and the gas mass **M** were big enough to start the collapse.

$$3NkT < \frac{3}{5}\frac{GM^2}{r}$$

N: number of atoms
k: Boltzmann's constant 1.38×10^{-23} Joules / Kelvin
T: temperature in Kelvin
G: gravity constant 6.67×10^{-11} (Newton)(meter^2) / kilogram^2
M: total mass in kilograms
r: radius in meters

Reflection Point 1: Interstellar Cloud

Milestone	Duration years galactic years	Diameter meters	Density kg / m^3	Core Temperature (Kelvin)	Surface Temperature (Kelvin)
1	2.13×10^6 9.47×10^{-3}	10^{17}	1.67×10^{-18}	10	10

Oh, you don't like just numbers. Think about this. If you squeeze a balloon or foam ball, the resistance you feel is like the thermal pressure from my gas pushing outward against the inward pull of gravity. Can you answer these questions?

1. What are the forces involved when I collapse?
2. What can cause me to collapse and become unstable (out of balance, $\mathbf{2E_K < E_P}$) ? You know, there are other stars out there.
3. Although I really don't like to have my mass calculated, I'll let you guess it. I challenge you to calculate my mass. Use "solar mass" units: one solar mass = 2×10^{30} kilograms. Assume each atom is a hydrogen atom (1.674×10^{-27} kilograms), and that I have a spherical shape.

Reflection Point 2: Protostar

Milestone	Duration years galactic years	Diameter meters	Core Density kg / m^3	Core Temperature (Kelvin)	Surface Temperature (Kelvin)
2	10^6 4.44×10^{-3}	10^{11}	0.001674	10^6	3,000
3	10^7 4.4×10^{-2}	10^{10}	16.74	5×10^6	4,000

A) What is Peter's luminosity as a protostar at milestone #2 and #3? Here's an equation to use: Draw a picture to help you visualize what you are calculating.

$L = (\sigma T^4) \cdot (4\pi r^2)$ which is power (energy per second) per unit area times my surface area.

L: luminosity in Watts

σ (sigma) is the Stefan-Boltzmann constant = 5.67×10^{-8} (Watts / meter^2) $\cdot$ Kelvin^4

T: surface temperature in Kelvin

π: (pi) is the ratio of a circle's circumference to its diameter = 3.14159…

r: radius in meters

B) How has Peter's luminosity and temperature changed from milestone 2 to milestone 3?

C) Calculate Peter's surface area $4\pi r^2$ for milestone 2 and milestone 3. How does it change?

D) What variable (surface temperature or radius) affected Peter's luminosity change the most? Why?

Reflection Point 3: Main Sequence Star

Milestone	Duration years galactic years	Diameter meters	Core Density kg / m^3	Core Temperature (Kelvin)	Surface Temperature (Kelvin)
4	10^{10} 44.4	1.4×10^9	10^5 kg/m^3	1.5×10^7	5,770

What are the main properties of a star?

While Sol was our Sun:

1. How many reaction cycles per second must occur for the Sun to radiate 4×10^{26} Watts?

   Hints: Each fusion reaction cycle yields 4.3×10^{-12} Joules. A Watt is a unit of power (Joules per second).

2. If each reaction cycle yields 4.3×10^{-12} Joules, how much mass per second is the Sun converting into energy?

   Hint: Use Einstein's equation $E = Mc^2$

   E = energy in Joules, M = mass in kilograms, c = speed of light (3×10^8 meters/second)

3. Explain why was Sol in equilibrium.

Reflection Point 4: Red Giant

Milestone	Duration years galactic years	Diameter present Sun diameters	Core Density kg / m^3	Core Temperature (Kelvin)	Surface Temperature (Kelvin)
5	10^8 0.44	3	10^7	5×10^7	4,000

The Sun's present diameter as a main sequence star is 1.392×10^8 meters.

A) What is the mean density of Sol at this milestone? The Sun's mass is 2×10^{30} kilograms. The density of water is 1000 kg/m^3.

Hint: the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

B) If Sol's diameter continues to increase, what will happen to his mean density? Draw a picture to explain your answer.

Reflection Point 5: Red Giant - before helium flash

Milestone	Duration years galactic years	Diameter present Sun diameters	Core Density kg / m^3	Core Temperature (Kelvin)	Surface Temperature (Kelvin)
6	10^5 4.44×10^{-4}	100	10^8	10^8	4,000

The Sun's diameter as a main sequence star is 1.392×10^8 meters.

A) Goliath said that his helium flash released 30,000 years worth of energy (milestone 4) in just a few seconds.
If the helium flash lasted 10 seconds, what is the average power that Goliath's helium flash radiated?
Hints: At milestone 4, Sol's luminosity was 4×10^{26} Watts (Joules per second). How many seconds are in a year?

Reflection Point 6: Red Giant – helium fusion after helium flash

Milestone	Duration years galactic years	Diameter present Sun diameters	Core Density kg / m^3	Core Temperature (Kelvin)	Surface Temperature (Kelvin)
7	5×10^7 0.22	10	10^7	2×10^8	5,000

The Sun's diameter as a main sequence star is 1.392×10^8 meters.
After the helium flash, Goliath's core shrank.
A) If the core's radius only changed by one meter, how much gravitational potential energy in *one kilogram of matter* (the mass of your textbook) became thermal energy to heat up the core?
As an approximation, assume that the acceleration of gravity is constant over the one meter change in radius.

($g = GM_{core} / r^2$). Use $M_{core} = 8 \times 10^{29}$ kg, and a core radius = 6.96×10^7 meters.

Reflection Point 7: Red giant becomes super giant

Milestone	Duration years galactic years	Diameter present Sun diameters	Core Density kg / m^3	Core Temperature (Kelvin)	Surface Temperature (Kelvin)
8	10^4 4.44×10^{-5}	500	10^8	2.5×10^8	4,000

The Sun's diameter as a main sequence star is 1.392×10^8 meters.

By this time, Goliath has ballooned into a super giant.
A) What is Goliath's average density? Assume his mass is still 2×10^{30} kilograms.

B) Calculate Goliath's luminosity. Express your answer in terms of the Sun's luminosity (4×10^{26} Watts)

Reflection Point 9: White Dwarf

Milestone	Duration years galactic years	Diameter present Sun diameters	Core Density kg / m^3	Central Temperature (Kelvin)	Surface Temperature (Kelvin)
10	$10^?$?	10^{-2}	10^{10}	starts at 3×10^8 and cools down	starts at 10^5 and cools down

The Sun's diameter as a main sequence star is 1.392×10^8 meters.

A) For a surface temperature of 10^5 Kelvin, what is the initial luminosity for a white dwarf?

B) How do you think the luminosity of the white dwarf will change over time?
Draw a graph to plot how the luminosity changes over time to support your explanation. Plot the luminosity for the temperatures 20,000 K, 15,000 K, 10,000 K, 5,000 K, and 2,500 K.
Hint: What variables will remain constant over time in the luminosity equation?

Explain

In about 500 words, write Page's column *A Star's Life* based on Sol's scrapbook, the Ranger Rick "Birth and Death of a Star" diagram, and your calculations of the properties of Sol throughout his life. As you compose your story, make connections to everyday life so that your readers understand answers to the following questions:

1. What are the primary characteristics of a star?
2. During the interview, Sol and his friends mentioned many variables: luminosity, temperature (core and/or surface), density, and diameter. Which of these could we (people on Earth) observe and measure with a telescope?
3. What was Sol's life long balance to maintain? How did that affect Sol's life over time?
4. During what phase of his life was Sol happiest? Why?
5. At the end, Sol mentioned entering a second life. What do you suppose his second life will be, and how long?
6. What are Sol's properties as a white dwarf?

Elaborate

What is Sol’s ultimate fate?